ERCOFTAC Autumn Festival 2024

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New insights on the self-similar behaviour of turbulent flows

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Imperial College London

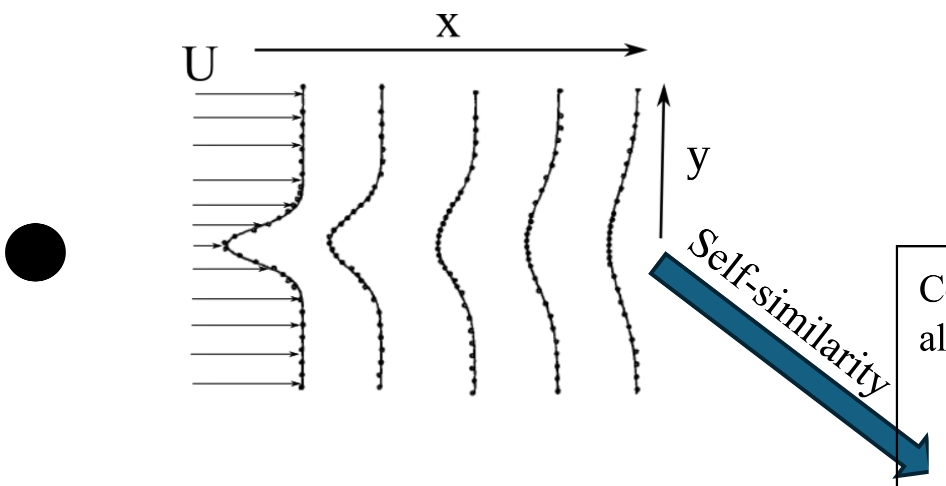
Self-similarity in fluid mechanics

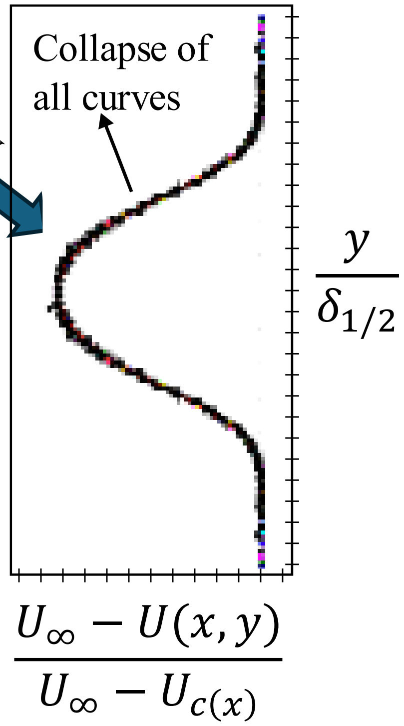
$$\boldsymbol{A}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{A}^{*}(\boldsymbol{x})\boldsymbol{f}(\frac{\boldsymbol{y}}{\boldsymbol{l}(\boldsymbol{x})})$$

Self-similarity in fluid mechanics

$$A(x,y) = A^*(x) f(\frac{y}{l(x)})$$

Turbulent Wake





$$\underbrace{A(x,y)}_{U_\infty - U(x,y)} = \underbrace{A^*(x,y)}_{(U_\infty - U_c(x))} \underbrace{f(\frac{y}{l(x)})}_{f(y/\delta_{1/2})}$$

$$U_\infty - U(x,y) = (U_\infty - U_c(x)) f(y/\delta_{1/2})$$

Pope 2002

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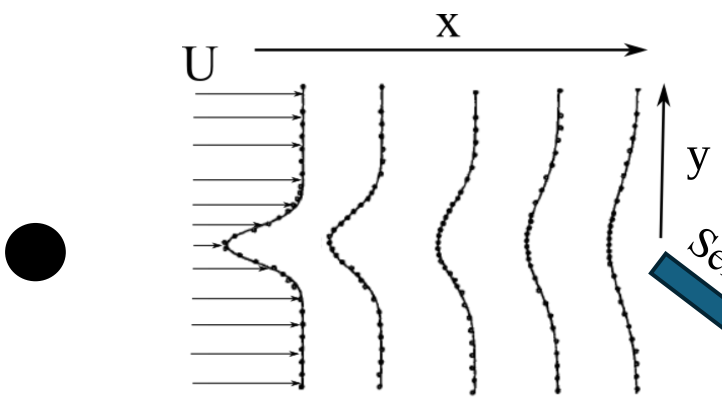

