Reduced Order Models for Transonic Potential Flows

ERCOFTAC Spring Festival 2024

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16/05/2024

Outline

1. Kratos MultiPhysics Group
2. Motivation
3. Transonic Full Potential Equation
4. Proper Orthogonal Decomposition
5. Projection methods ROM - HROM
6. 2D Application Case - Naca 0012
7. 3D Application Case - Onera M6 Wing
8. Conclusions
9. Ongoing - future work
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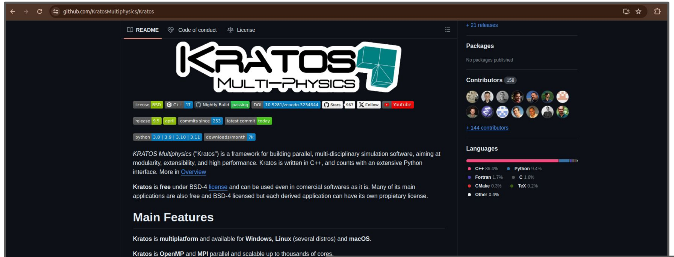
Kratos MultiPhysics Group



KRATOS Multiphysics ("Kratos") is a framework for building parallel, multi-disciplinary simulation software, aiming at modularity, extensibility, and high performance. Kratos is written in C++, and counts with an extensive Python interface. More in Overview

Kratos is **free** under BSD-4 license and can be used even in comercial softwares as it is. Many of its main applications are also free and BSD-4 licensed but each derived application can have its own propietary license.

Main Features

Kratos is **multiplatform** and available for **Windows, Linux** (several distros) and **macOS**.

Kratos is **OpenMP** and **MPI** parallel and scalable up to thousands of cores.

Kratos provides a core which defines the common framework and sever
can be extended in diverse fields.

Its main applications are:

- DEM for cohesive and non cohesive spheric and non spheric particl
- Fluid Dynamics Provides 2D and 3D incompressible fluids formulati
- Fluid Structure Interaction for solution of different FSI problems
- Structural Mechanics Providing solution for solid, shell and beam st dynamic behavior
- Contact Structural Mechanics For contact problems used along the

Kratos Team

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Kratos MultiPhysics Group

Main Features

Kratos is multiplatform and available for Windows, Linux (several distros) and macOS. OpenMP and MPI parallel and scalable up to thousands of cores.

Provides a core which defines the common framework and several application which work like plug-ins that can be extended in diverse fields.

Kratos github site

GiD®
The personal pre and
post processor

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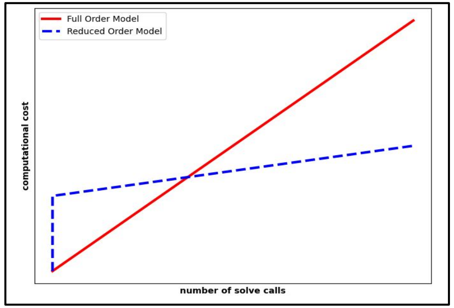
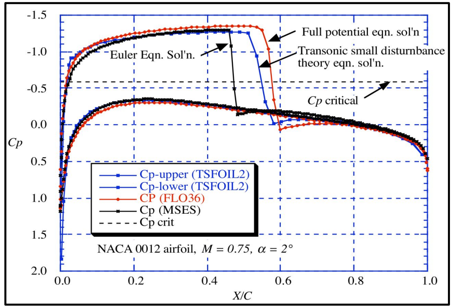
Motivation

1. Commercial aircrafts normally fly in the transonic regime.
2. There is a need for high-fidelity models, but the computational cost is unfeasible for realistic problems.
3. The full potential equation is the lowest fidelity layer that can capture transonic behavior and can be used in preliminary design and optimization problems.





[6]

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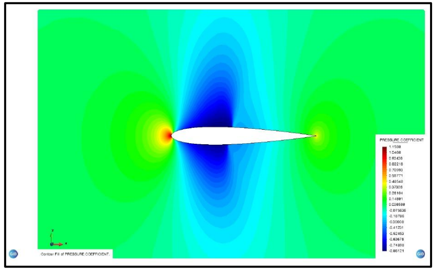
Transonic Full Potential Equation

