Optimal growth in the turbulent pipe

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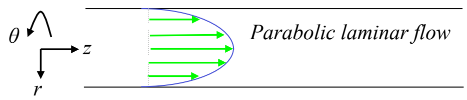
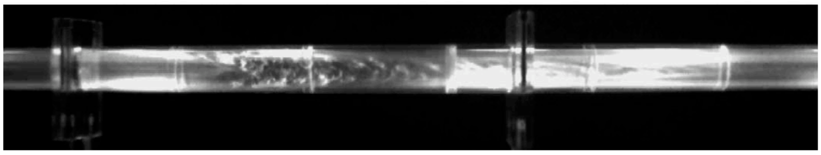


D, diameter
U, mean axial flow (constant)

$$Re = UD / \nu$$

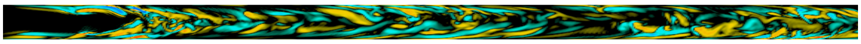
Simulation, axial vorticity



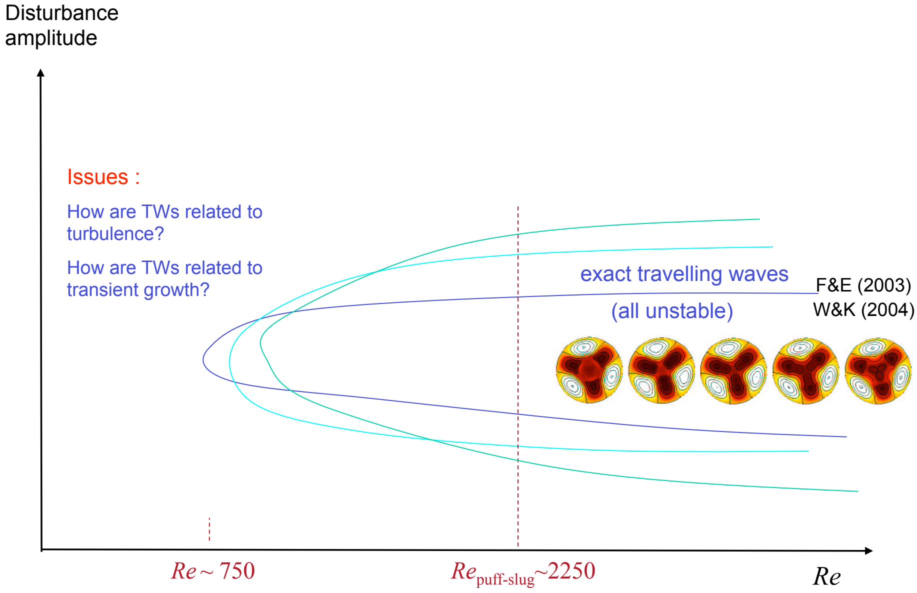
Experiment, illumination of flakes

Localised “puff”

Expanding “slug”



Recent Question: Are there coherent structures in pipe turbulence?

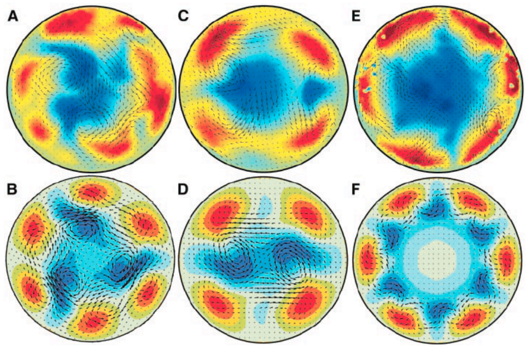


Lab. experiment

Hof *et al*. 2004, Science

Exact solutions

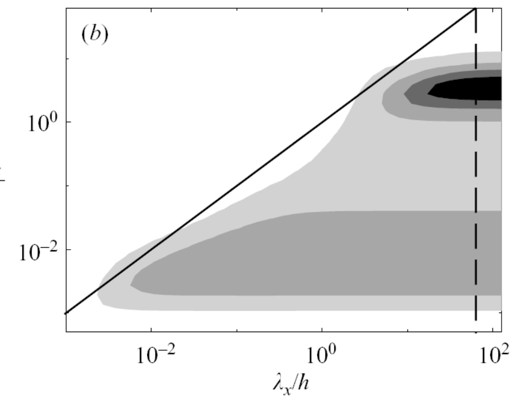
Faisst & Eckhardt 2003, PRL



Can these structures really be associated with TWs?

