

# **IMPERIAL**

**Department of  
Chemical Engineering**

## **Direct numerical simulations of Multiphase Flows in Stirred Vessels and Static Mixers**

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# Multiphase flows

## Challenges

### Multiphase flows are discontinuous

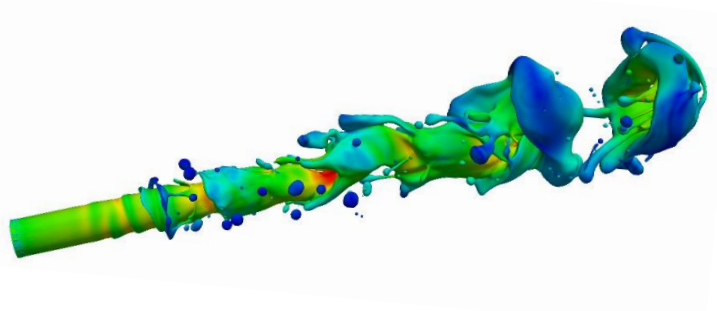
- Phase-interface extremely thin and exhibits a singular surface tension force
- Fluid properties jump across the interface

### Multiphase flows are multi-scale

- From microscopic droplets to meters (oceanic flows)
- Complex topology changes
- Turbulence (atomisation)



Multi-scale: oceanic flows



Discontinuous: Atomisation



Multi-physics: dirty interfaces

### Multiphase flow are multi-physics

- **Surfactant-laden** interfaces featuring Marangoni stresses (dirty interfaces)
- Non-isothermal, phase change

**Numerical simulations facilitate isolation of mechanisms**

# Multiphase Flows

## Governing Equations

- N-S equations:

$C_s = 0.2$ , Smagorinsky-Lilly coefficient  
Large Eddy Simulation

$$\nabla \cdot \tilde{\mathbf{u}} = 0,$$

$$\tilde{\rho} \left( \frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} \right) = -\nabla \tilde{p} + \frac{1}{Re} \nabla \cdot \tilde{\mu} (\nabla \tilde{\mathbf{u}} + \nabla \tilde{\mathbf{u}}^T) - \nabla \cdot (\tilde{\rho} C_s^2 \tilde{\Delta}^2 |\tilde{s}|) (\nabla \tilde{\mathbf{u}} + \nabla \tilde{\mathbf{u}}^T) - \frac{1}{Fr} \tilde{\rho} \tilde{\mathbf{g}} + \tilde{\mathbf{F}}_{fsi}$$

$$+ \frac{1}{We} \left( \int_{\tilde{A}_e} \tilde{\sigma} \tilde{\mathbf{k}} \mathbf{n} + \nabla_s \tilde{\sigma} \right) \delta_f (\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_f) d\tilde{A}_e,$$

$\Delta \equiv \min(\Delta x, \Delta y, \Delta z)$

Solid-fluid interaction force

- Interface behaviors

Hybrid front-tracking/level-set method<sup>[1]</sup>

- Rotating impeller

Direct-Forcing method<sup>[2]</sup>

Tangential component:  
Marangoni stresses

Normal component:  
mean interfacial tension

[1] Shin, S., Chergui, J., Juric, D., Kahouadji, L., Matar, O. K., & Craster, R. V. (2018). A hybrid interface tracking–level set technique for multiphase flow with soluble surfactant. *Journal of Computational Physics*, 359, 409-435.

[2] Fadlun, E. A., Verzicco, R., Orlandi, P., & Mohd-Yusof, J. (2000). Combined immersed-boundary finite-difference methods for three-dimensional complex flow simulations. *Journal of computational physics*, 161(1), 35-60.

# Multiphase Flows

## Governing Equations

- Surfactant transport:

Bulk: 
$$\frac{\partial \tilde{C}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{C} = \frac{1}{Pe_b} \nabla^2 \tilde{C},$$

Interface: 
$$\frac{\partial \tilde{\Gamma}}{\partial t} + \nabla_s \cdot (\tilde{\Gamma} \tilde{\mathbf{u}}_t) = \frac{1}{Pe_s} \nabla_s^2 \tilde{\Gamma} + \tilde{J},$$

Ad/des flux: 
$$\tilde{J}_{a/d} = Bi [k \tilde{C}_0 (1 - \tilde{\Gamma}) - \tilde{\Gamma}],$$

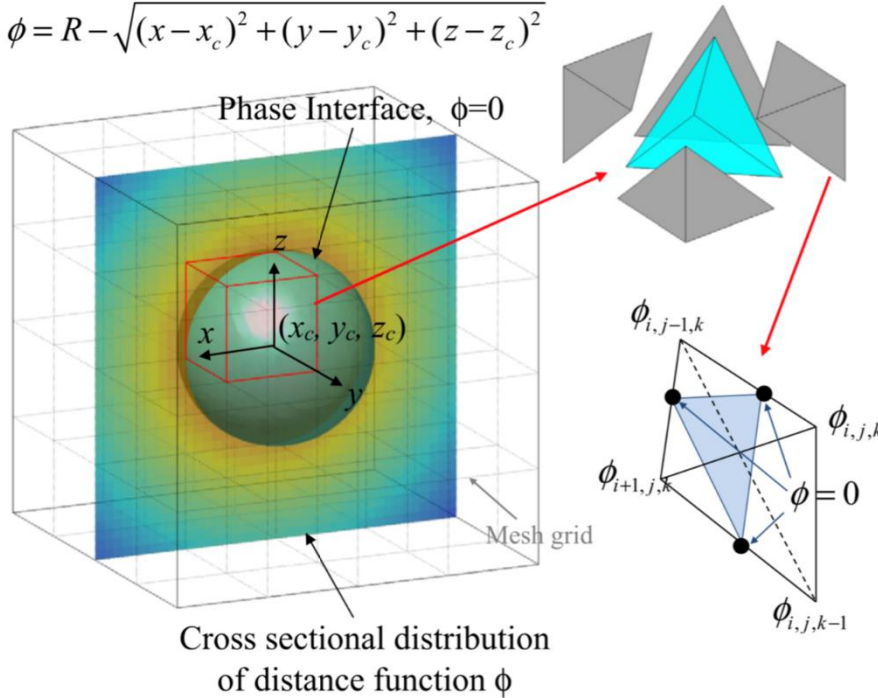
Diffusive flux : 
$$\tilde{J}_{diff} = -\frac{1}{Pe_b h} \mathbf{n} \cdot \nabla \tilde{C}|_{sub}$$

Langmuir relation: 
$$\tilde{\sigma} = \max[0.05, 1 + \beta_s \ln(1 - \tilde{\Gamma})].$$

# Multiphase Flows

## Numerical Method

$$\phi = R - \sqrt{(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2}$$



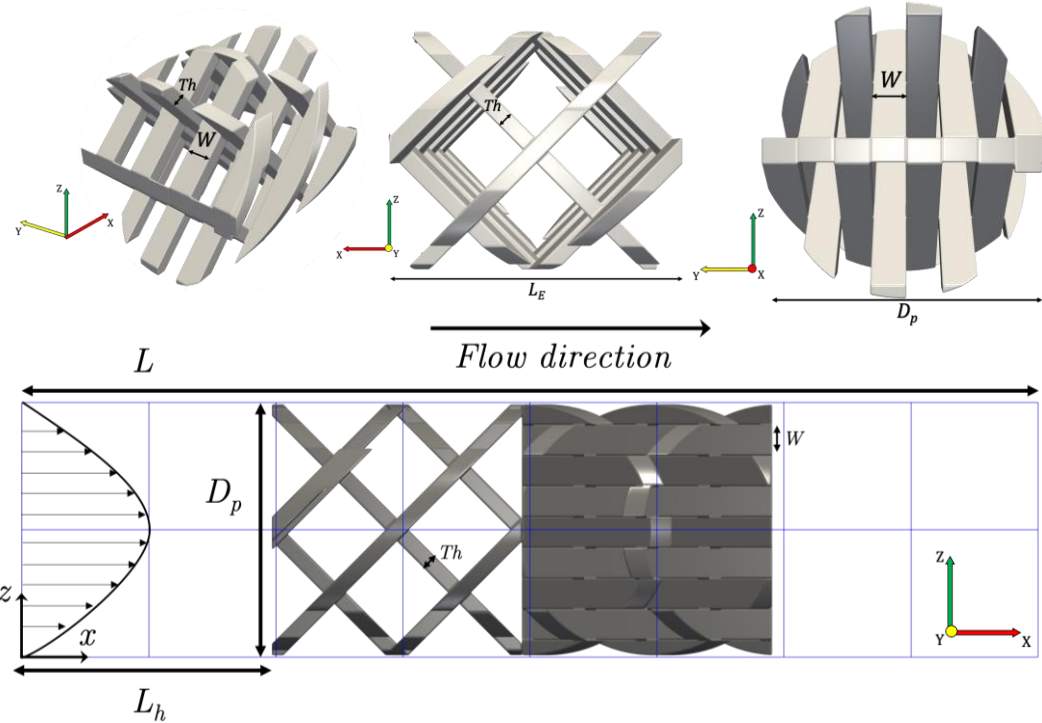
### Hybrid **front-tracking/level-set** methods

- Accurate calculation of surface tension forces
- It can handle complex topological changes
- Mass conservation
- Lagrangian interface tracking