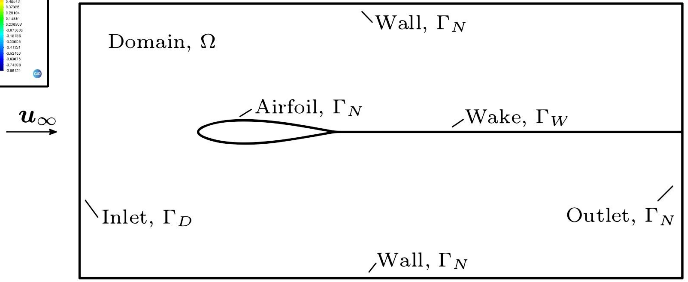
Potential Flow Problem

$$\nabla \cdot (\rho \mathbf{u}) = 0$$

Boundary Conditions

$$\begin{cases} \phi = \phi_\propto & \text{on} \quad \Gamma_D \\ \hat{n} \cdot (\rho \mathbf{u}) = q & \text{on} \quad \Gamma_N \\ \hat{n} \cdot (\rho_u \mathbf{u_u} - \rho_l \mathbf{u_l}) = 0 & \text{on} \quad \Gamma_W \\ |\mathbf{u_u}|^2 - |\mathbf{u_l}|^2 = 0 & \text{on} \quad \Gamma_W \end{cases}$$





Stabilization method:

Upwind density based

Solution method:

FEM + Newton Raphson + Line search [3][5][6]

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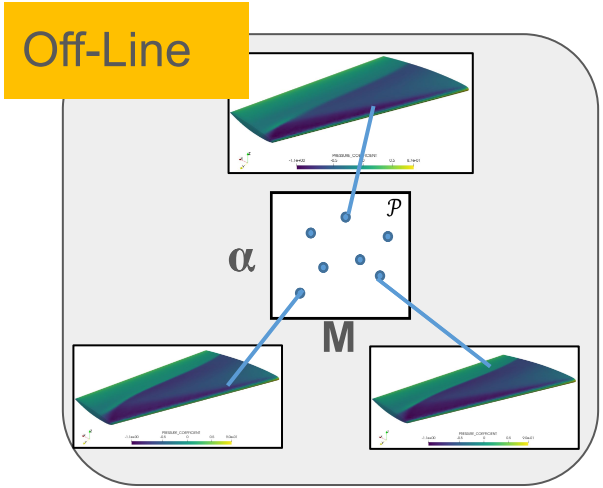
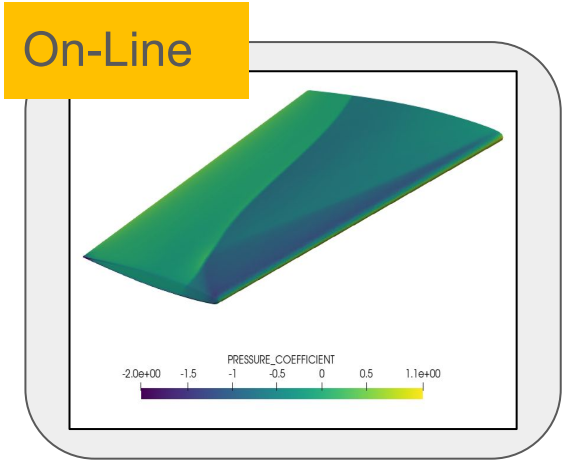
Proper Orthogonal Decomposition

$$\mathbf{u}(\boldsymbol{\mu}) \ , \quad \boldsymbol{\mu} \in \mathcal{P} \subset \mathbb{R}^p$$



ROM



α = 3.06 º
M = 0.839

$\mu = (\alpha, \mathbf{M}, ...geometry) \in \mathrm{P}$
Solve the FOM using
finite elements to find u(μ)