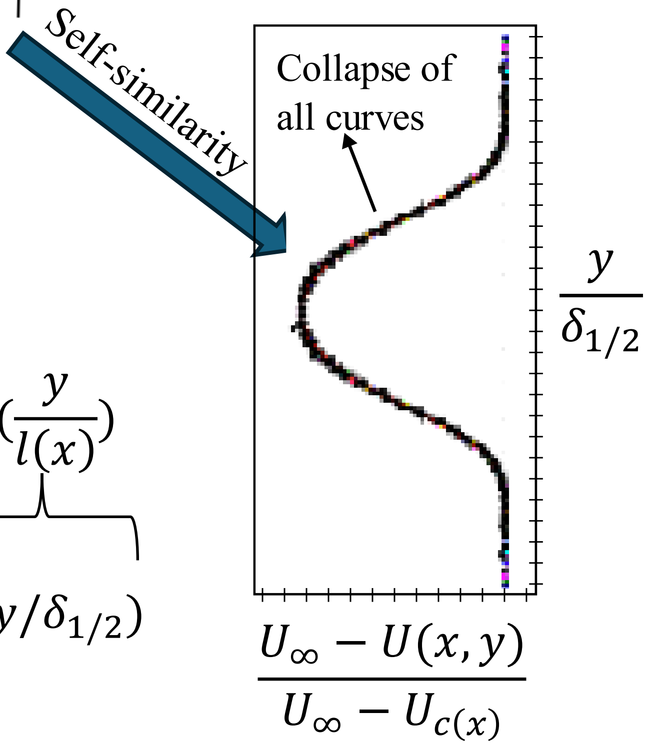

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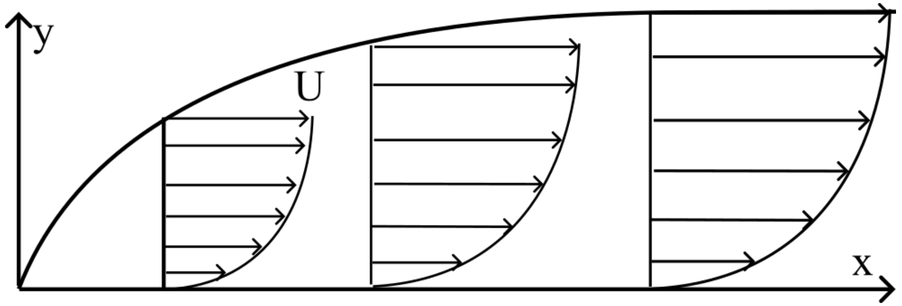
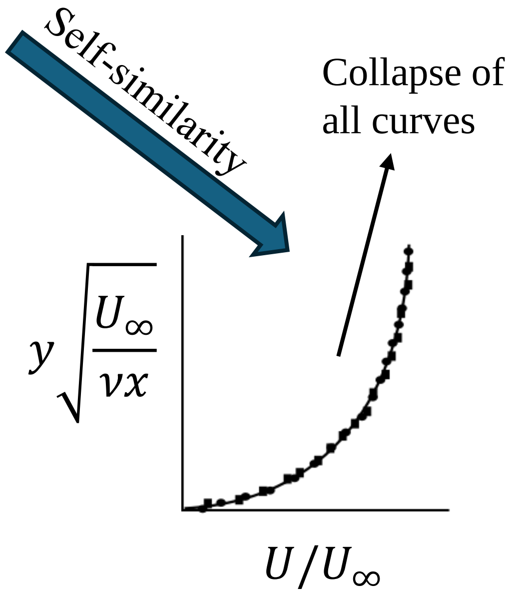
Pope 2002

Laminar Boundary Layer





$$\frac{U}{U_\infty} = f\left(y\sqrt{\frac{U_\infty}{\nu x}}\right)$$

Blasius (1908)

Self-similarity in fluid mechanics

$$\boldsymbol{A(x,y) = A^*(x) f(\frac{y}{l(x)})}$$

- Reduction of the variables of pde's (pde ➔ ode)

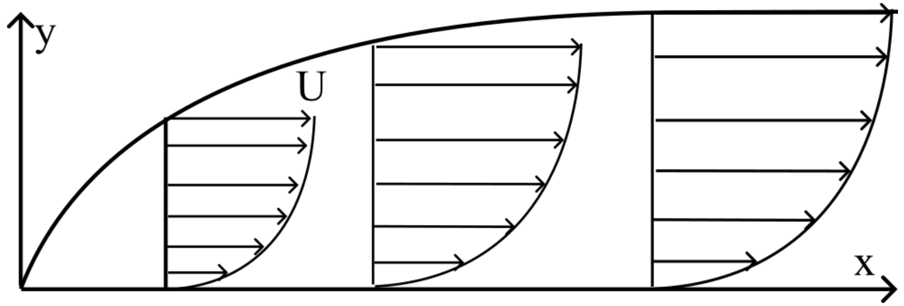
$$\Psi_y \Psi_{xy} - \Psi_x \Psi_{yy} - \nu \Psi_{yyy}$$

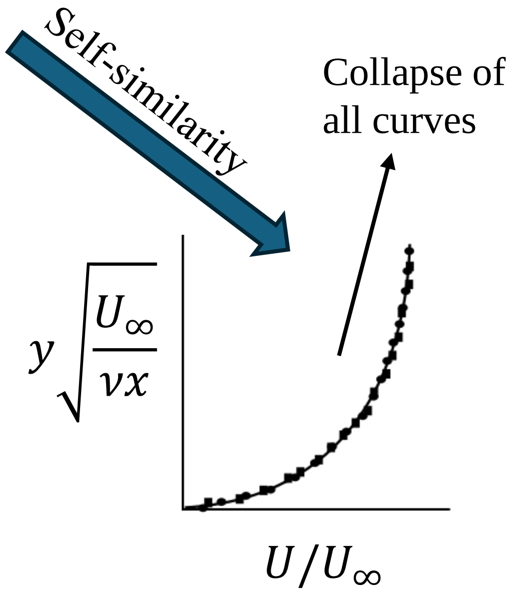
Blasius equation ➡

$$2f''' + ff'' = 0$$

Blasius (1908)

Laminar Boundary Layer





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Blasius (1908)