# Static Mixers

## Geometry



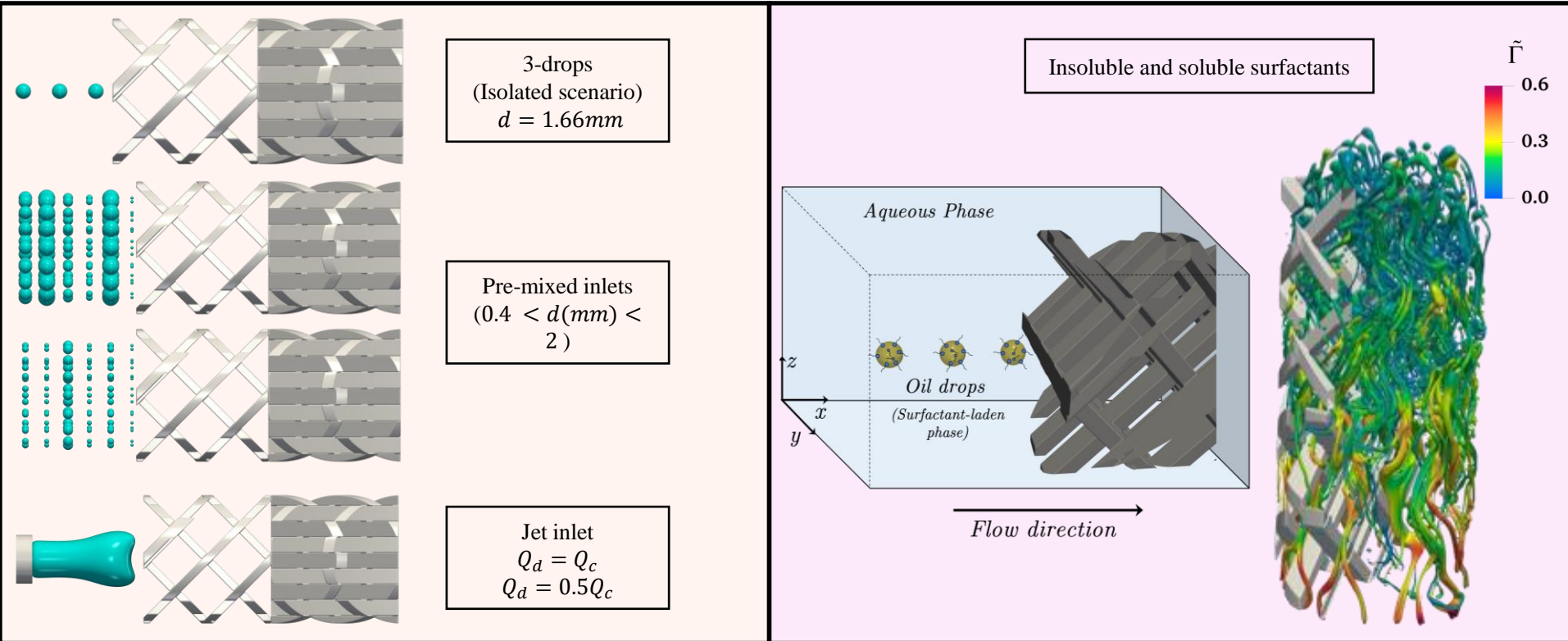
*Liu et al. (2005) and Rama Rao et al. (2007) specifications*

Feature		
Pipe diameter $D_P$ (m)		0.01575
Number of crossbars		8
Aspect ratio $L_E/D_P$		1
Entry length $L_h$ (m)		0.01575
Number of elements $n_E$		2
Bar thickness $T_h$ (m)		0.001
Bar width $W$ (m)		0.0019
Length $L$ (m)		0.064
Grid details	Subdomains	12 × 6 × 6
	Cells per subdomain	128 × 64 × 64

	Continuous	Dispersed
Density $\rho$ ( $kg/m^3$ )	1364	960
Viscosity $\mu$ ( $Pa \cdot s$ )	0.615	0.0984
Surface tension $\sigma$ ( $N/m$ )		0.036
Flow rate $Q$ ( $m^3/s$ )		9.0e-6
Re		1.63

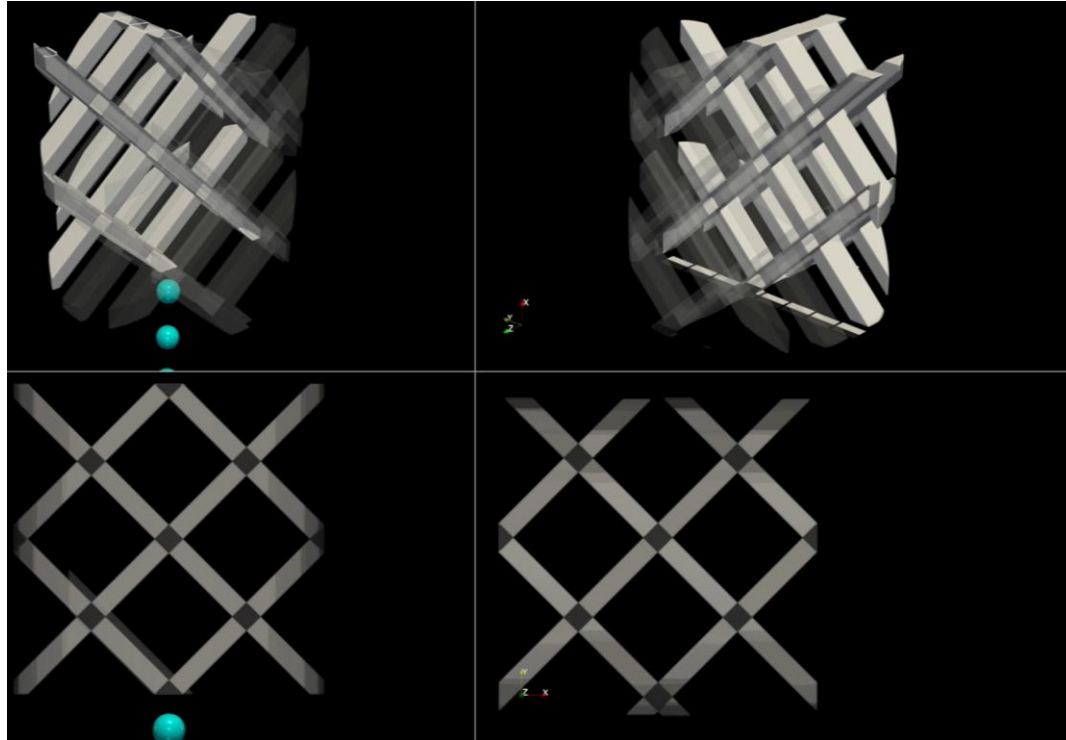
# Static Mixers

## Case Studies



# Static Mixers

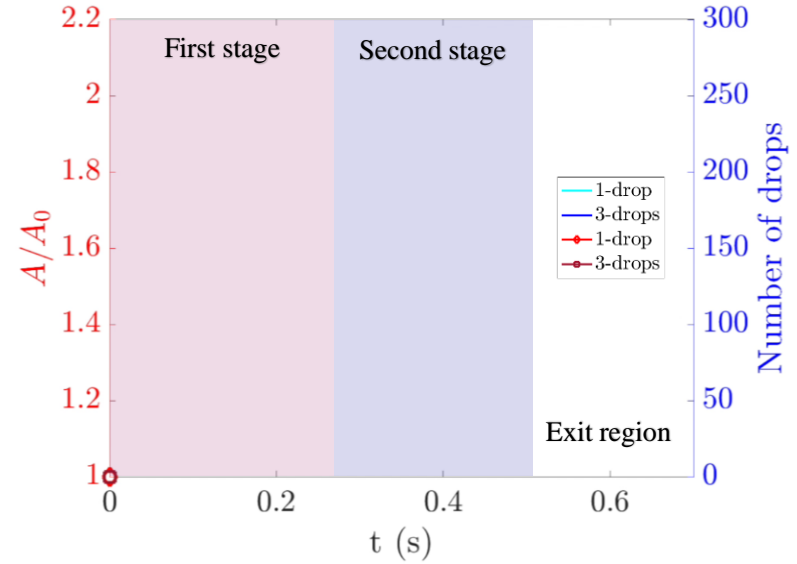
## Two-stage Breakup



3-drops

Direct numerical simulations of liquid–liquid dispersions in a SMX mixer under different inlet conditions

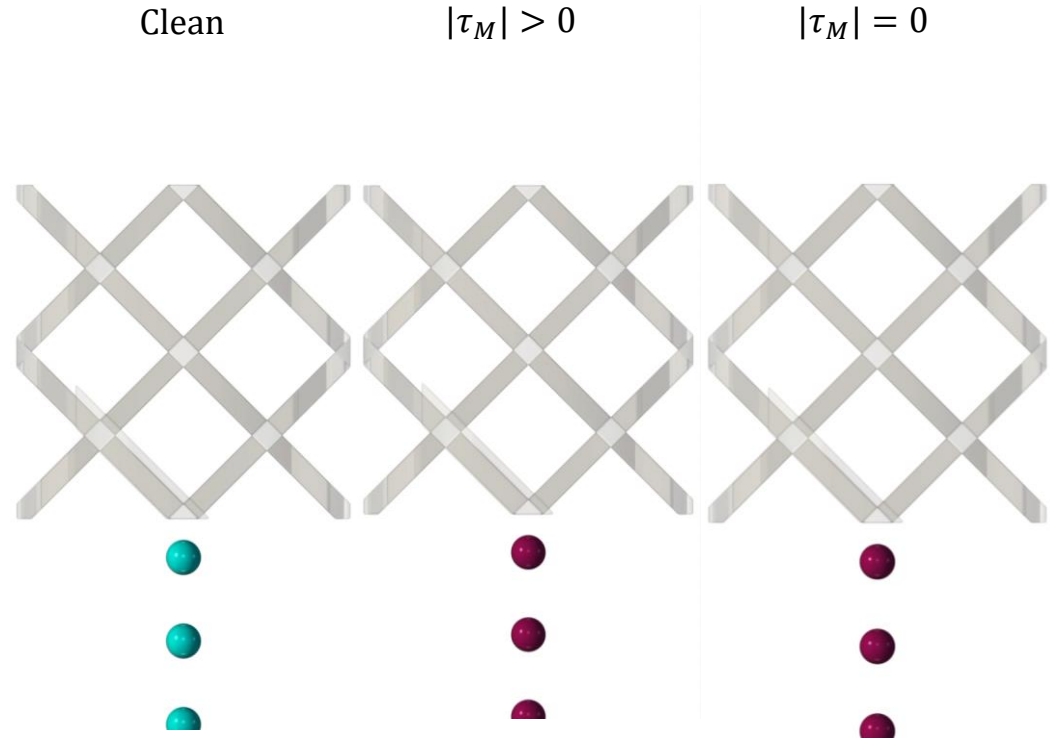
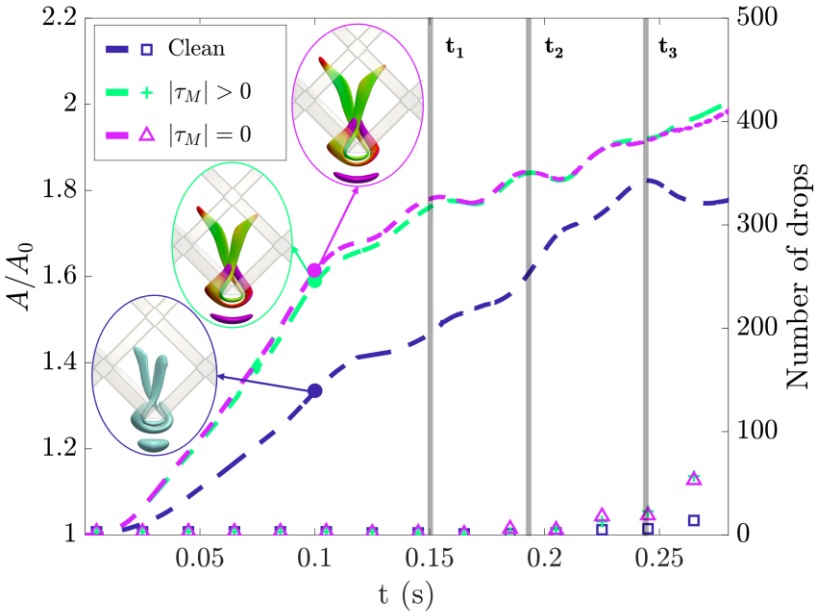
Juan Pablo Valdes <sup>a,\*</sup>, Lyes Kahouadji <sup>a</sup>, Fuyue Liang <sup>a</sup>, Seungwon Shin <sup>b</sup>, Jalel Chergui <sup>c</sup>, Damir Juric <sup>c,d</sup>, Omar K. Matar <sup>a</sup>