S

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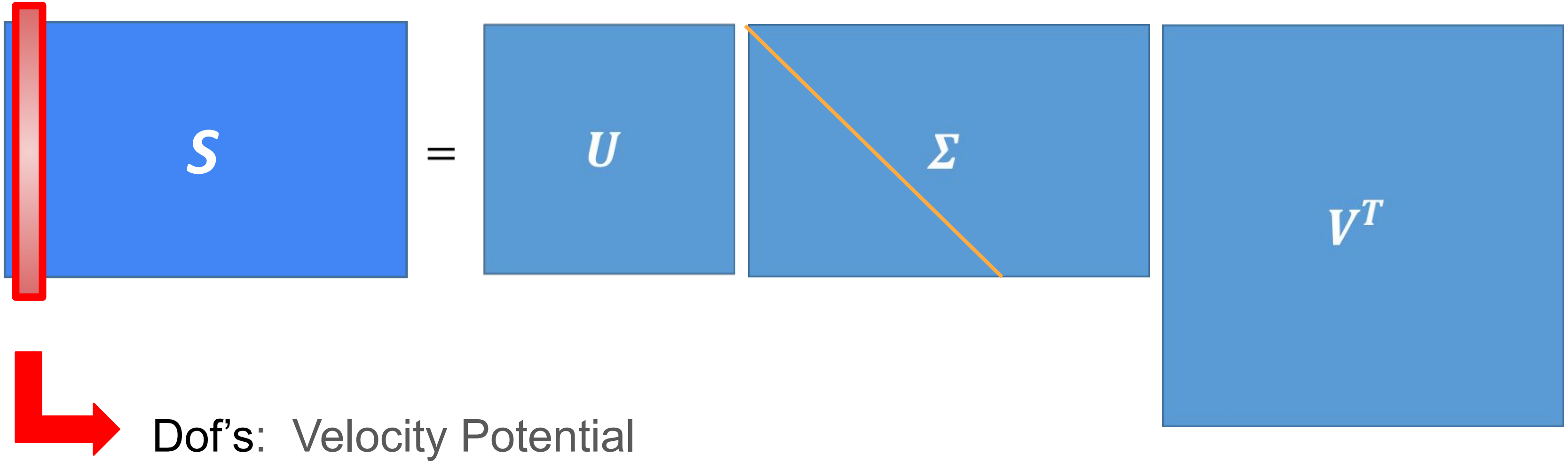
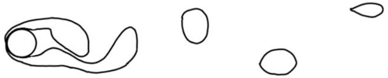
Proper Orthogonal Decomposition

Take the SVD of $\boldsymbol{S} = \boldsymbol{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} \approx \boldsymbol{U}_k\boldsymbol{\Sigma}_{\mathrm{k}}\mathbf{V}_{\mathrm{k}}^{\mathrm{T}}$



Dof's: Velocity Potential
Auxiliary Velocity Potential (Wake)



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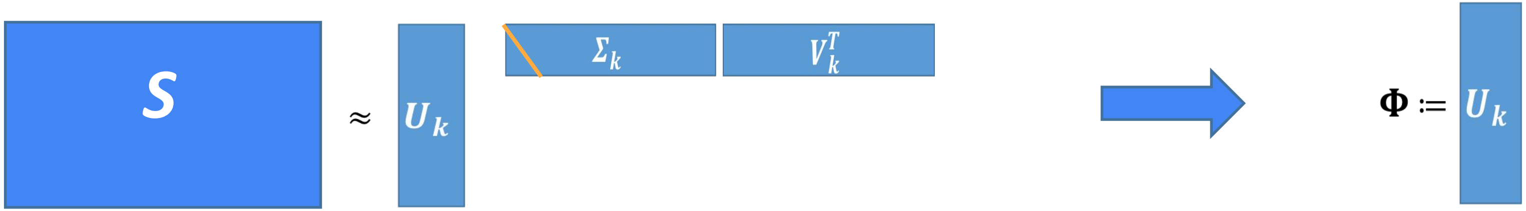
Proper Orthogonal Decomposition

Given $\mathbf{A} \in \mathbb{R}^{nxn}, \quad n \geq m$
The asymptotic complexity of computing its SVD is[8]:

Standard SVD → $O(m^2n)$

Randomized SVD → $O(mnk)$

Take the SVD of $\boldsymbol{S} = \boldsymbol{U\Sigma}\mathbf{V}^{\mathrm{T}} \approx \boldsymbol{U}_k\boldsymbol{\Sigma}_{\mathbf{k}}\mathbf{V}_{\mathbf{k}}^{\mathrm{T}}$



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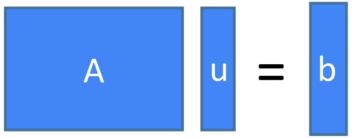
Projection Methods - ROM

Full Order Model (FOM):

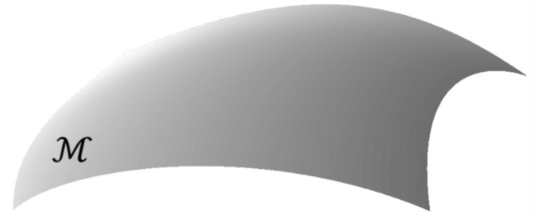
$\boldsymbol{r}(\boldsymbol{u};\boldsymbol{\mu}) = \mathbf{0}$

$\boldsymbol{u} \in \mathbb{R}^n$: state vector

$\boldsymbol{\mu} \in \mathcal{P} \subset \mathbb{R}^p$: parameters vector



Solution manifold: $\mathcal{M} = \{\, \boldsymbol{u}(\boldsymbol{\mu}) \mid \boldsymbol{\mu} \in \mathcal{P} \,\} \subset \mathbb{R}^n$