- 'Balance' of the dynamic equations

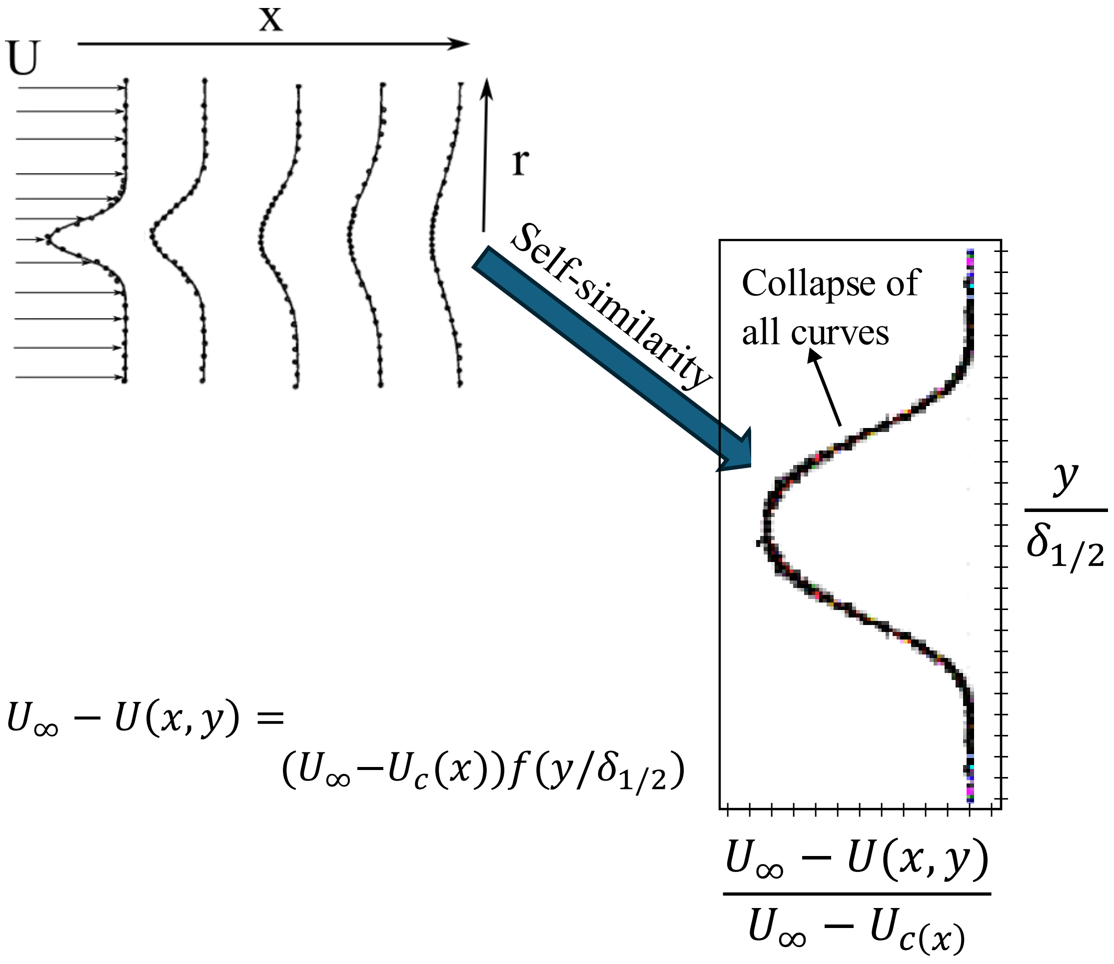
$$UU_x + VU_y = -(\overline{u'v'})_y$$

Plug-in of self-similar forms ➔

$$\left[-\frac{U_\infty L}{U_s^2}\frac{dU_s}{dx}\right] f + \left[\frac{U_0}{U_s}\frac{dL}{dx}\right]\xi f' = [1]\, g'$$

Townsend (1976)

Turbulent Wake



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- Concept of eddy viscosity $\overline{u'v'} = -\nu_T U_y$

Pope (2002)

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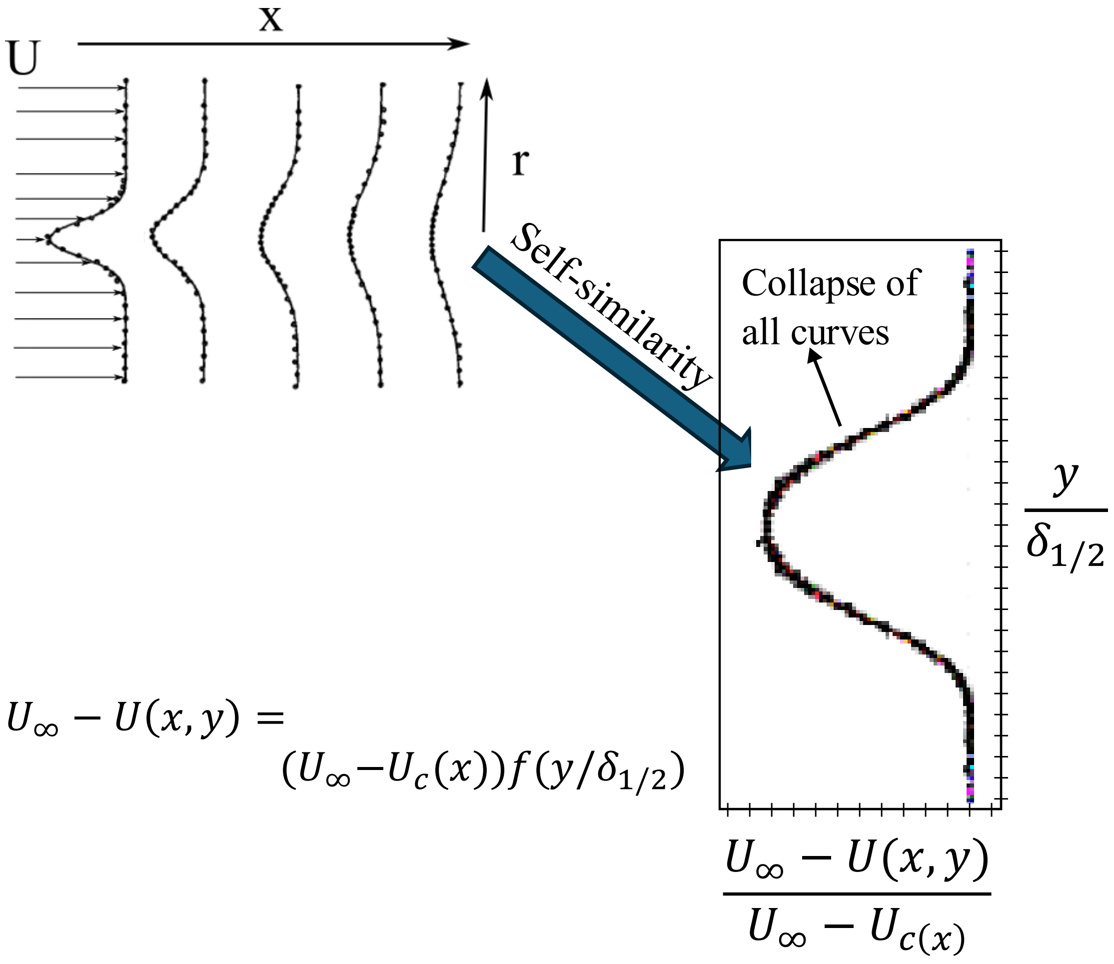