In a turbulent channel

del Alamo & Jimenez (2006)



Maximum transient growth G
, from light to dark, 1.5(+1)4.5.

Flow profile in Pipe

Invert θ,z-averaged N-S for U

$$0 = -\partial_z P + \frac{1}{Re}\left(\frac{1}{r} + \partial_r\right)(\nu_\tau \partial_r U) \qquad B = -\partial_z P$$

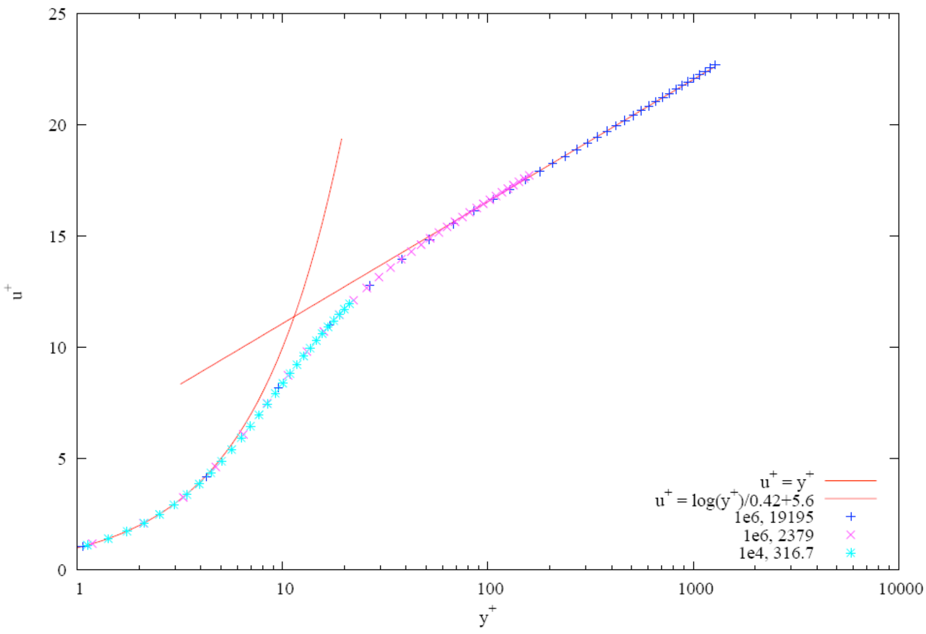
$$\nu_\tau = 1 + E(y)$$

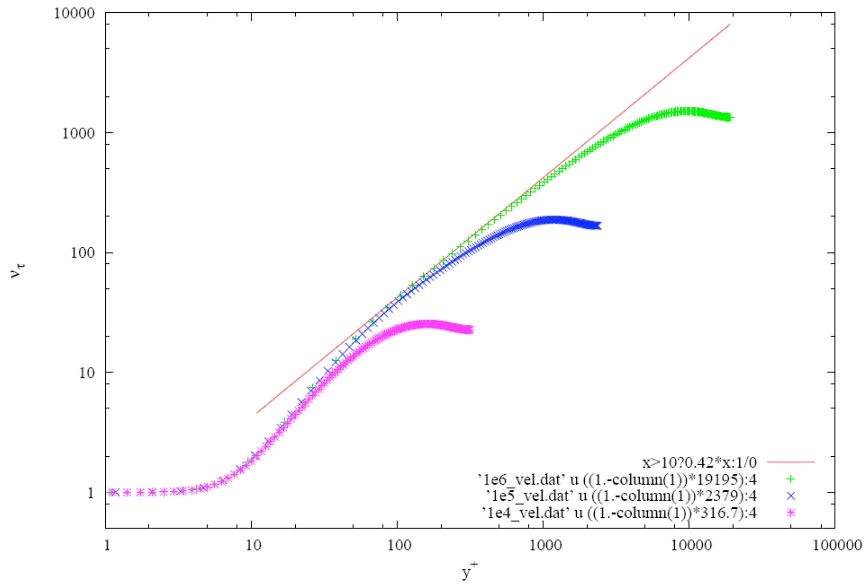
Reynolds & Tiederman (1967)

