Optimal Disturbances of 3D Boundary Layers

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Outline

- **Motivation**
- **Theory**
  - Model
  - Governing Equations
- **Results**
- **Conclusions**

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Motivation

- Part of EU-project TELFONA
  - Objective is to demonstrate the ability to predict aircraft performance in flight based on wind tunnel tests and CFD results
- Need to understand effect of free-stream turbulence
  - High levels of free-stream turbulence in wind tunnels
  - Low levels of free-stream turbulence in free flight
- Receptivity model needed
  - Which boundary-layer disturbances will result from the penetration of external perturbations into the boundary-layer

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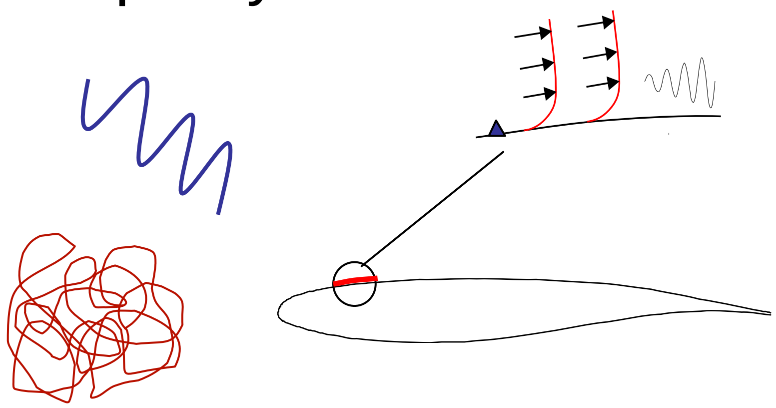
Motivation

- As first step to a receptivity model one can ask…
  - **Which disturbances are most dangerous?**
- Optimal disturbances
  - Those initial disturbances which are associated with the maximum energy growth
- Optimal disturbances could then be used to determine receptivity coefficients



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Model

- To model the boundary layer of a real wing we use the Falkner-Skan-Cooke similarity solutions
  - Velocity at edge: $U^e = Cx^m$

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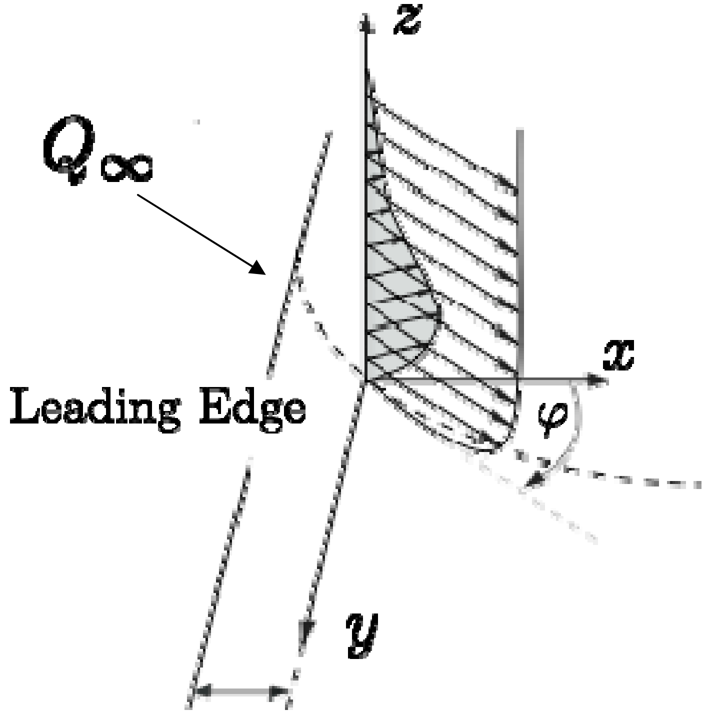