$$U_\infty - U(x,y) = (U_\infty - U_c(x)) f(y/\delta_{1/2})$$

Outline

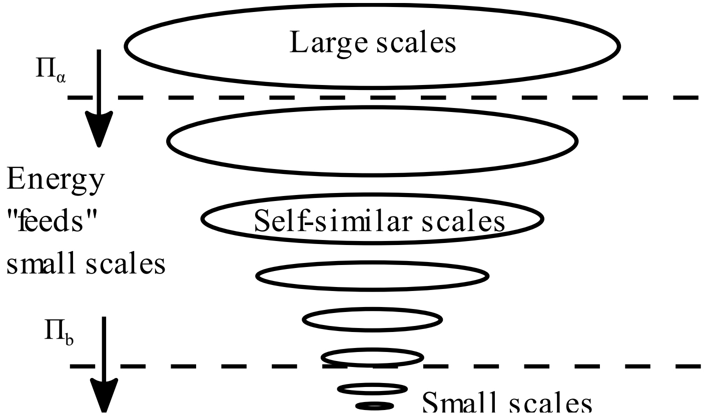
- Large-scale correction to K41 using self-similar dynamics
- Data-driven extraction of self-similarity

K41 framework

General Energy Budget

K41 Assumptions

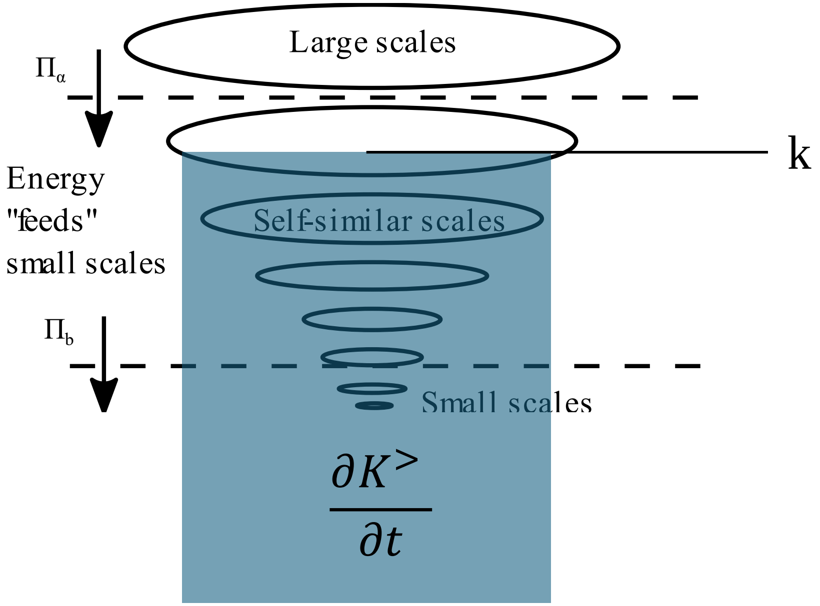
$$\frac{\partial K^{>}}{\partial t} = \Pi - \epsilon^{>}$$



Kuchler *et al.* 2023
Kolmogorov (1941)

K41 framework

General Energy Budget	K41 Assumptions
High-pass Kinetic Energy $K^{>} = \int_{k}^{\infty} E dk$ ↑ $\frac{\partial K^{>}}{\partial t} = \Pi - \epsilon^{>}$	