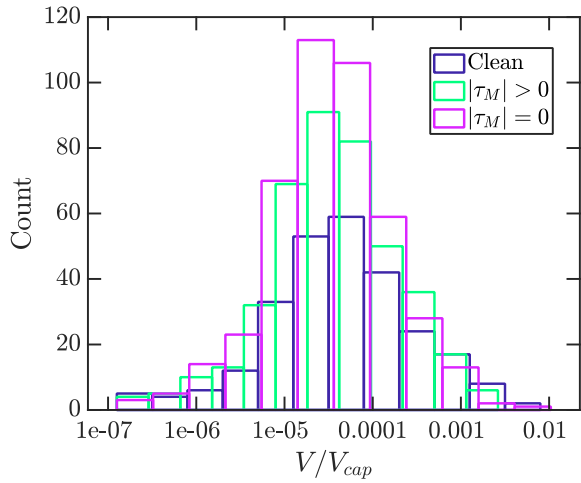
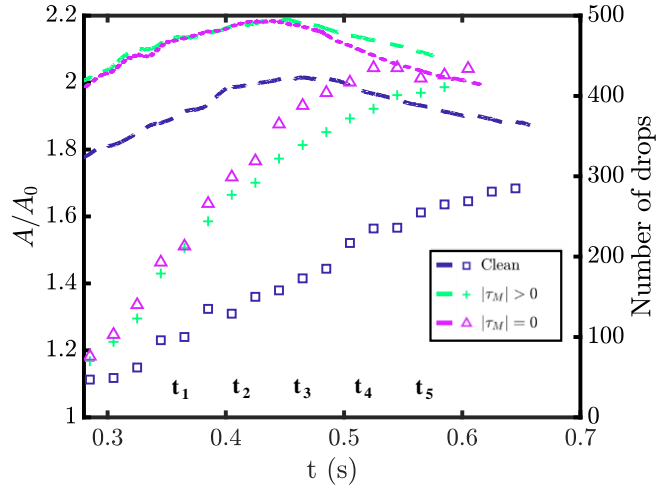
# Static Mixers

## Dispersion Process: 1<sup>st</sup> Stage



# Static Mixers

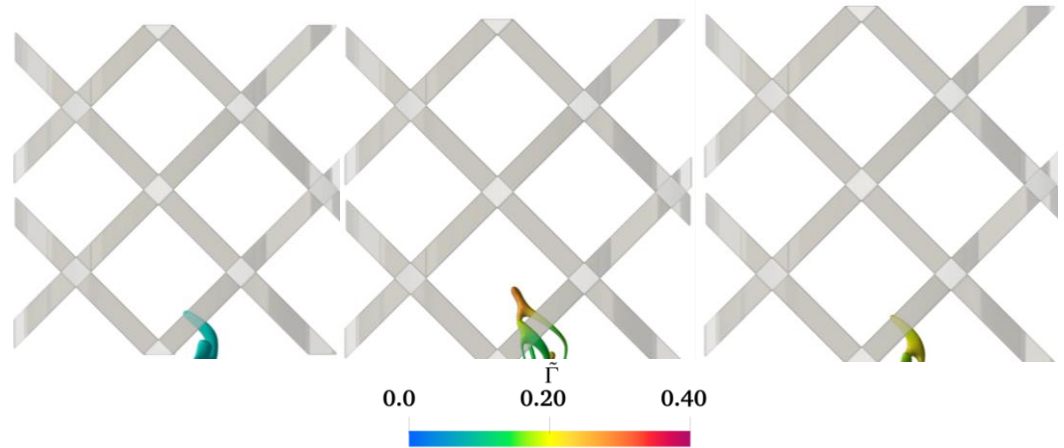
## Dispersion Process: 2<sup>nd</sup> Stage



Clean

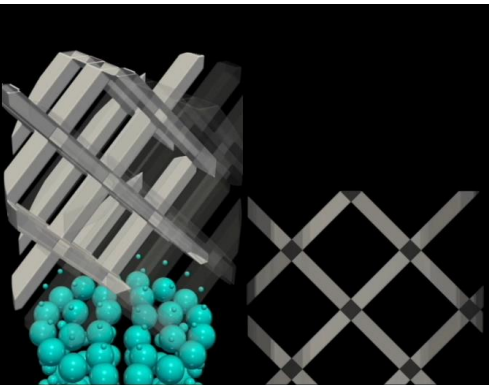
$|\tau_M| > 0$

$|\tau_M| = 0$

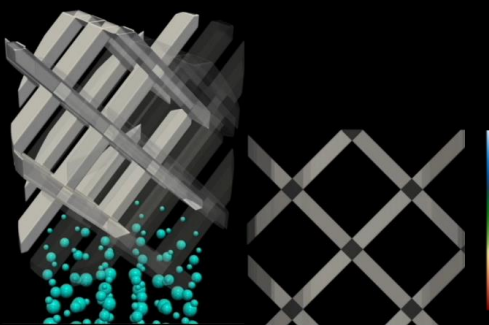


# Static Mixers

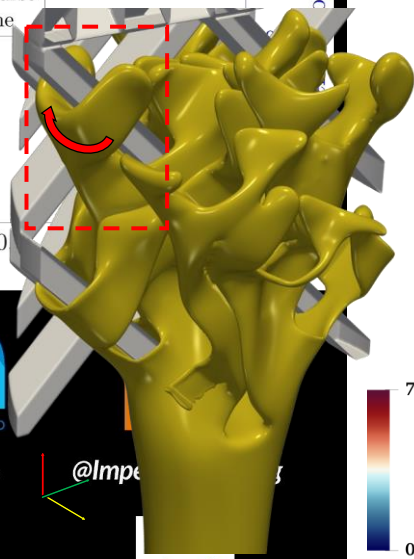
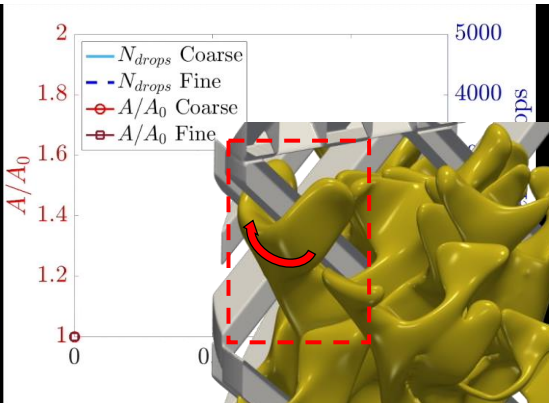
## Complex Inlets