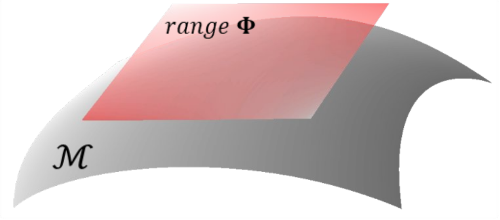


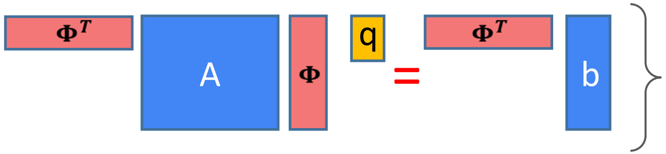
Let $\boldsymbol{u} \approx \boldsymbol{\Phi}\, \mathbf{q}$

Reduced Order Model (ROM):

$\boldsymbol{\Phi}^T \boldsymbol{r}(\boldsymbol{\Phi}\, \mathbf{q};\boldsymbol{\mu}) = \mathbf{0}$

$\boldsymbol{q} \in \mathbb{R}^k$: reduced state vector





A MUCH SMALLER SYSTEM!

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Projection Methods - ROM

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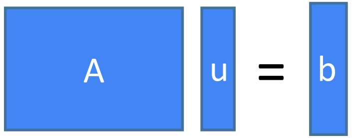

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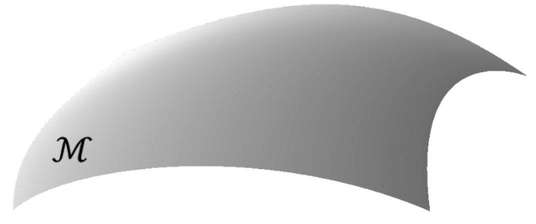

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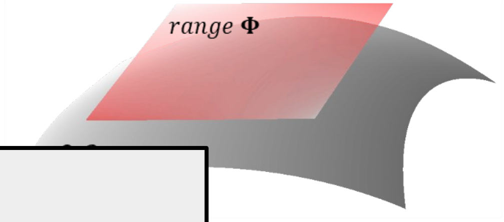


PROBLEM:

STILL EXPENSIVE TO MOUNT THE SYSTEM

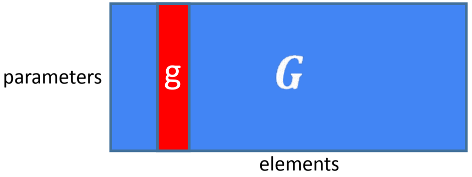
A MUCH SMALLER SYSTEM!

Projection Methods - HROM

The goal is to find a **subset of elements and their corresponding weights** by solving an optimization problem[9].

$$
\begin{aligned}
(\boldsymbol{E},\boldsymbol{W}) &= \arg min \,\|\boldsymbol{\zeta}\|_{\boldsymbol{0}} \\
s.t. \quad & \|\boldsymbol{G}\mathbf{1} - \boldsymbol{G}\boldsymbol{\zeta}\|_2^2 \le \epsilon \|\boldsymbol{G}\mathbf{1}\|_2^2 \\
& \zeta_i \ge 0
\end{aligned}
$$

Where $\boldsymbol{G} = \boldsymbol{G}(\boldsymbol{\Phi},\boldsymbol{R})$



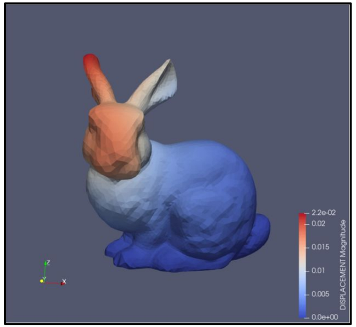
NP-HARD. Solving via greedy procedure

$$
\begin{aligned}
(\boldsymbol{E},\boldsymbol{W}) &= \arg min \left\| \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{g}_{\boldsymbol{i}} - \sum_{\boldsymbol{i}\in \boldsymbol{E}} \boldsymbol{g}_{\boldsymbol{i}} \boldsymbol{\omega}_{\boldsymbol{i}} \right\|_2^2 \\
s.t. \quad & \omega_i > 0
\end{aligned}
$$