$$E(y) = \frac{1}{2}\left\{1 + \frac{\kappa^2 R^2 B}{9}[2y - y^2]^2(3 - 4y + 2y^2)^2\left[1 - \exp\left(\frac{-yR\sqrt{B}}{A^+}\right)\right]\right\} - \frac{1}{2}$$

McKeon et al (2005) κ=0.42

B'=0.56 → A^+=27





Calculate growth using method of Schmid & Henningson (2001)

To get eigenvectors of N-S linearised about base state U ...

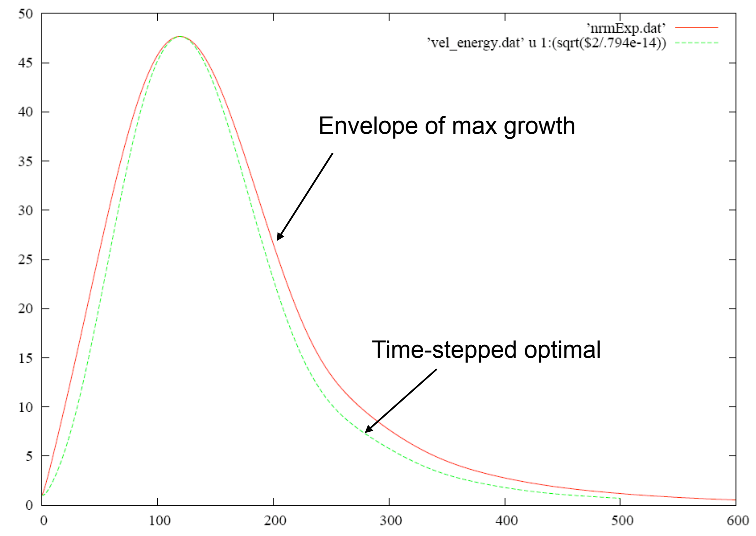
$$u(x,t) \sim \exp(\sigma t)u(x)$$

$$\begin{bmatrix} \text{N-S} & \\ \nabla\cdot u & 0 \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = -\sigma \begin{bmatrix} I & & & \\ & I & & \\ & & I & \\ & & & 0 \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix}$$

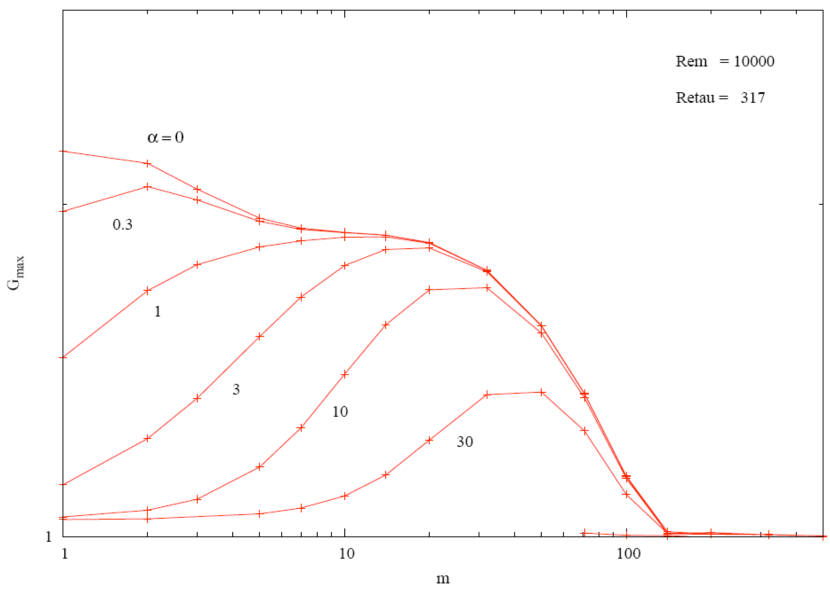
$A\,x = -\sigma\,B\,x$ B simple but singular