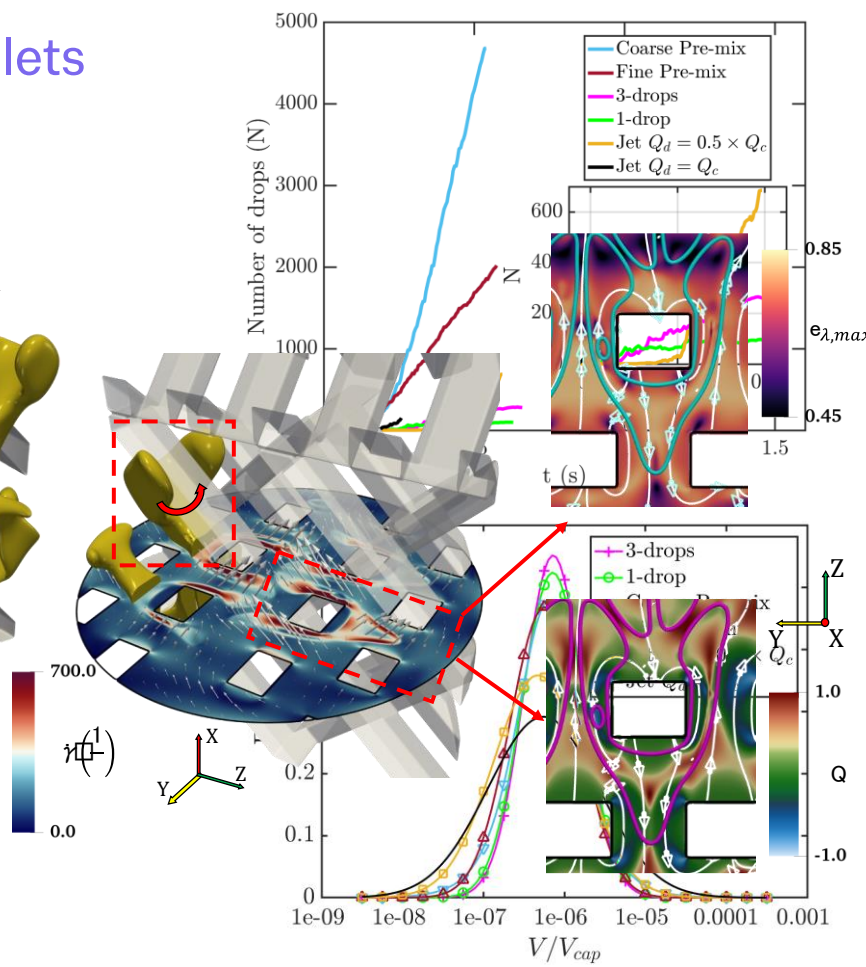
Coarse mix



Fine mix

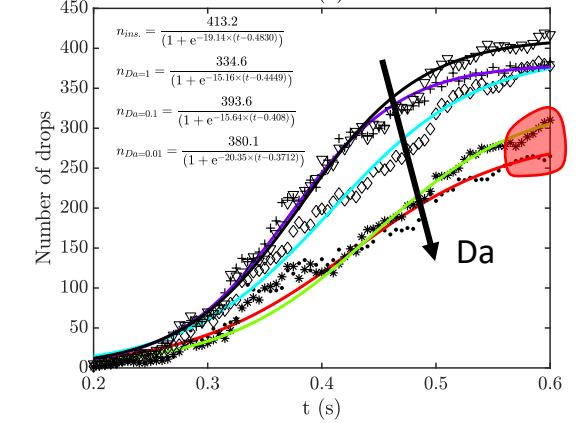
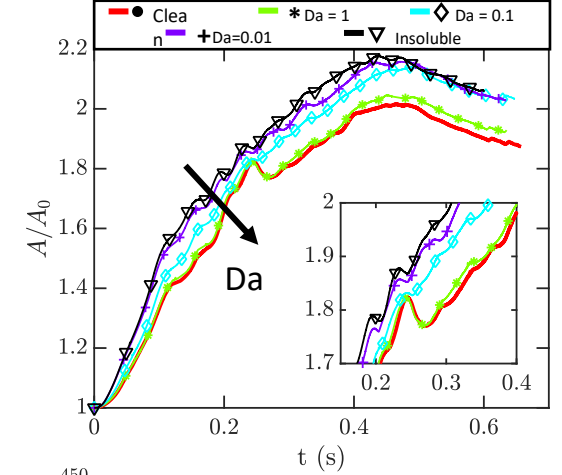
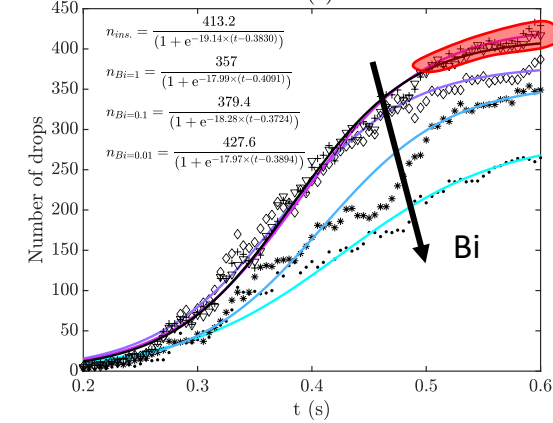
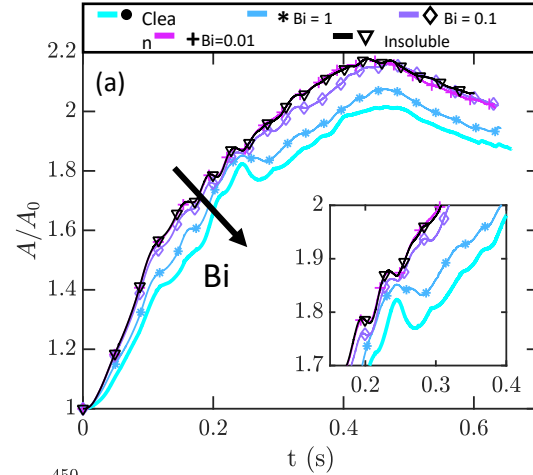
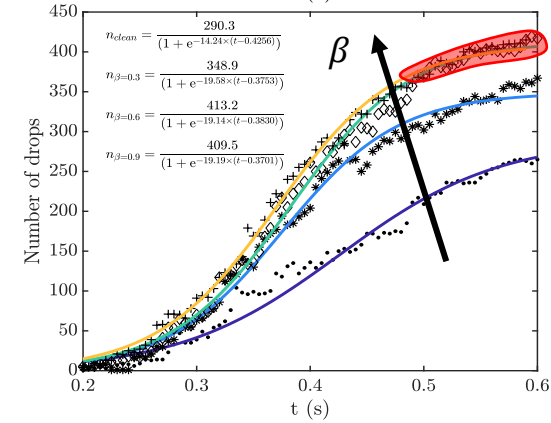
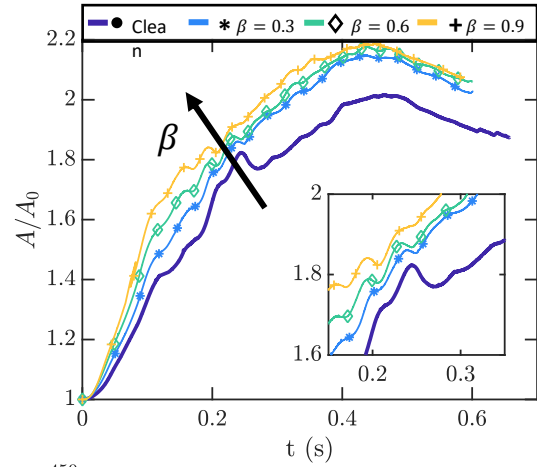


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 MIERE @PREMIERE\_UKRI  
 LISN @LisnLab



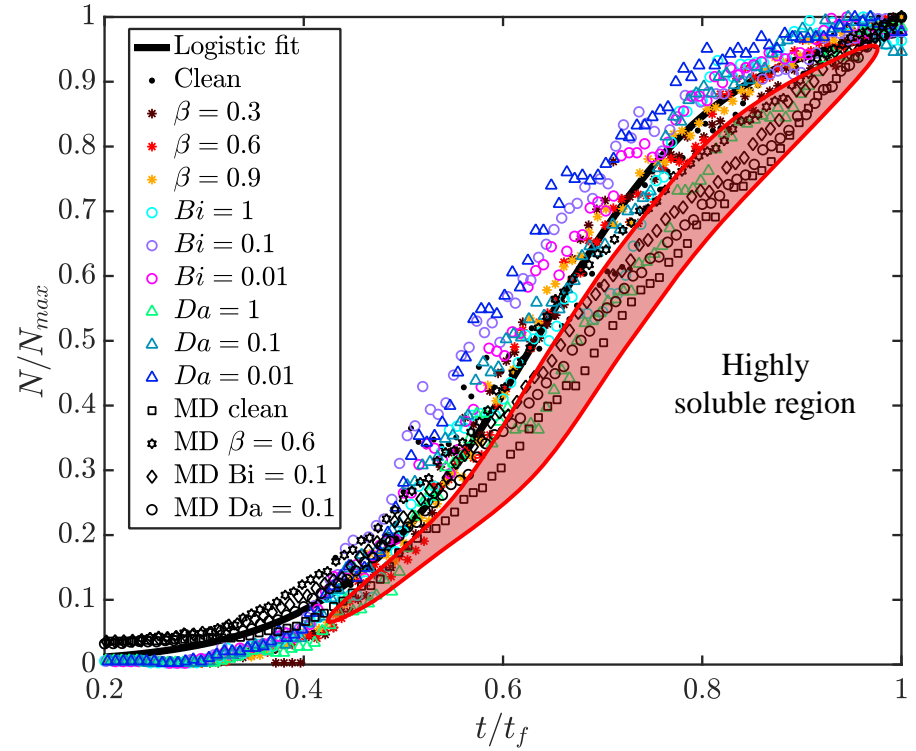
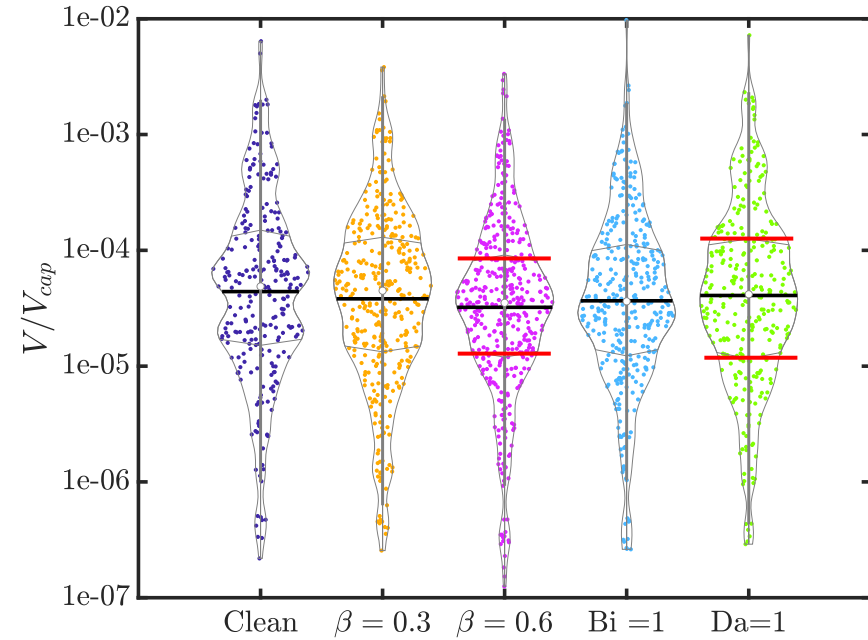
# Static Mixers