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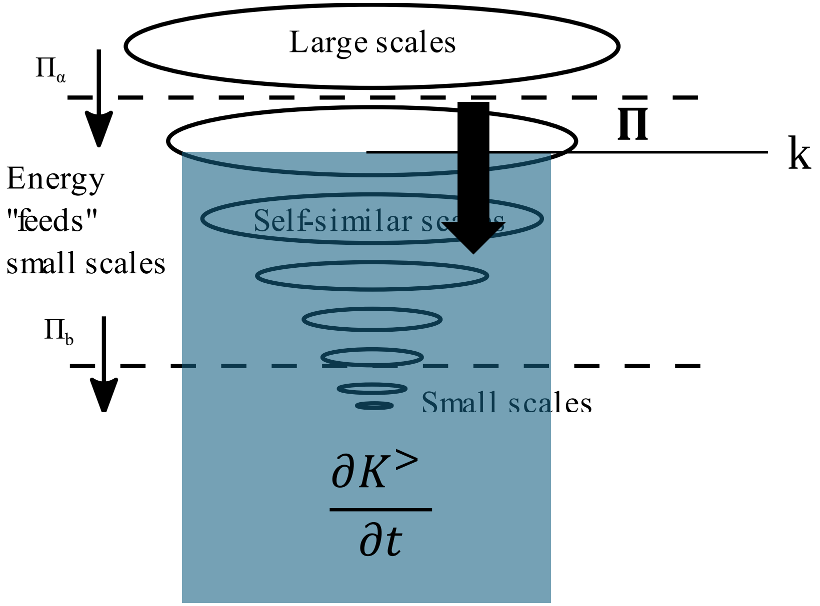

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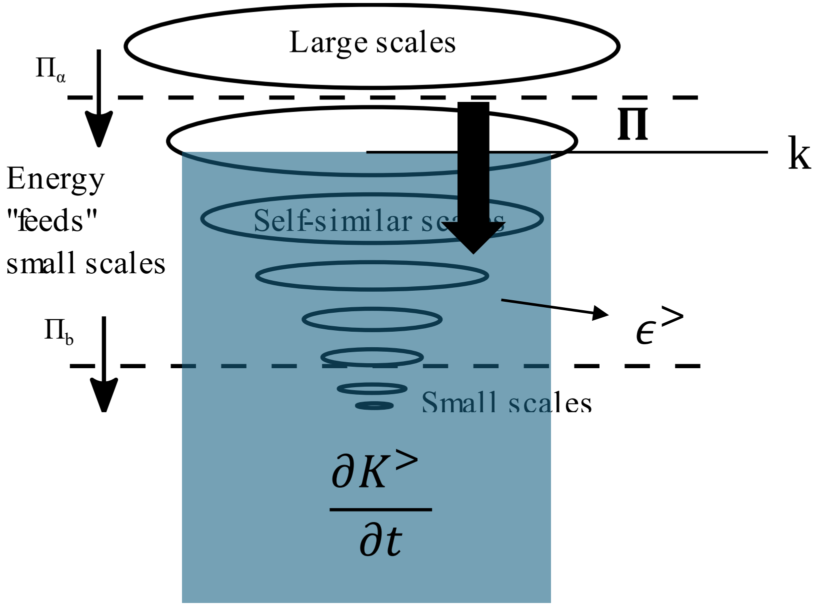

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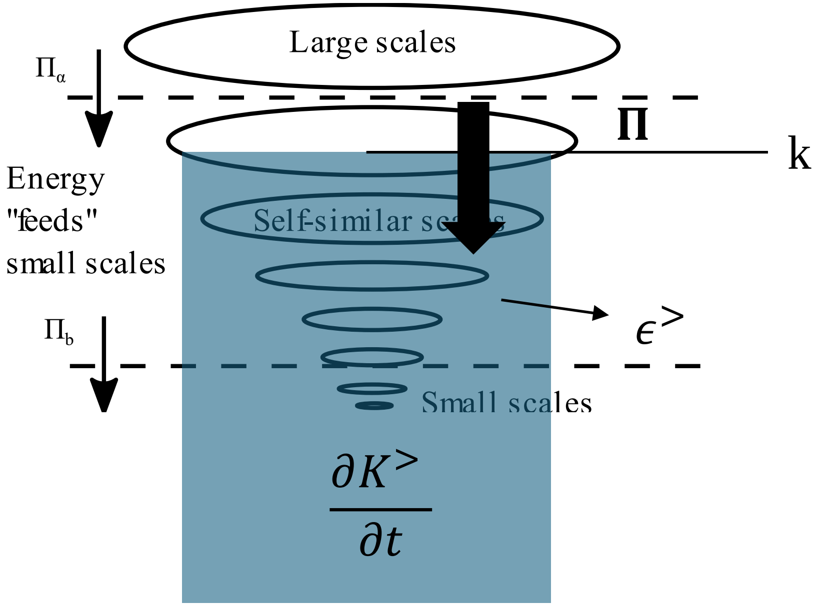
1) Universal Equilibrium Range

$$Re \to \infty, \quad kL \to \infty \longrightarrow \frac{\partial K^{>}}{\partial t} \approx 0$$