Governing Equations

- We want to monitor growth of disturbances
  - Follow the disturbances as they evolve in space
- Starting with the linear, incompressible disturbance equations derived from Navier-Stokes equations

$$\cancel{\frac{\partial \mathbf{u}'}{\partial t}} + (\mathbf{u}' \cdot \nabla)\bar{\mathbf{u}} + (\bar{\mathbf{u}} \cdot \nabla)\mathbf{u}' = -\frac{1}{\rho}\nabla p' + \nu \nabla^2 \mathbf{u}'$$

$$\nabla \mathbf{u}' = 0$$

- Aim is to derive a parabolic set of equations

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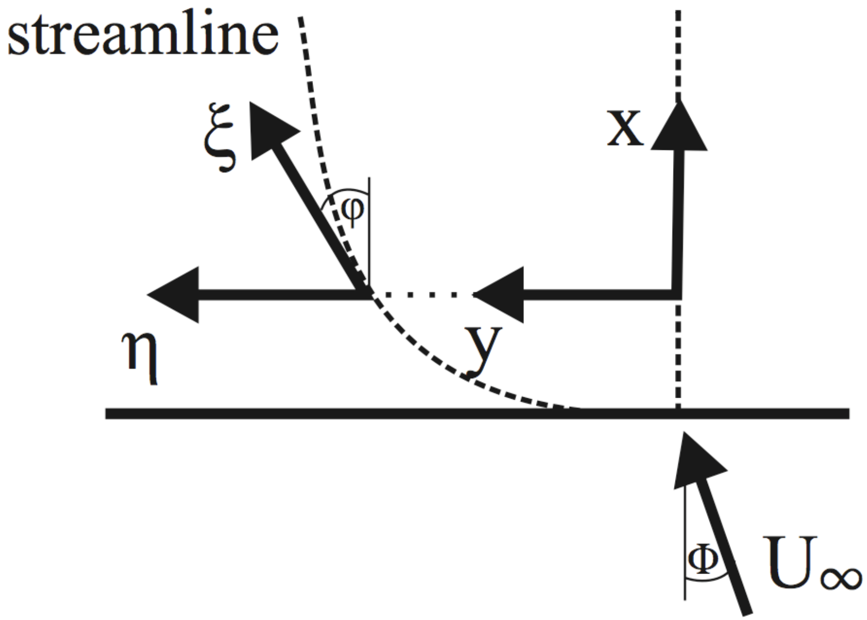
Governing Equations

- Scaling needed
- Assume disturbances:
  - Aligned with streamline
  - Periodic in spanwise
  - Weakly varying, non-oscillatory in streamwise direction
- Express in non-orthogonal coordinate system



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- Now disturbances are assumed to be of the form

$$\mathbf{q}'(\xi,\eta,\zeta) = \hat{\mathbf{q}}(\xi,\zeta)\exp(i\beta\eta)$$
$$\mathbf{q} = (u, v, w, p)$$

Governing Equations

- Applying a scaling appropriate to the assumptions made on the disturbances
- Neglecting terms of order higher than Re_δ^{-1}

➔ Parabolic set of equations in (ξ, η, ζ)

- Transforming back to cartesian coordinates results in the Parabolised Stability Equations ($\alpha = -\tan(\varphi)\beta$)
- Initial value problem, solved by marching downstream
- Optimise energy via adjoint-based optimisation

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Results

- Validation of 2.5D Code by comparing results for a Blasius BL and a “swept Blasius BL”