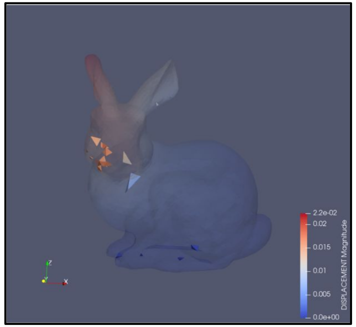
Projection Methods - HROM

Assembly comparison FOM vs HROM:

$$\left(\prod_{e=1}^{n\,elem} A_e\right)\boldsymbol{u} = \prod_{e=1}^{n\,elem} b_e \quad \Longrightarrow \quad \left(\sum_{e\in E} \boldsymbol{\Phi}_e^T \boldsymbol{A}_e \boldsymbol{\Phi}_{\mathbf{e}}\, \omega_e\right)\boldsymbol{q} = \sum_{e\in E} \boldsymbol{\Phi}_{\mathbf{e}}^T \boldsymbol{b}_e \omega_e$$



FOM Simulation



HROM Simulation

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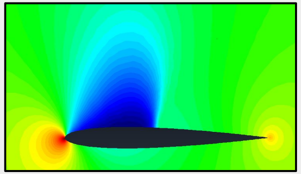
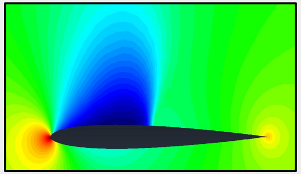
2D Application case - Naca 0012

Parameters range:

Angle of attack : [1º - 2º]
Mach : [0.70 - 0.75]