2) "Away" from dissipative eddies

$$k\eta \to 0 \longrightarrow \epsilon^{>} \approx \epsilon$$



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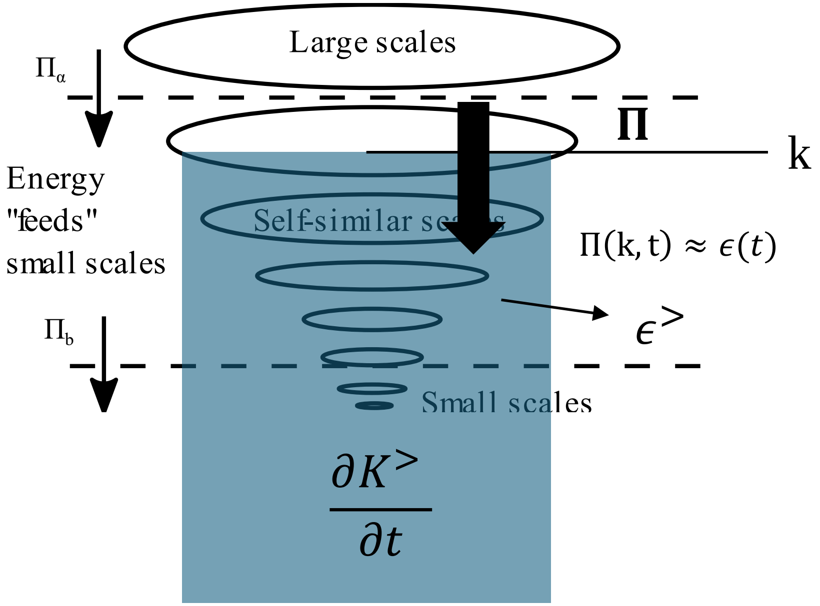

$$\boldsymbol{\Pi(k, t) \approx \epsilon(t)}$$

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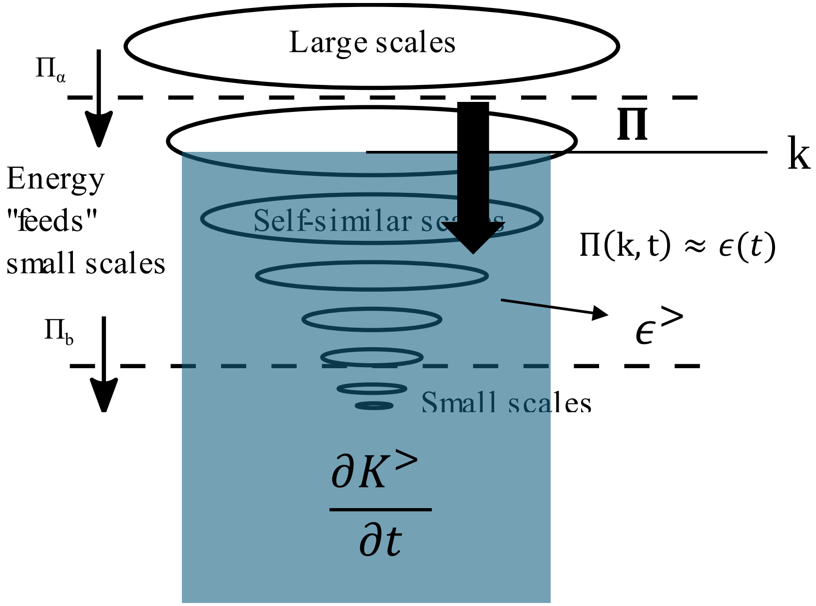

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-5/3 law

$$E \propto \Pi^{2/3} k^{-5/3}$$

Kuchler *et al.* 2023
Kolmogorov (1941)

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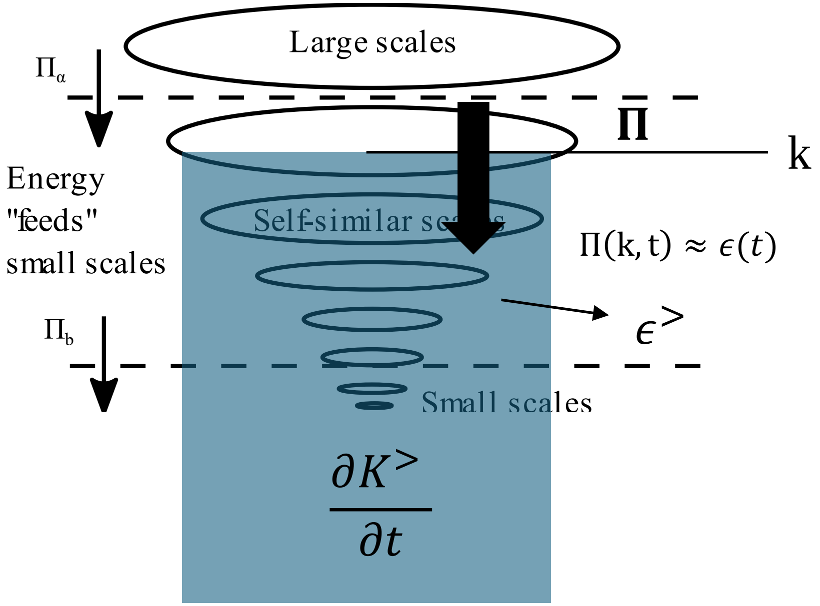

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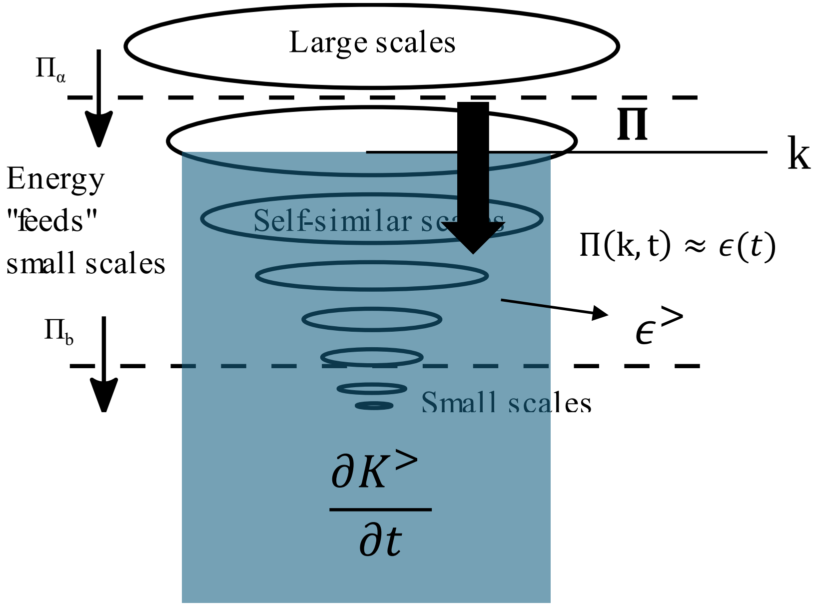