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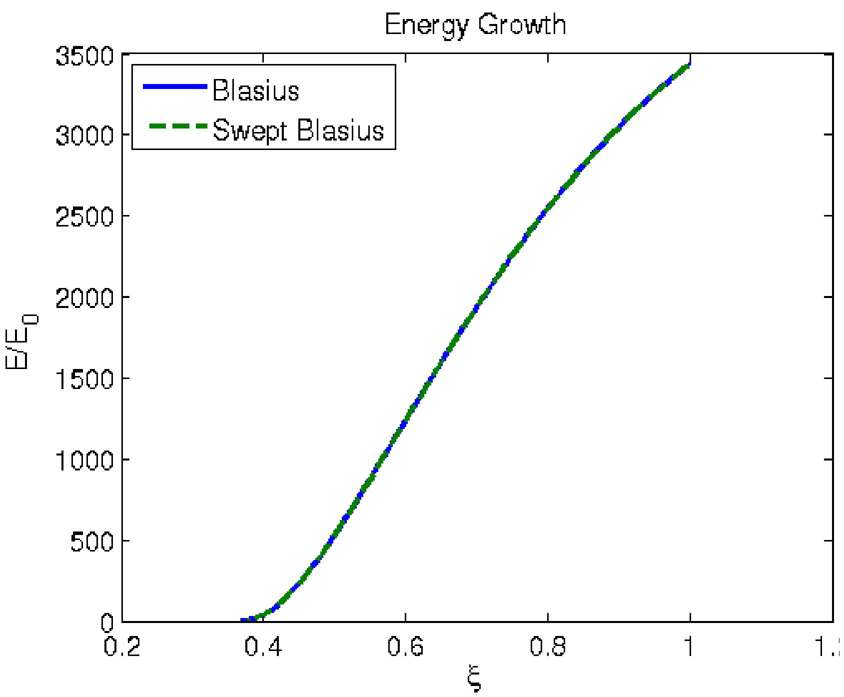
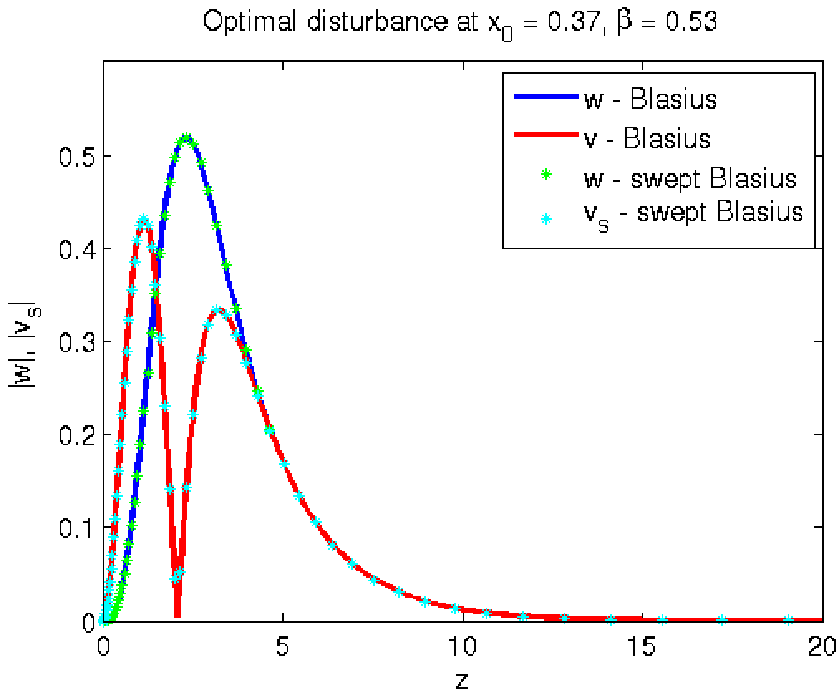
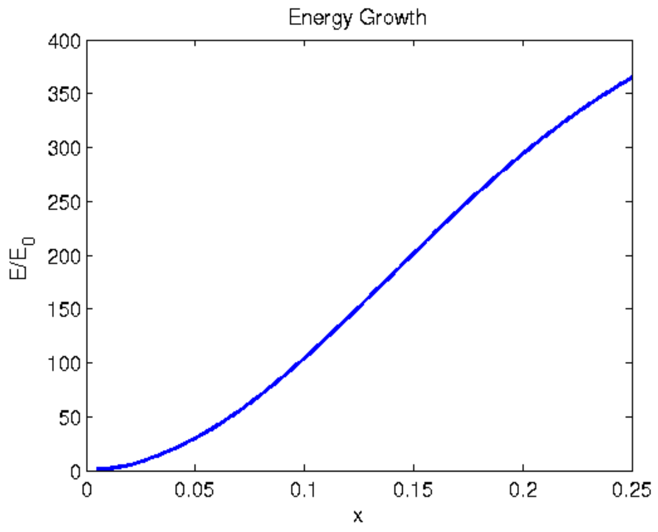