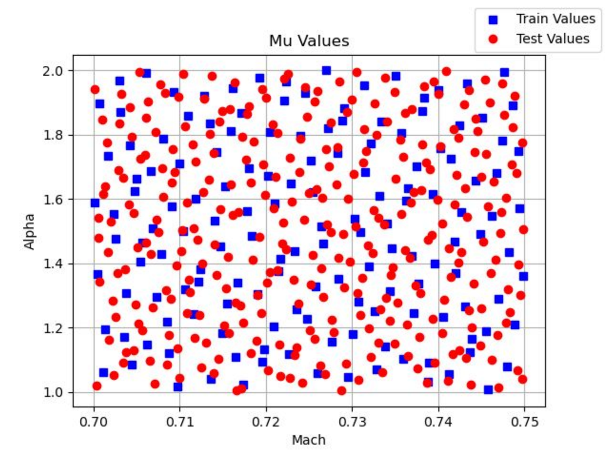


1.984º, 0.735



1.828º, 0.742

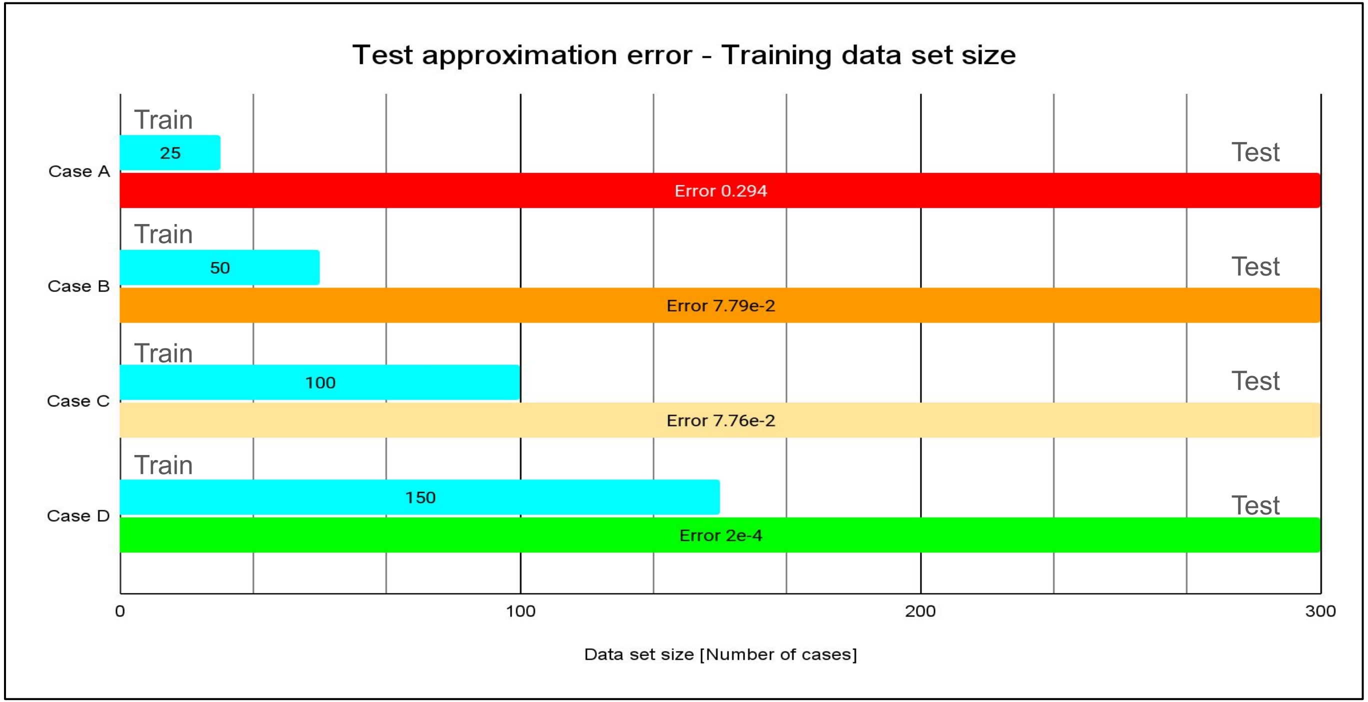
Train set 150 - Test set 300



Halton sequence

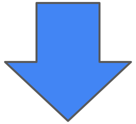
Coarse mesh ~ 4100

2D Application case - Naca 0012



FOM

ROM



~ 1 sec

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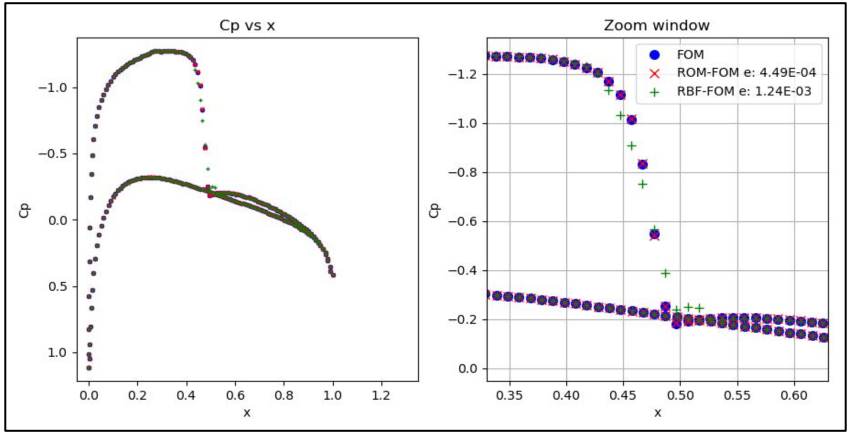
2D Application case - Naca 0012