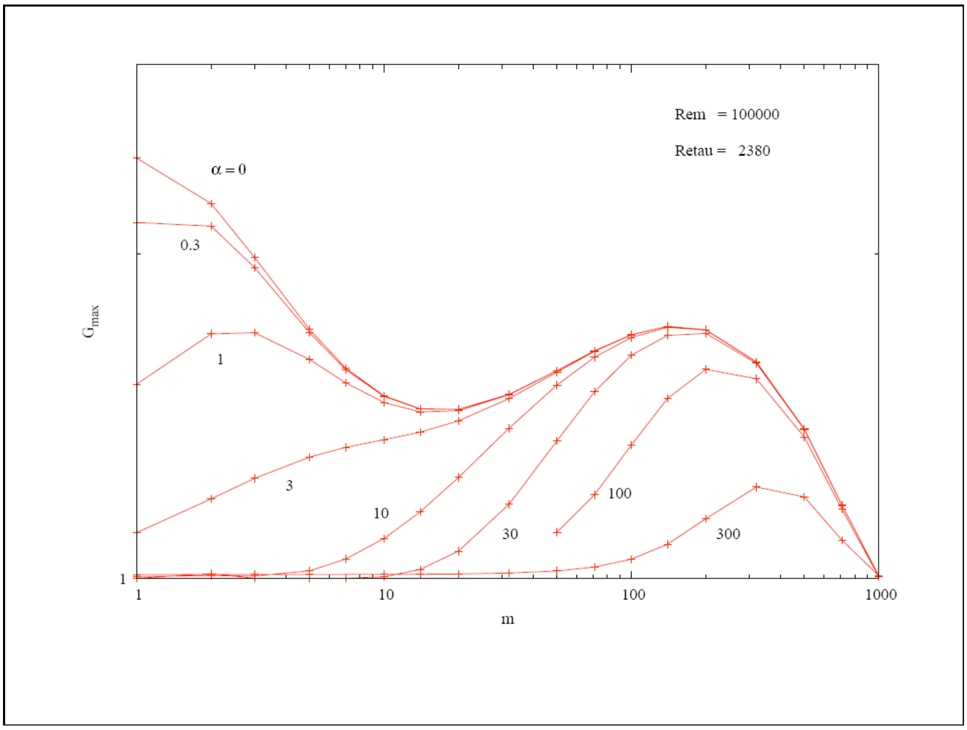
$\lambda\,x = (A^{-1}\,B)\,x$ $\lambda = -(1/\sigma)$