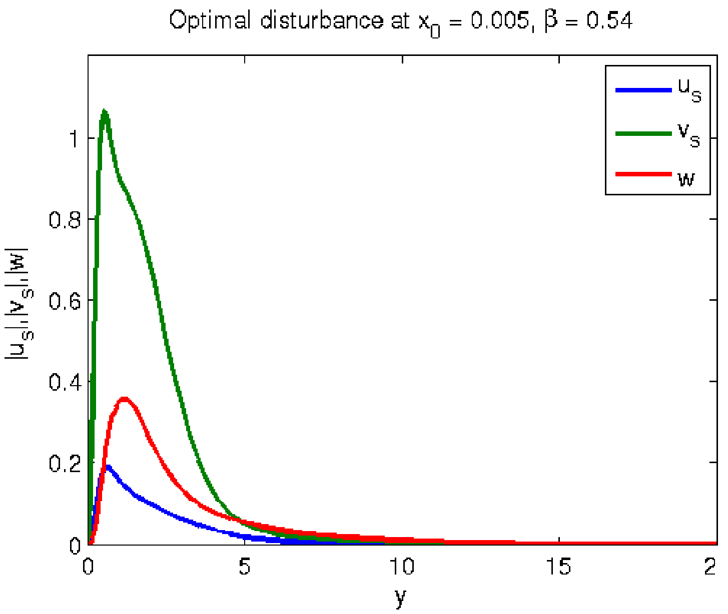


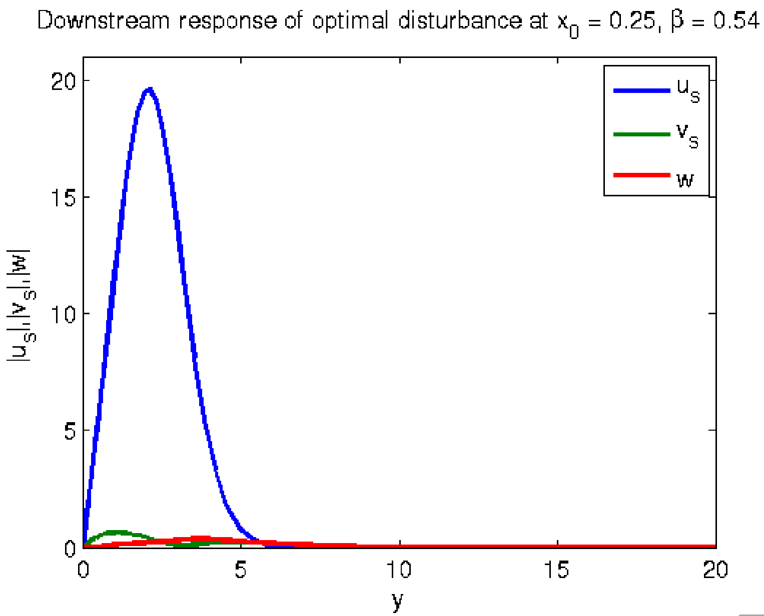
- Results for Blasius from Levin (2003)

Results

- Optimal disturbance of FSC - boundary layer with m = 0.1
- $x_0 = 0.005$, $x_f = 0.25$