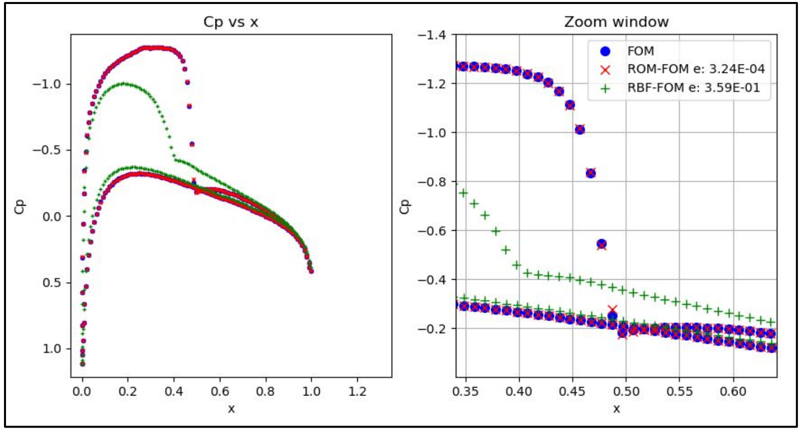
150 training cases

Case D

***α* = 1.997º**
M = 0.740

Case C

100 training cases



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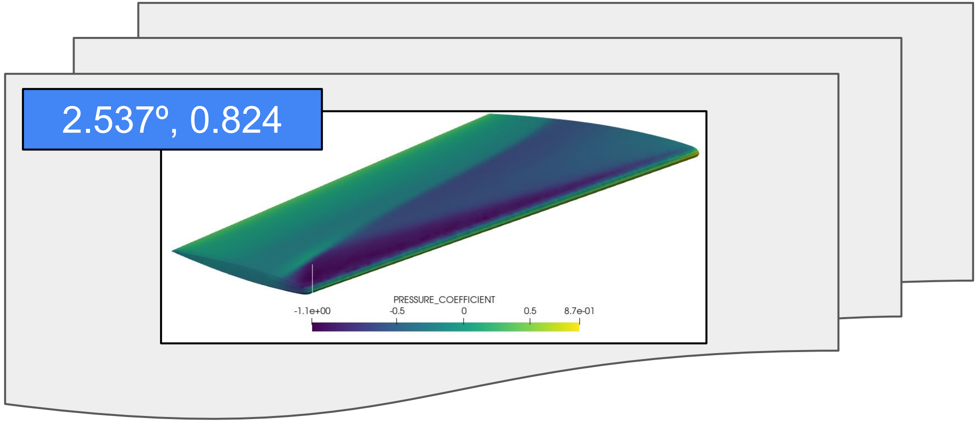
3D Application case - Onera M6 Wing[4]

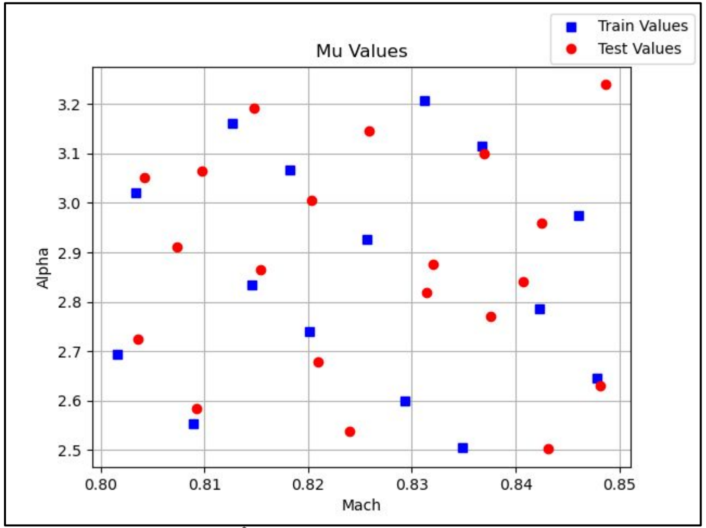
Parameters range:

Angle of attack : [2.5º - 3.25º]

Mach : [0.80 - 0.85]

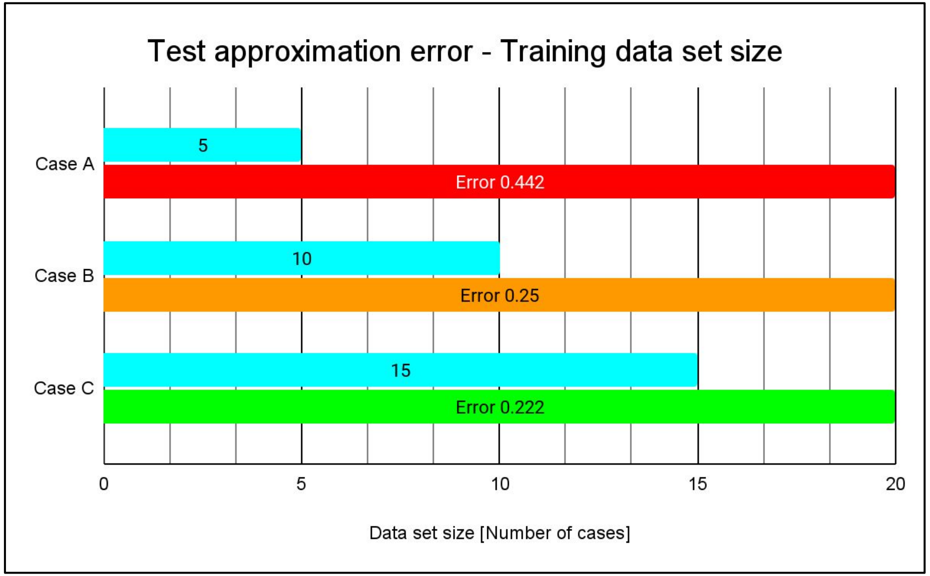


Train set 15 - Test set 20



Halton sequence

3D Application case - Onera M6 Wing



Coarse mesh ~ 170.000

3D Application case - Onera M6 Wing

Case C

Approximation Test:

α = 3.064

M = 0.809

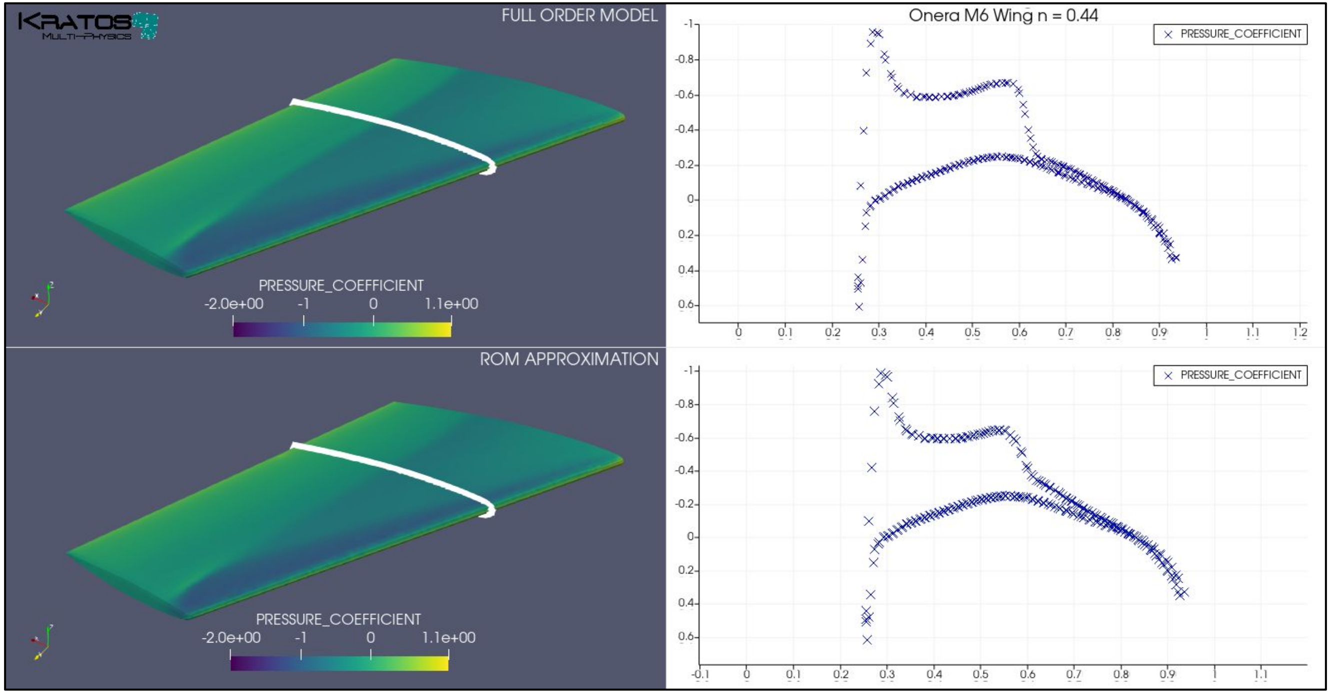
FOM: **+7 min**

ROM: **59 sec**

Approx error:

fom - rom: **1.78%**

15 training cases



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Conclusions

- Reduced order models POD based have been trained for both two-dimensional and three-dimensional cases.
- Both cases have been used to obtain solutions for new parameters.
- It has been observed that the solving time is reduced, especially with the use of HROM, while maintaining accuracy.
- Some problems may arise when the method of solving the potential problem is slightly modified, for example, with viscosity corrections.
- The validity range of the Full Potential Equation is very small, although it is sufficient for exploring the application of the method and obtaining preliminary results.

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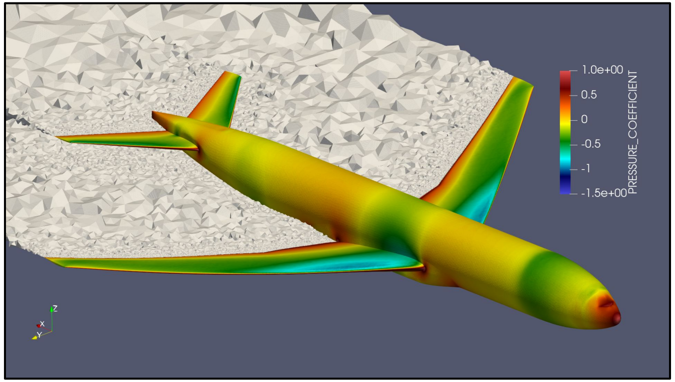
Ongoing - Future work

1. Finish implementation HROM Transonic Full Potential.(*)
2. Testing 3D plane models.
3. Testing Transonic Aerodynamic Shape Optimization.
4. Testing Multi-fidelity and ANN methods.



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Generalitat de Catalunya

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Thank you! Questions?

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