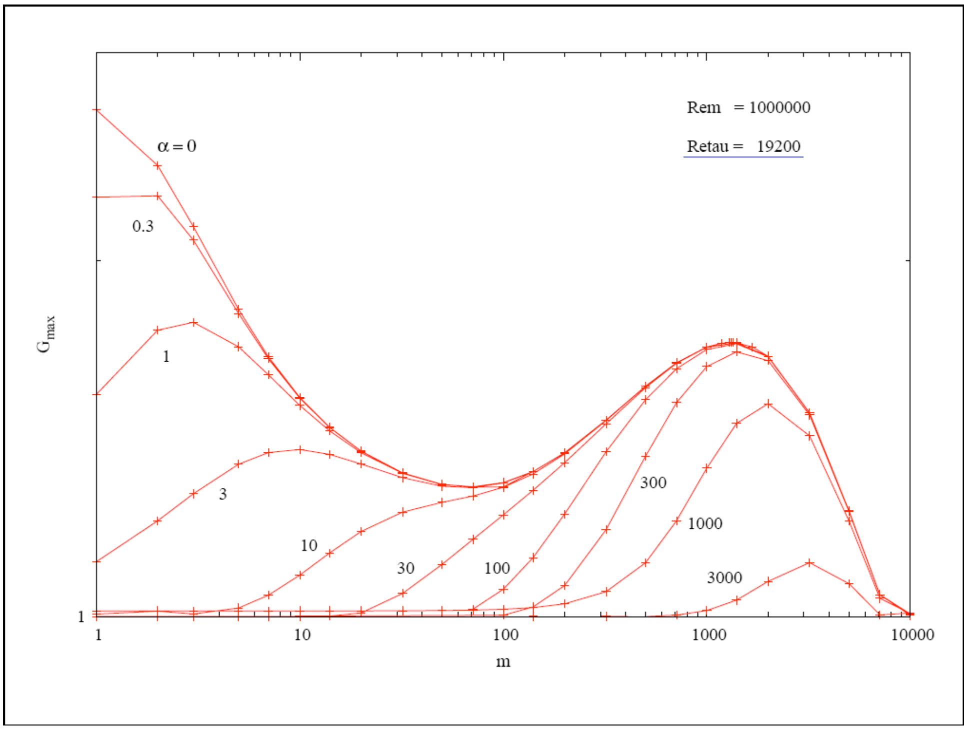
Check method, laminar profile

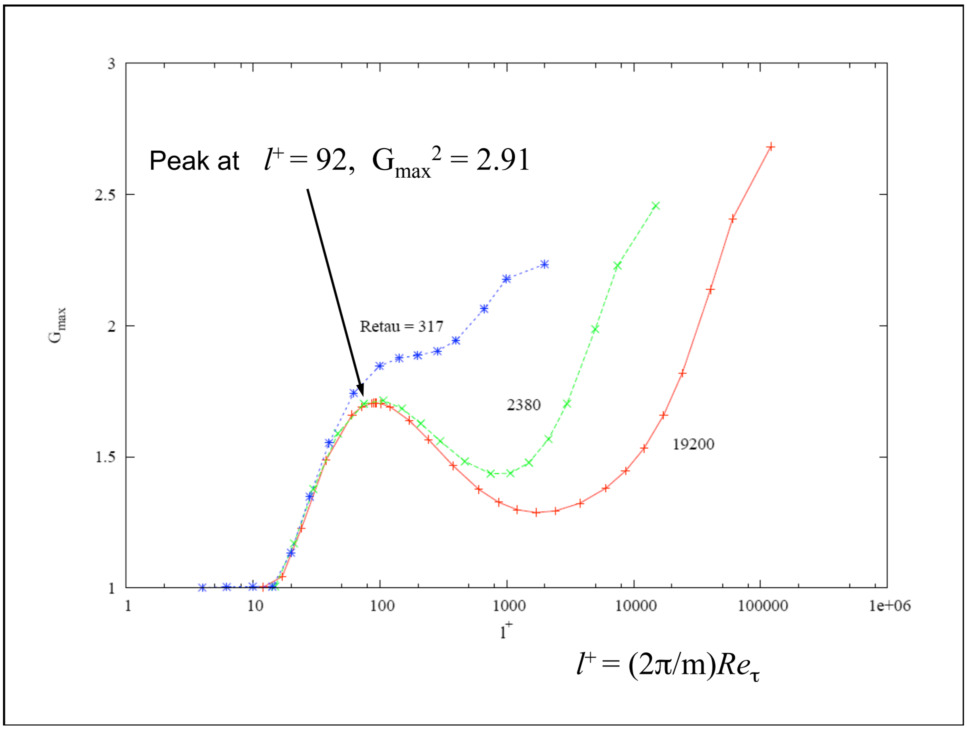


Growth for turbulent profile, low Re_τ

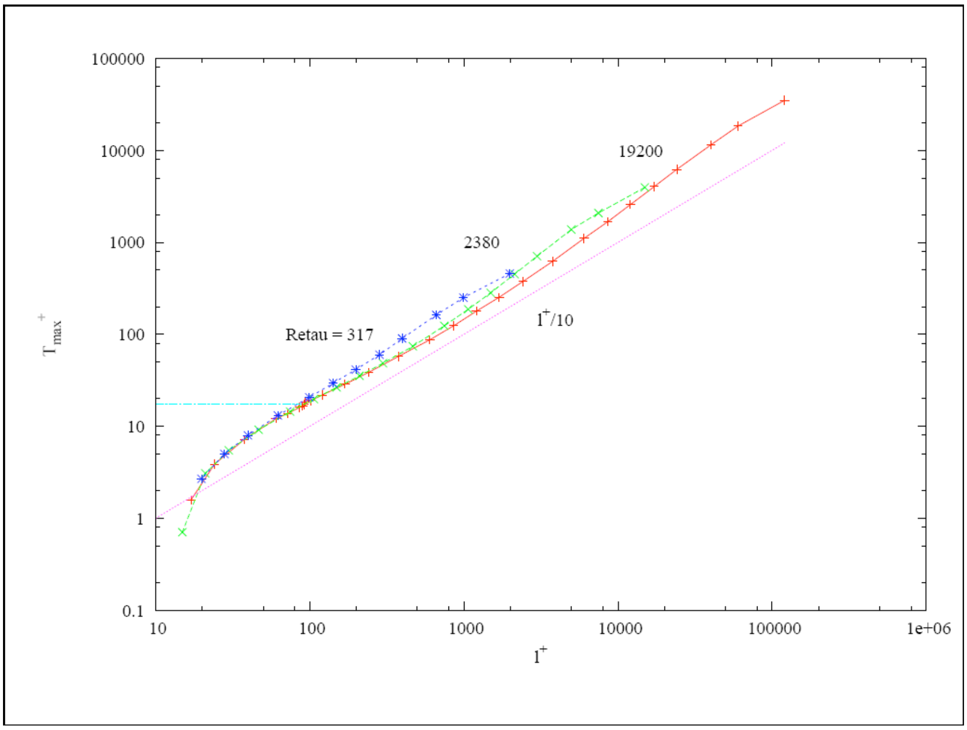



Rem = 100000
Retau = 2380
α = 0
0.3
1
3
10
30
100
300
G_{max}