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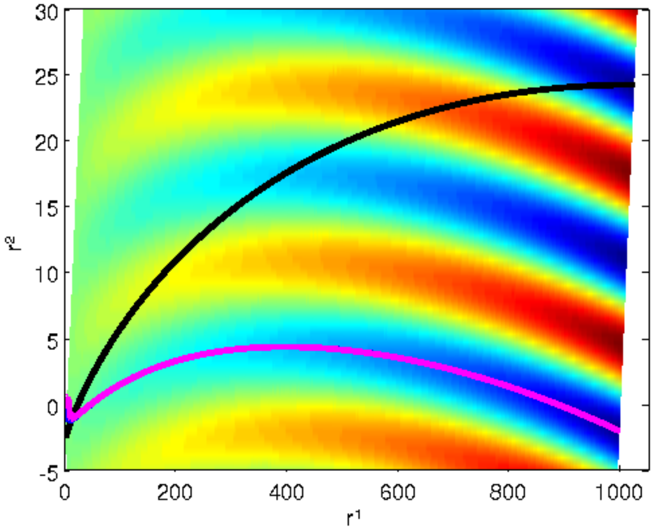
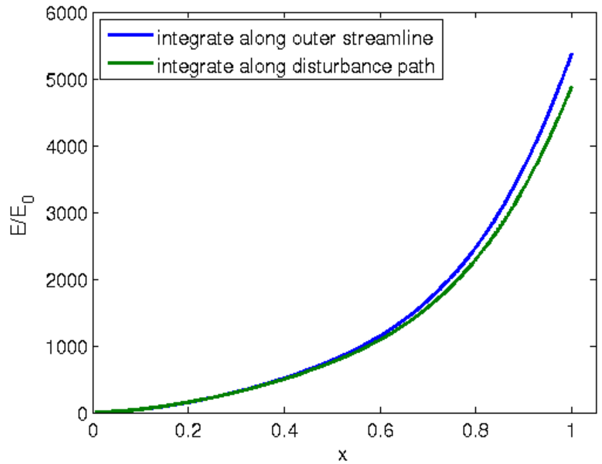
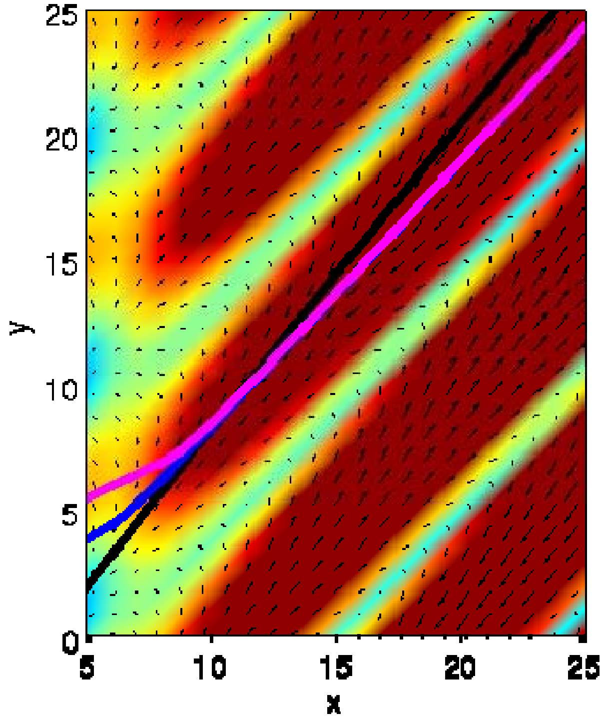
Results