## Parametric Study



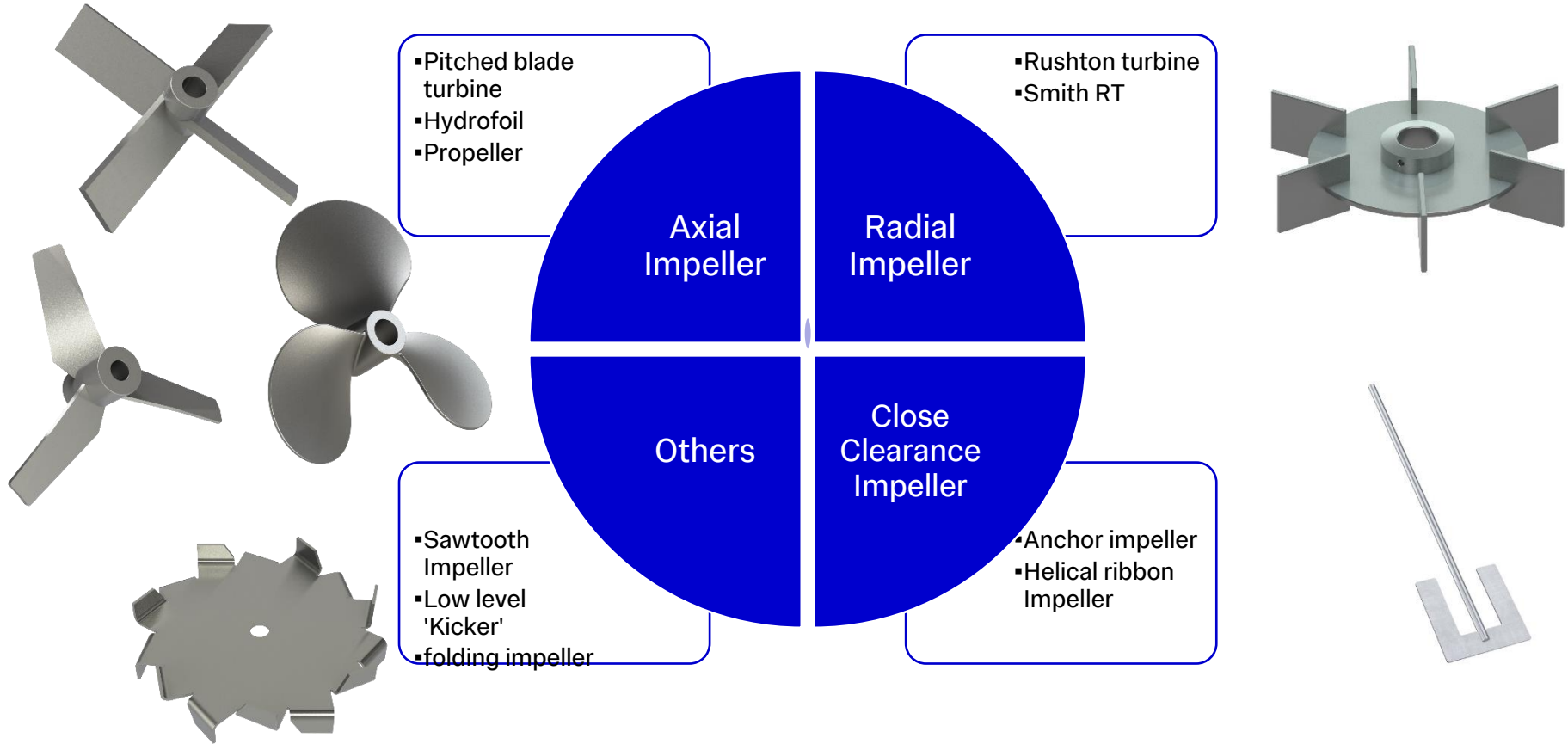
# Static Mixers

## Parametric Study



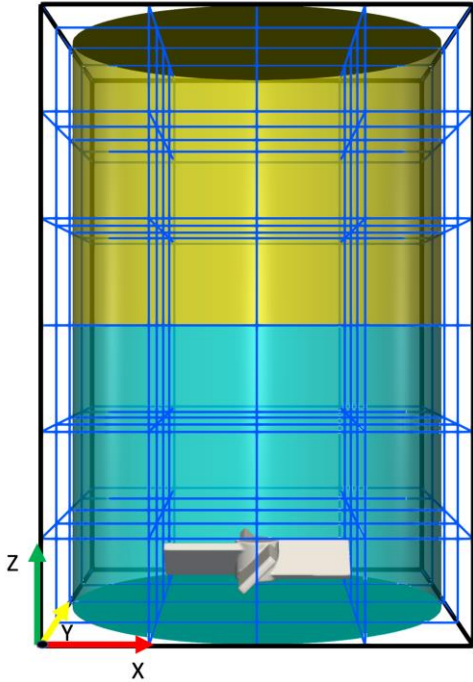
# Stirred Vessels

## Impeller Types



# Stirred Vessels

## Simulation Setup

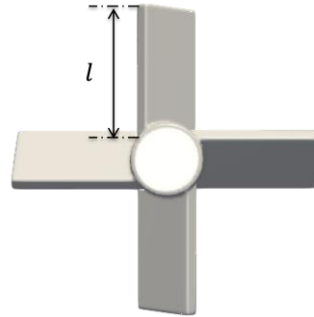
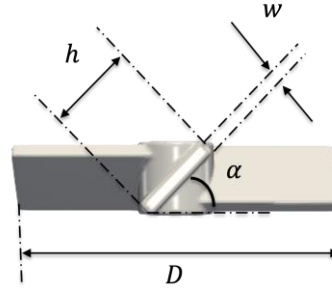


### Tank dimensions [cm]

Diameter $T$	8.5
Height, $H$	12.75

### Computational resolution

64x64x64 meshes in  
4x4x6 subdomain

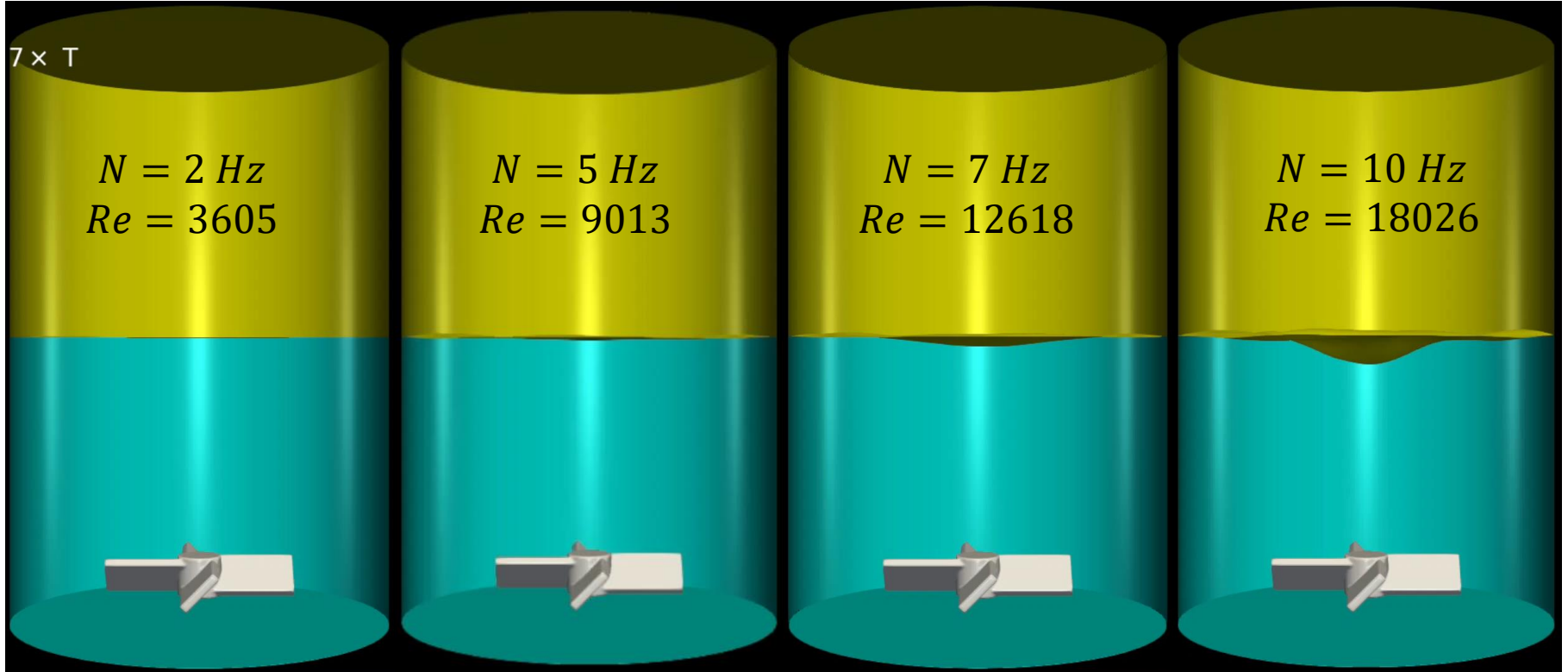


Impeller Geometry [cm]	
diameter, $D$	4.25
height, $h$	1
thickness, $w$	0.2
length, $l$	2.5
clearance, $C$	1
inclined angle, $\alpha$ [°]	45
rotation speed, $N$ [rps]	5
Physical properties	
surfactant-free interfacial tension, $\sigma_s$	0.035
oil viscosity, $\mu_o$ [Pa · s]	$5.4 \times 10^{-3}$
water viscosity, $\mu_w$	$1.0 \times 10^{-3}$
oil density, $\rho_o$ [kg/m <sup>3</sup> ]	824
water density, $\rho_w$	998