m
1
10
100
1000


Rem = 1000000
Retau = 19200
α = 0
0.3
1
3
10
30
100
300
1000
3000
G_{max}
m
1
10
100
1000
10000

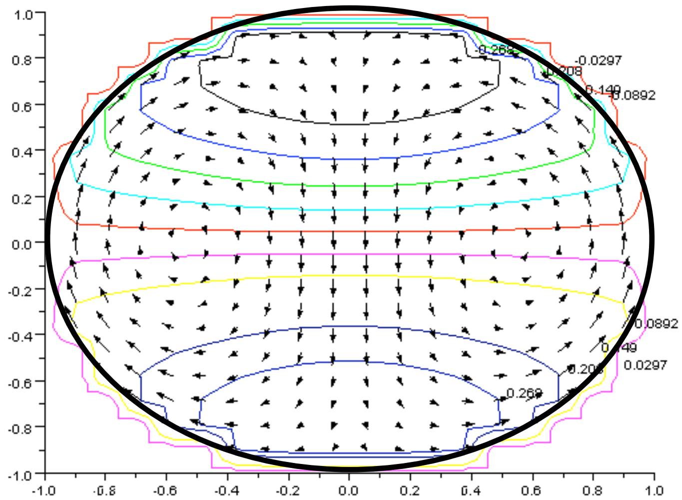


Peak at $l^+ = 92$, $G_{max}^2 = 2.91$
Retau = 317
2380
19200
G_{max}
l^+
$l^+ = (2\pi/m)Re_\tau$