-5/3 law

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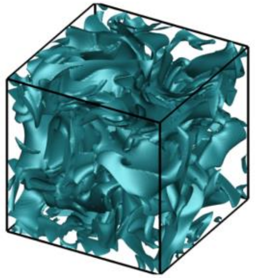
What happens at larger, out-of-equilibrium eddies?

Kuchler *et al.* 2023
Kolmogorov (1941)

Balanced cascade assumption

Steiros (2022) PRE



Self-similarity

$$\boldsymbol{\Pi(\kappa, t) = g(\kappa)\epsilon(t)}$$

With κ= kL

Balanced cascade assumption

Steiros (2022) PRE

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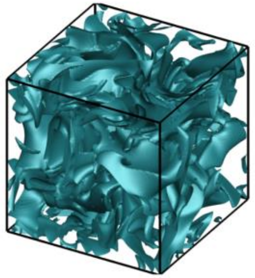
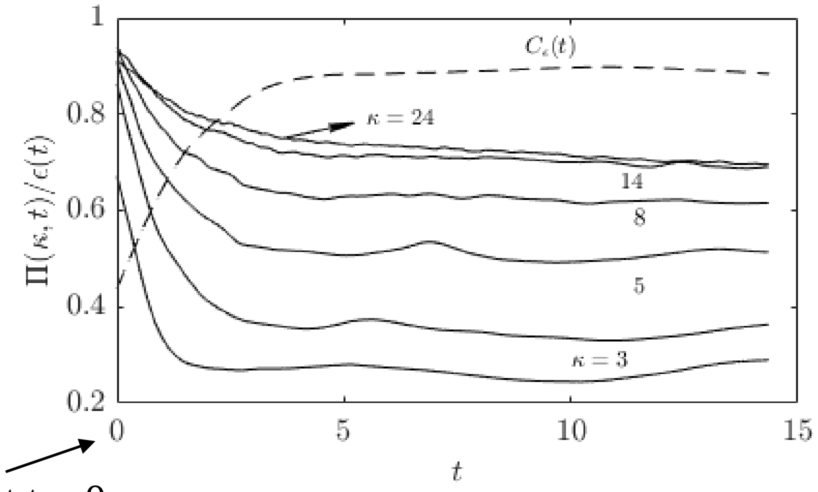


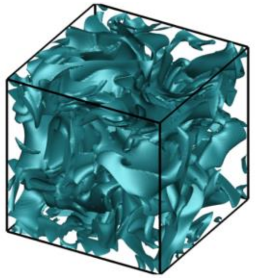


Forcing stops at t = 0

at t > 3 ➔ Π/ϵ = const
Non-equilibrium

Balanced cascade assumption

Steiros (2022) PRE



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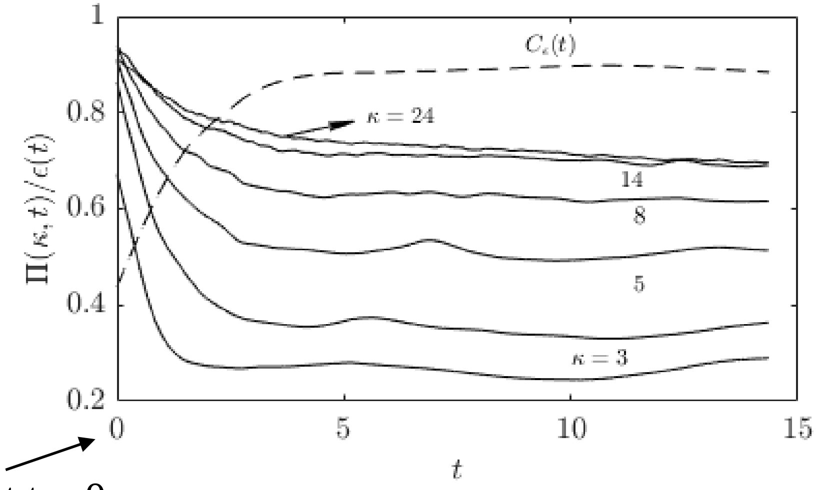

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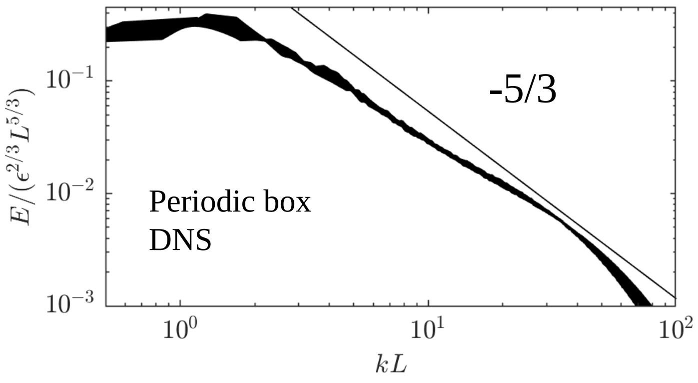
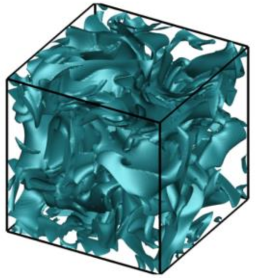