# Stirred Vessels

## Effect of Impeller Speed



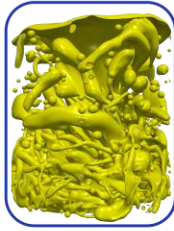


# Stirred Vessels

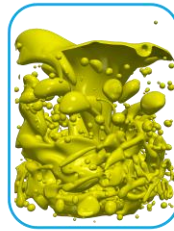
## Drop Size Distributions



$f = 10 \text{ Hz}, Re = 18026$



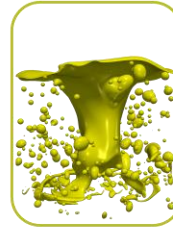
$f = 9 \text{ Hz}, Re = 16224$



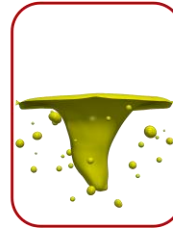
$f = 8 \text{ Hz}, Re = 14421$



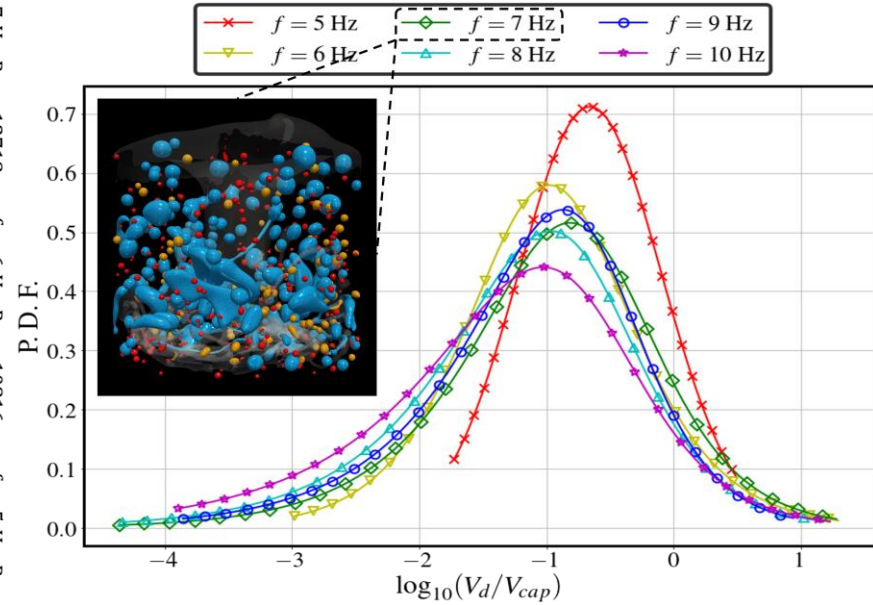
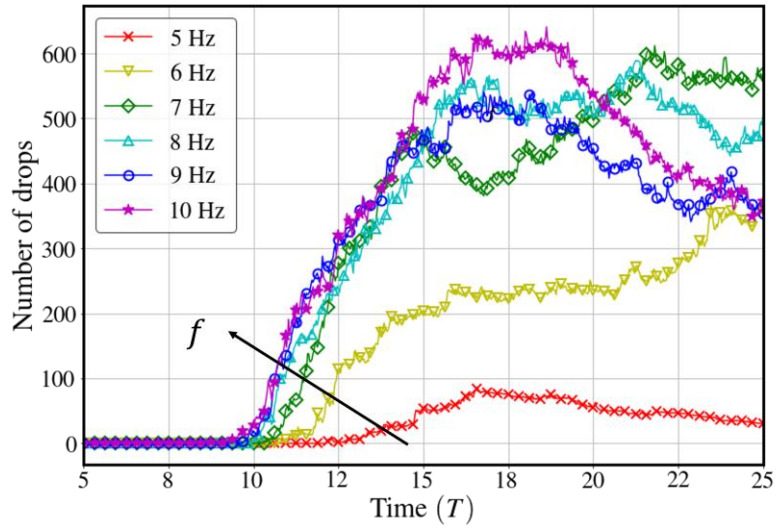
$f = 7 \text{ Hz}, Re = 12718$



$f = 6 \text{ Hz}, Re = 10816$



$f = 5 \text{ Hz}, Re = 9031$



# Stirred Vessels

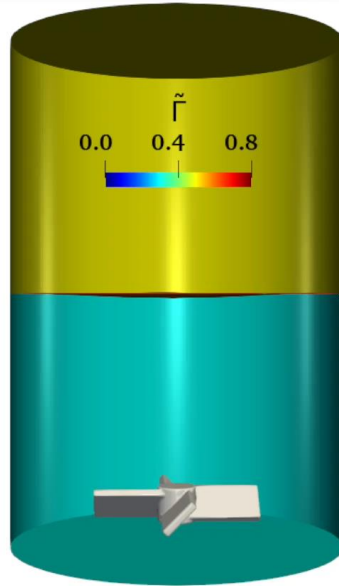
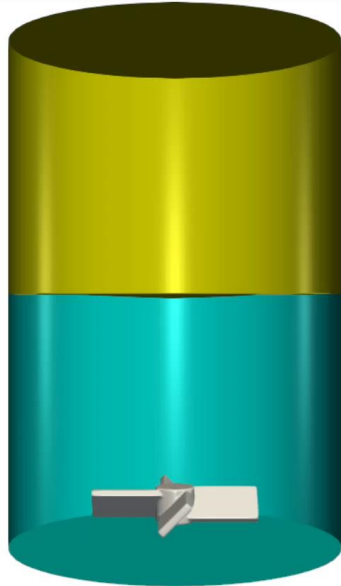
## Effect of Surfactant on Interfacial Dynamics

$$5 \times T$$

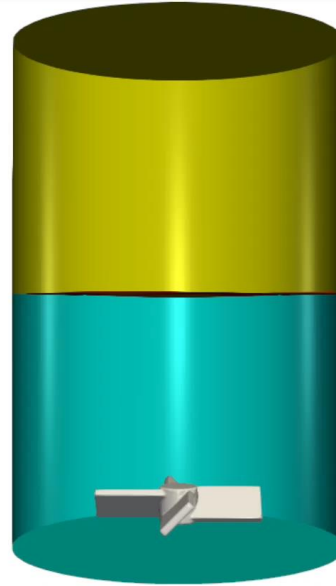
Surfactant-free

Surfactant-laden I

Surfactant-laden II



$$|\tau_M| = 0$$



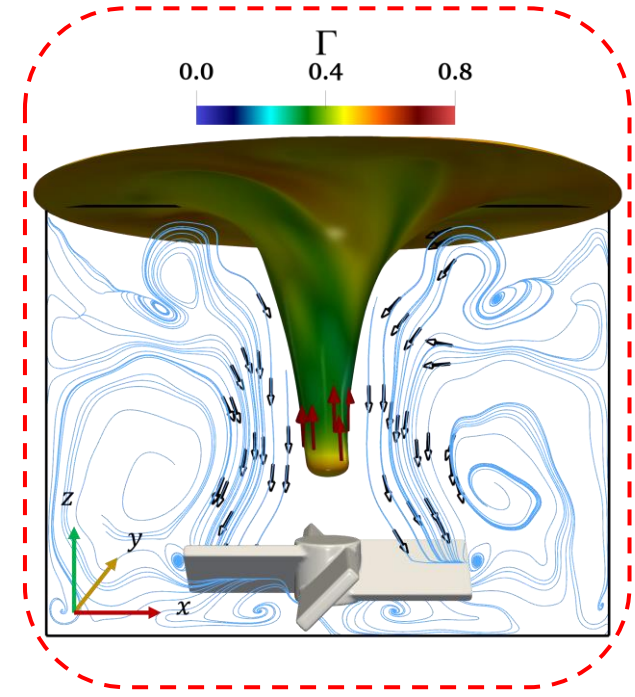
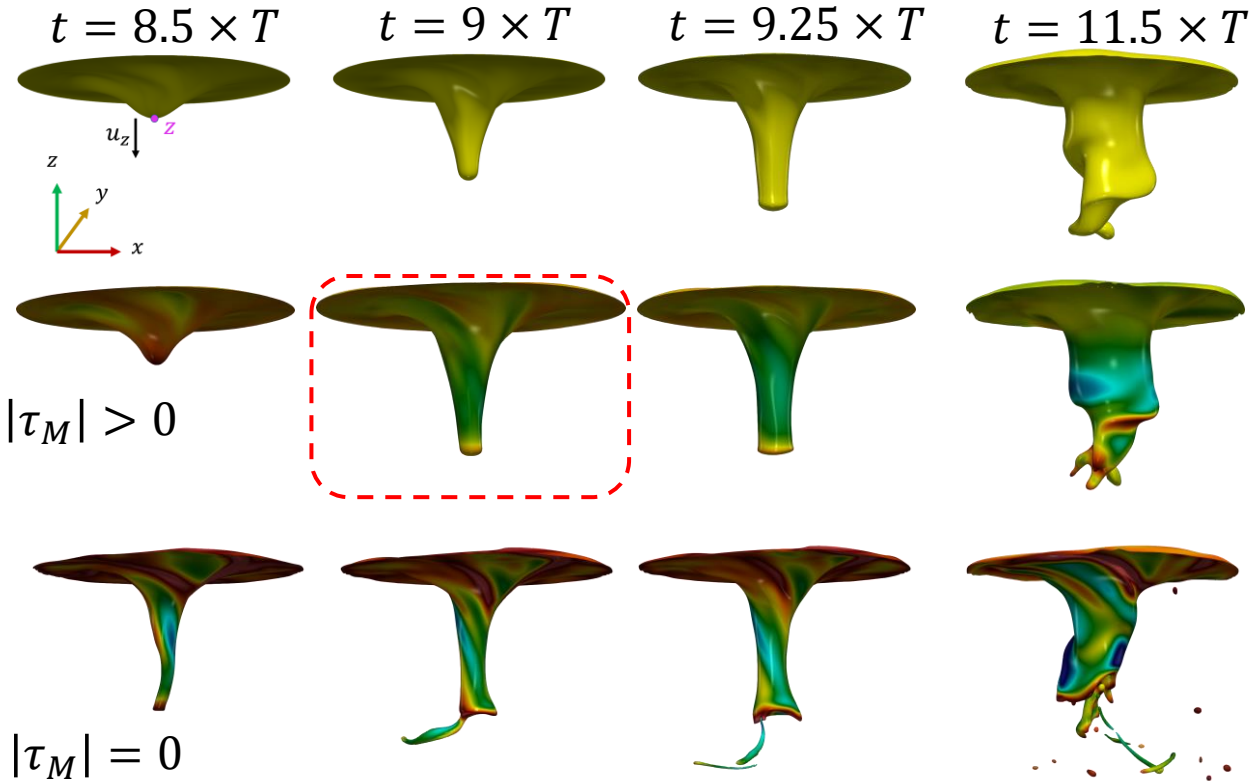
**Step 1**  
Interface deformation