- Disturbances not exactly aligned with streamline







$x_0 = 0.005$, $x_f = 1$

ß = 0.34

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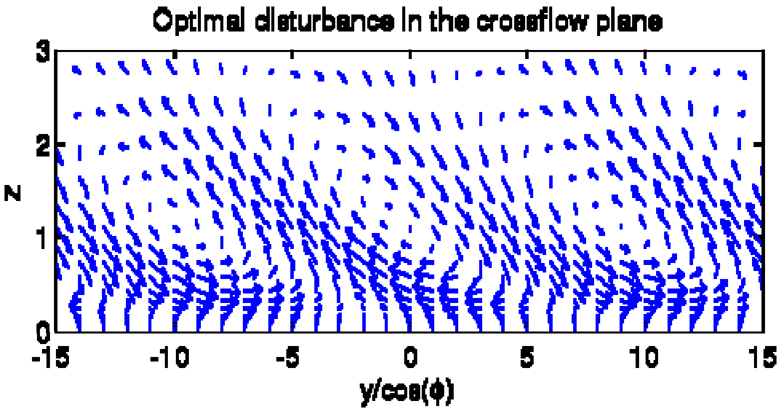
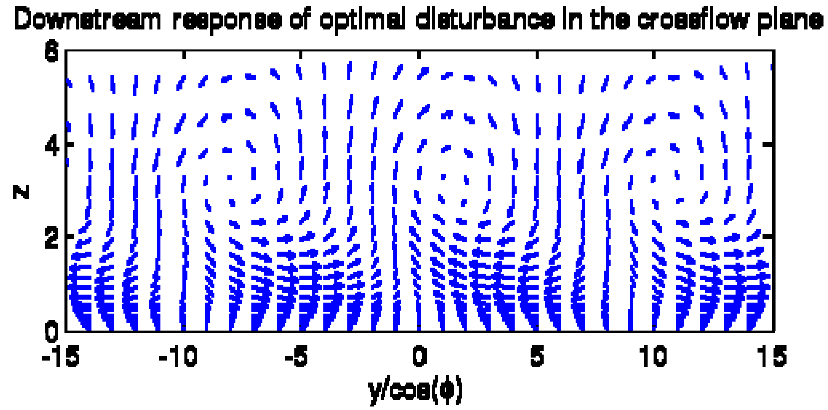
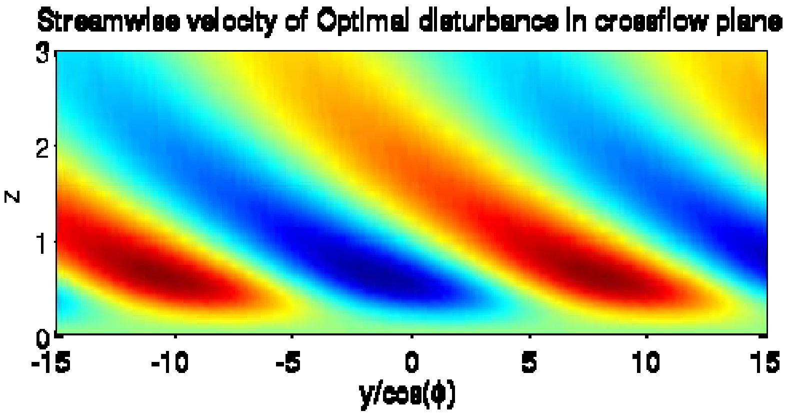
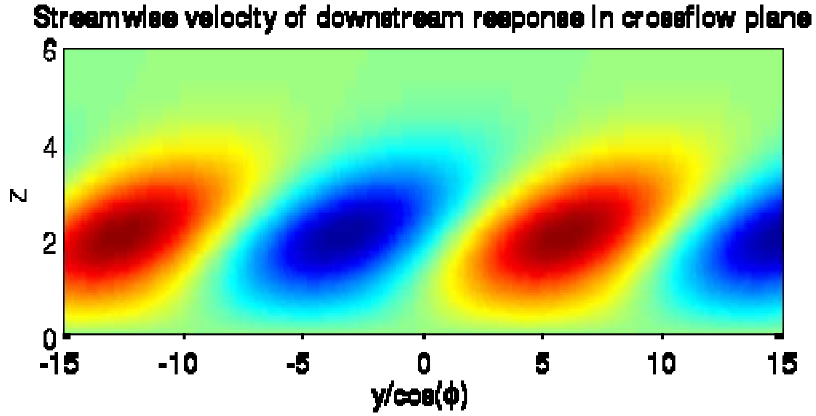
Results