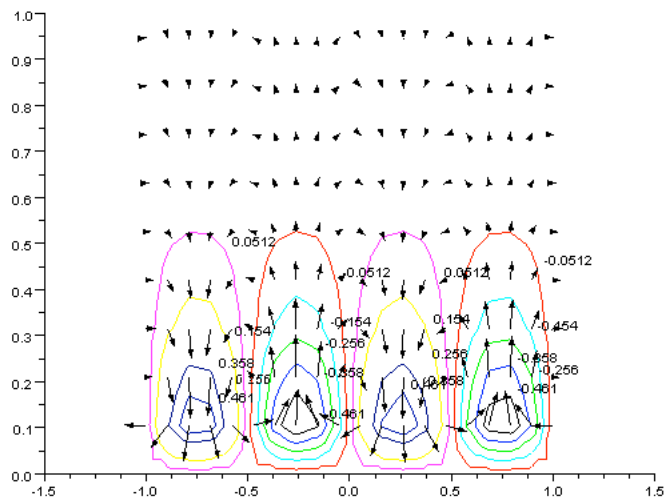


19200
2380
Retau = 317
$l^+/10$
T^+_{max}
l^+

Outer-scale mode, m=1



Inner-scale (wall) mode, $l^{+}=92$



Conclusions

1. Found the optimal modes for turbulent pipe flow.
2. Separate peaks for inner- and outer-scale modes at larger Re.
3. Inner-scale peak has same wavelength (l^{+}=92) as for the channel flow.
4. Compare with experiment?...

150 *J. P. Monty, J. A. Stewart, R. C. Williams and M. S. Chong*

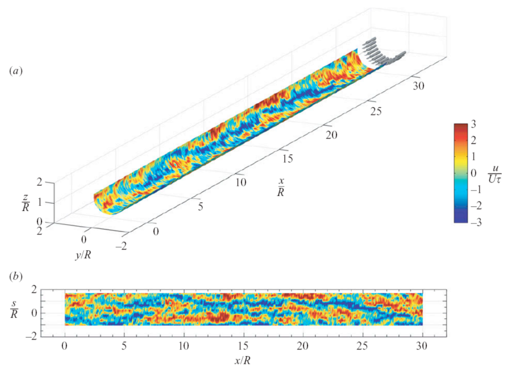
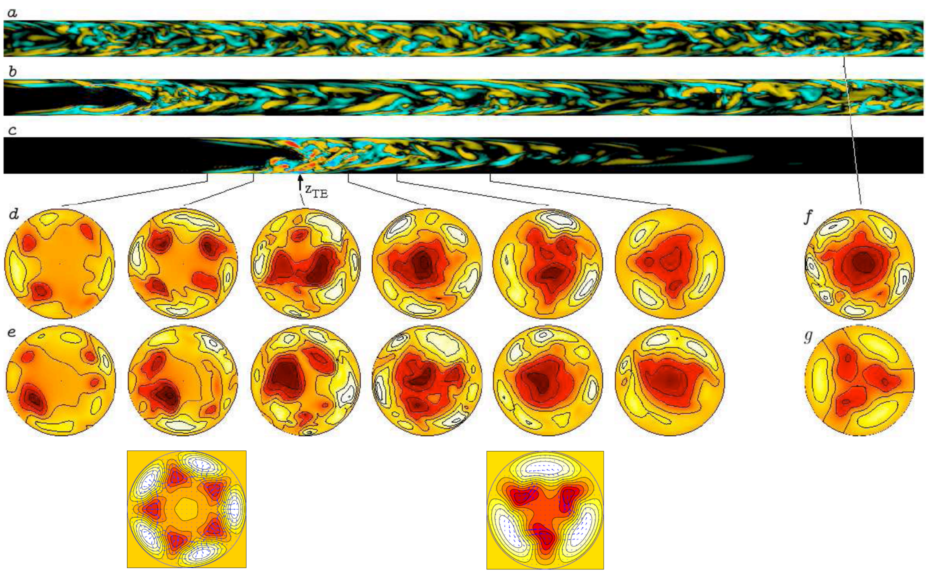
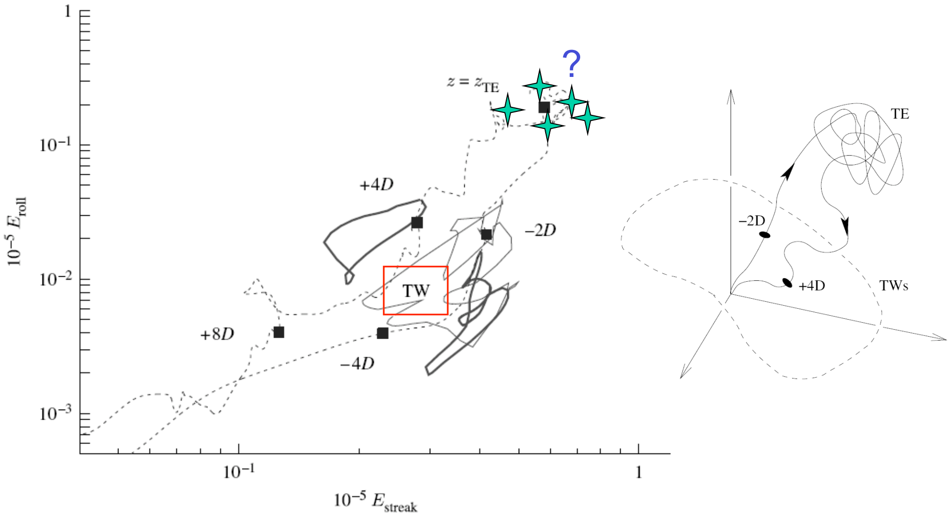