**Step 2 & 3**  
Ligament and drop formation

**Step 4**  
Cessation of interface-impeller contact

# Stirred Vessels

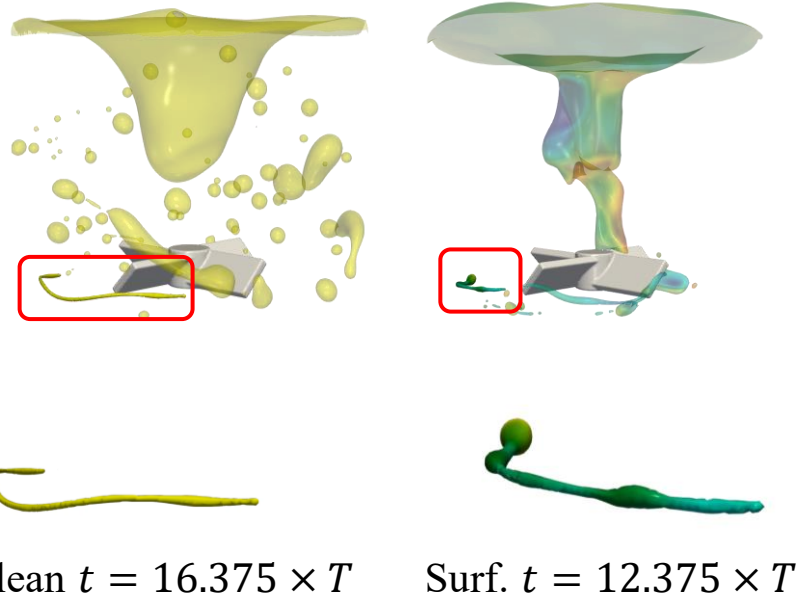
## Step 1: Interface Deformation



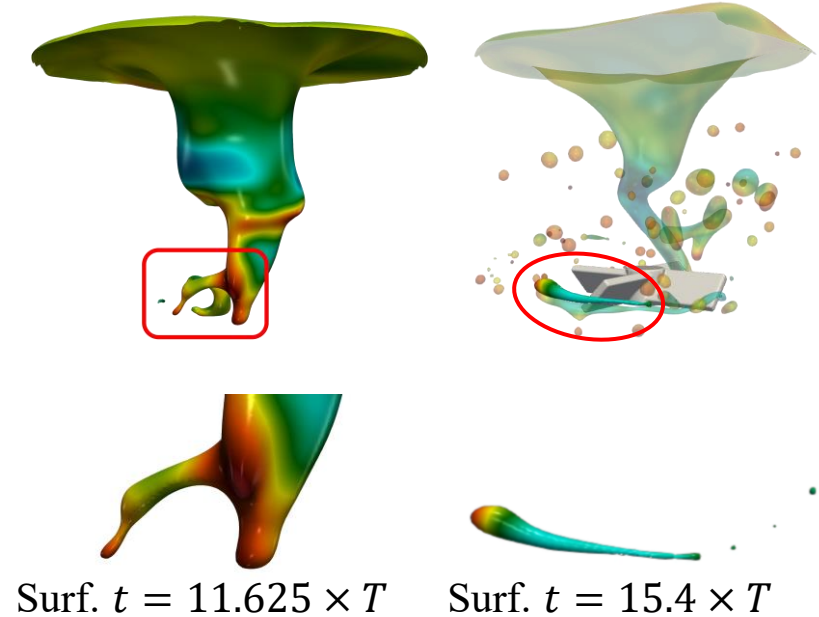
# Stirred Vessels

## Steps 2 & 3: Drop Dispersion

Mechanism 1:  
Capillary instability

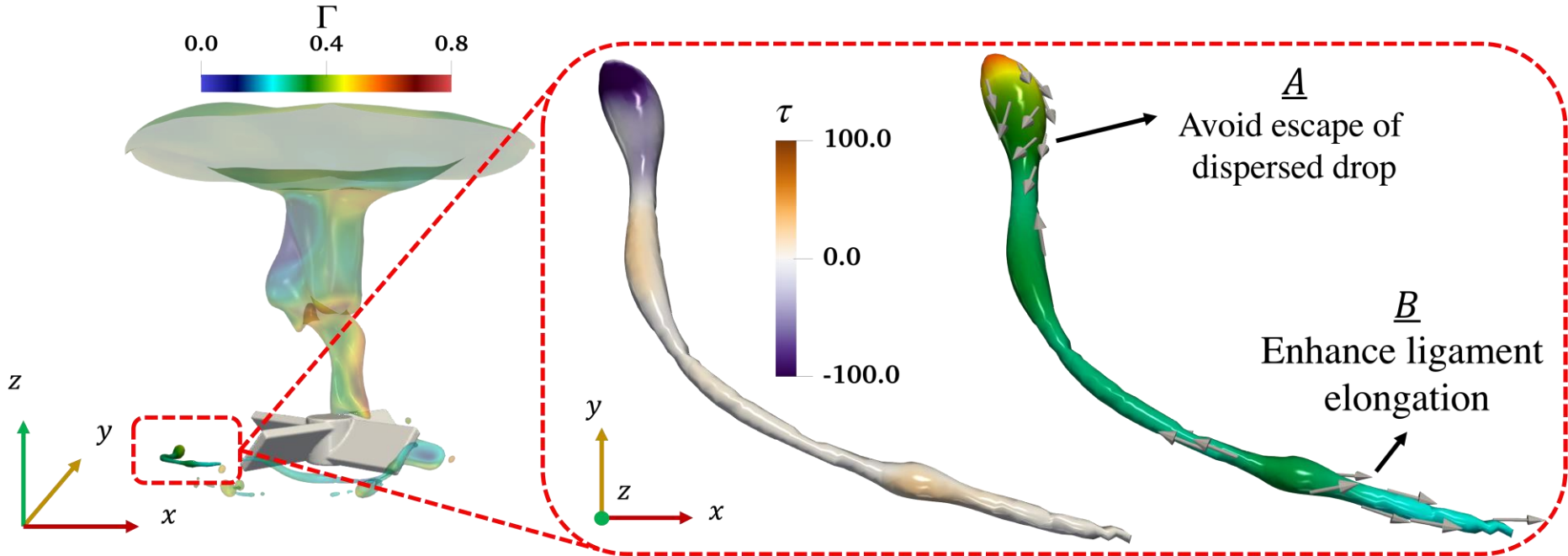


Mechanism 2:  
Surfactant-enhanced tip streaming



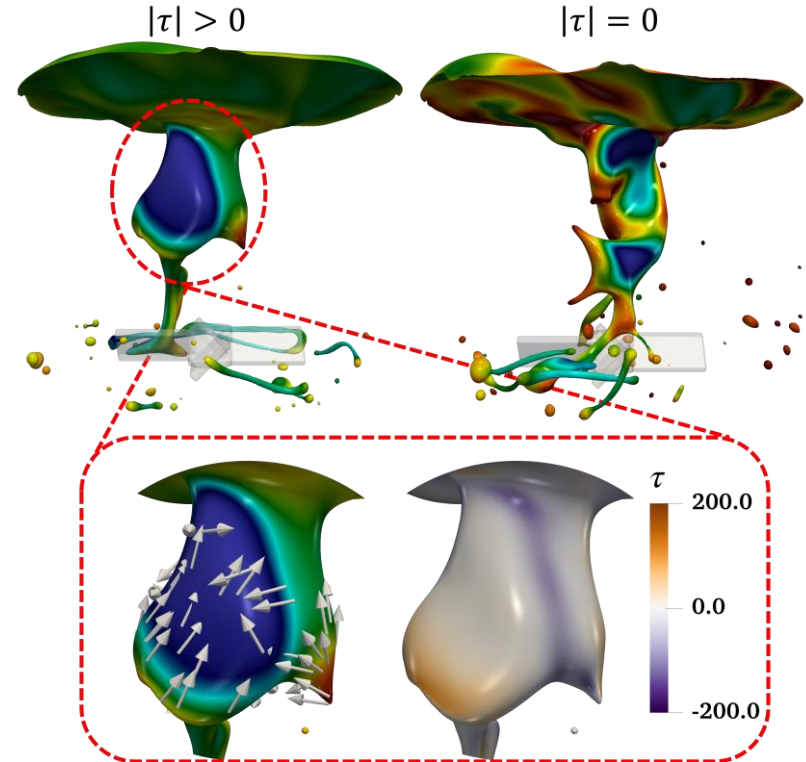
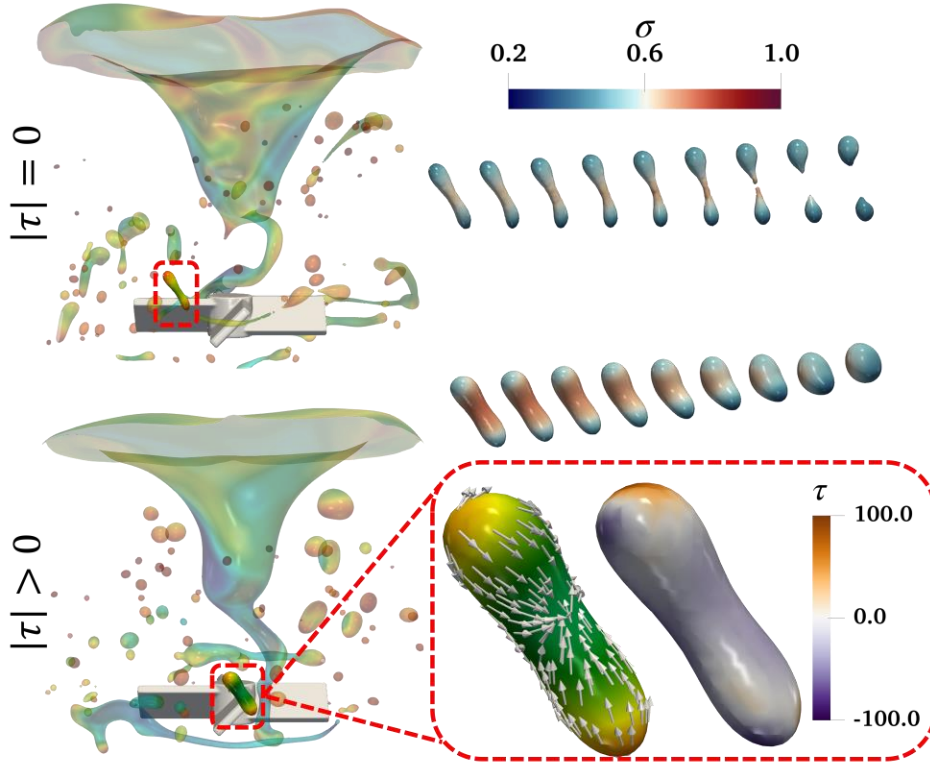
# Stirred Vessels