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Results

- Comparison between the evolving optimal disturbance and an crossflow mode ß = 0.34

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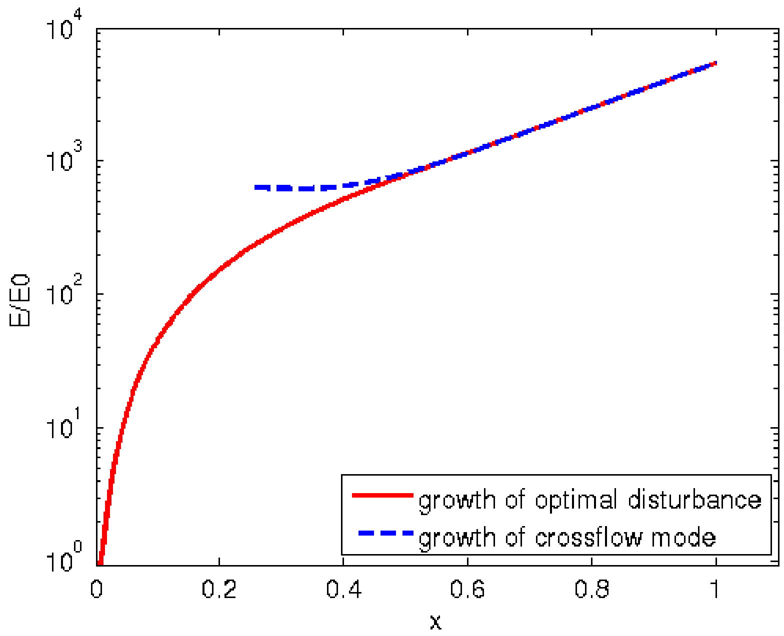

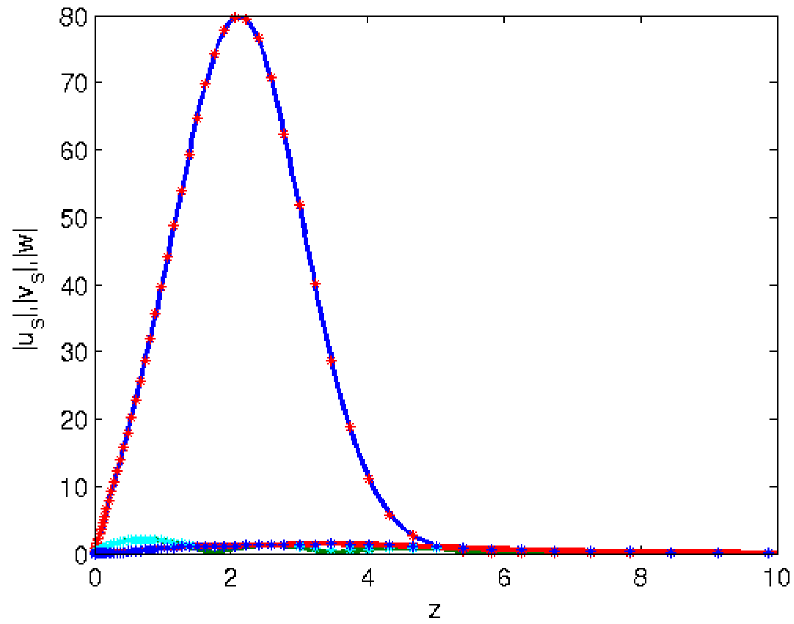

- Excellent agreement

Conclusions & Outlook

- Disturbances are not exactly aligned with the outer streamline
- Validation with Blasius / swept Blasius shows perfect agreement
- Optimal disturbance take form of tilted vortices in crossflow plane
- Transforms into crossflow mode when entering supercritical domain

- Parameter studies - different pressure gradients, include non-stationary disturbances, leading edge, …
- Comparison with DNS
- Project onto free stream turbulence

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