FIGURE 2. Contour plots of streamwise velocity fluctuations measured in the pipe at $Re_\tau = 3472$. The streamwise velocity has been scaled with the friction velocity, U_τ. (*a*) The velocity field in the true coordinate system; (*b*) a transformed view of the field in Cartesian coordinates.

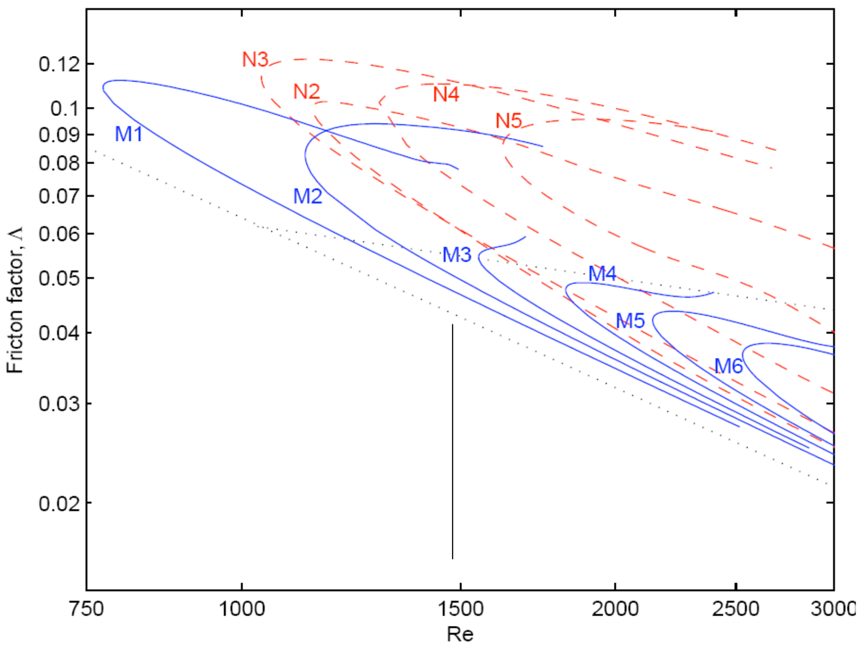


Energy in puff, at different locations relative to trailing edge



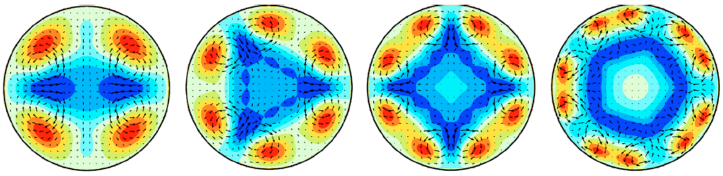
$$E_{roll} = \pi \sum_{m \neq 0} \int (|u_{m,r}|^2 + |u_{m,\theta}|^2)\, r\, dr$$

$$E_{streak} = \pi \sum_{m \neq 0} \int |u'_{m,z}|^2 r\, dr$$



Pringle, Duguet & Kerswell (2008)

Faisst & Eckhardt 2003, PRL



$m=2$ $m=3$ $m=4$ $m=5$

Exact travelling wave solutions:

- fast streaks near walls
- slow streaks in interior
- unstable