## Steps 2 & 3: Marangoni Effects



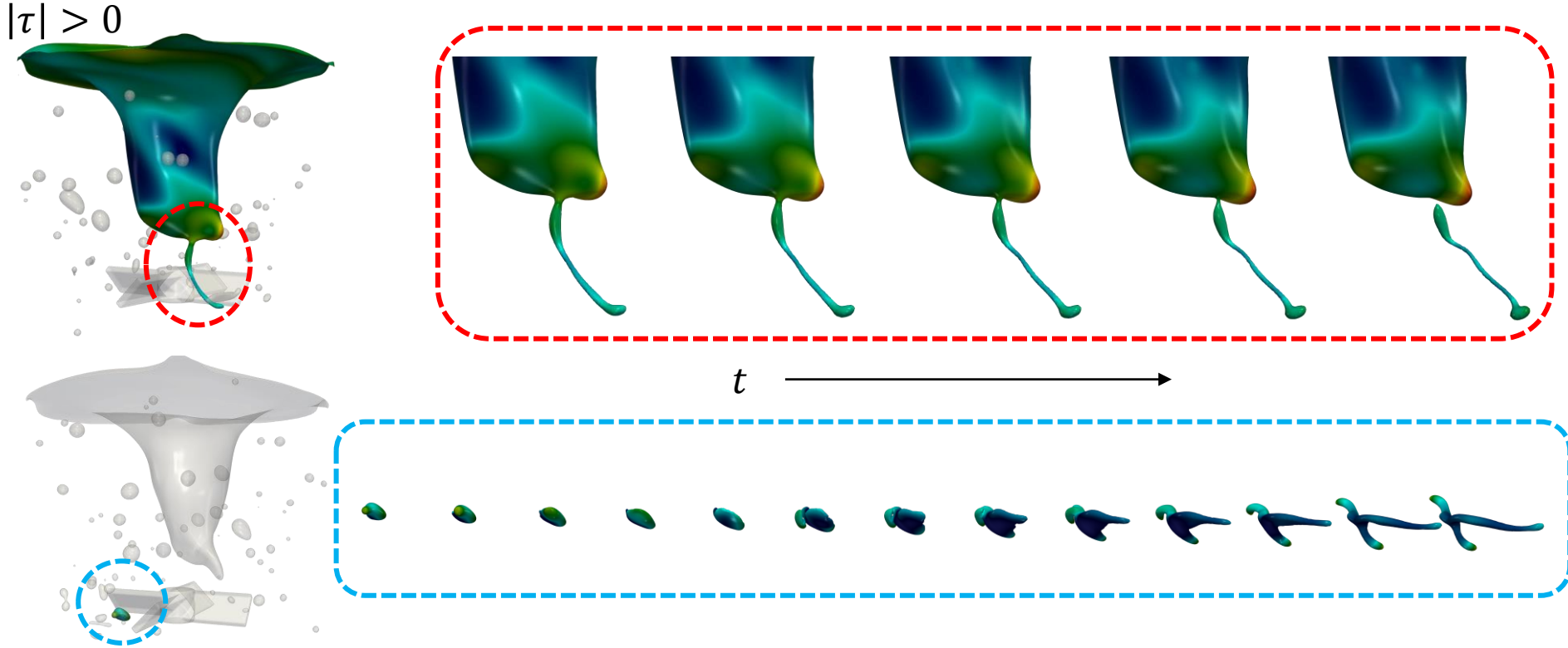
# Stirred Vessels

## Steps 2 & 3: Marangoni Effects



# Stirred Vessels

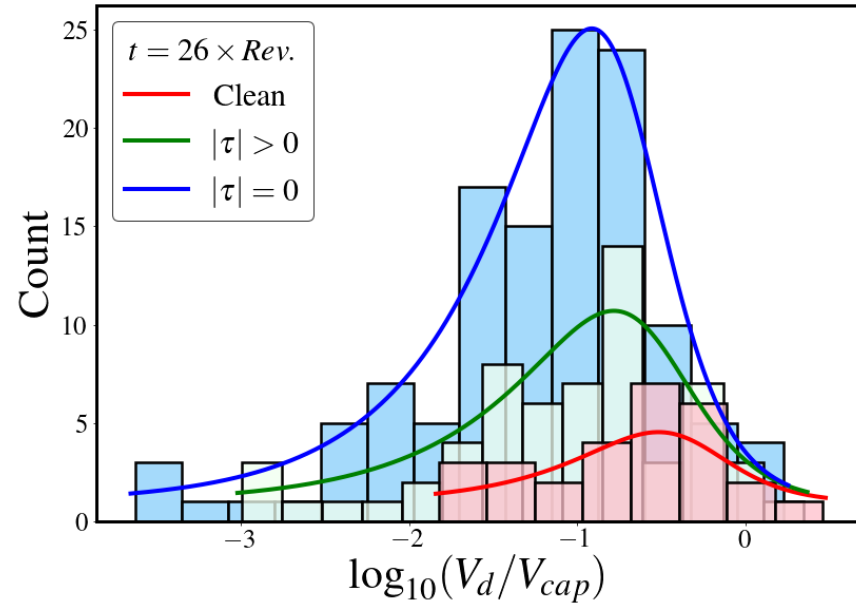
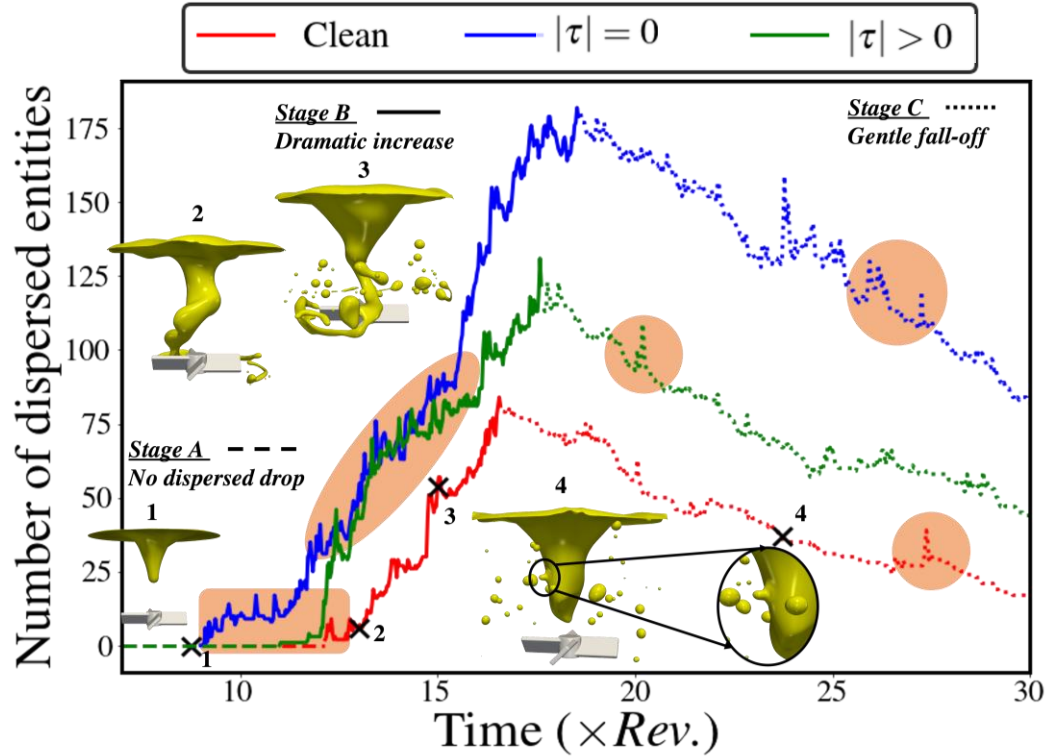
## Steps 4: Cessation of Interface-Impeller Contact





# Stirred Vessels

## Drop Number & Size Distribution





# Acknowledgements