Repercussion: A correction to -5/3 for non-equilibrium

$$E \propto \Pi^{2/3}k^{-5/3} \rightarrow \frac{E}{\epsilon^{2/3}L^{2/3}} \propto \underbrace{\kappa^{-5/3}}_{\text{K41}} \underbrace{g^{2/3}(\kappa)}_{\text{Large scale correction}}$$

Balanced cascade assumption

Steiros (2022) PRE



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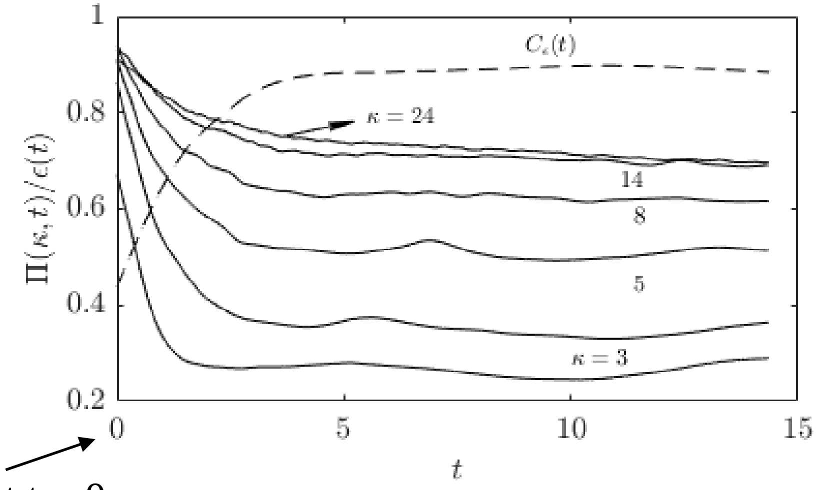

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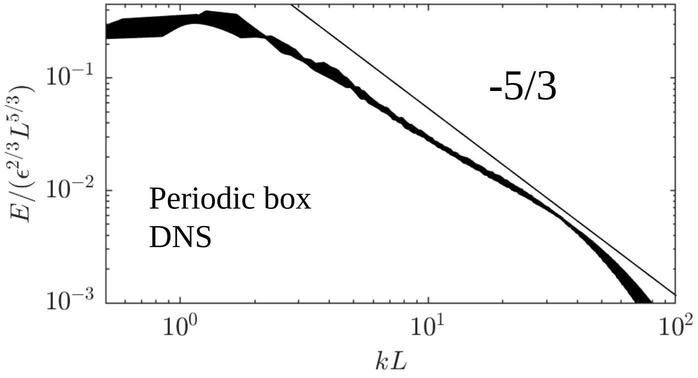

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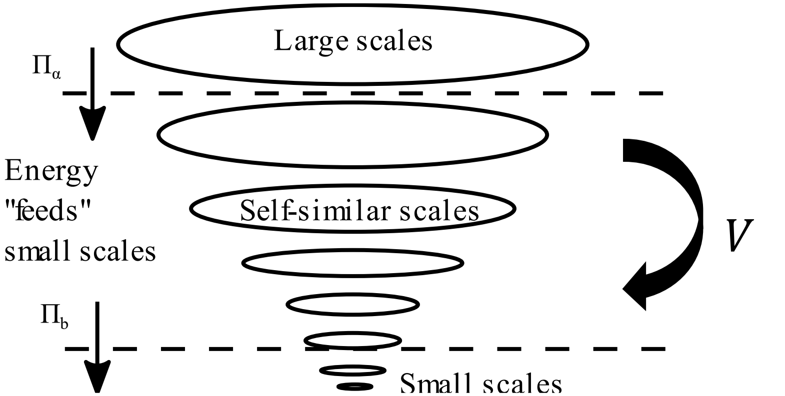
How to calculate g?

A constraint to the cascade

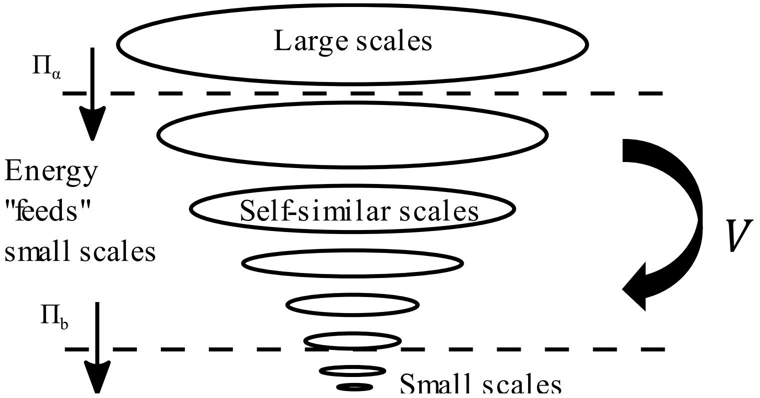
Steiros (2022) PRE



Lumley (1992):
No production or dissipation
means simple transport of energy

A constraint to the cascade

Steiros (2022) PRE



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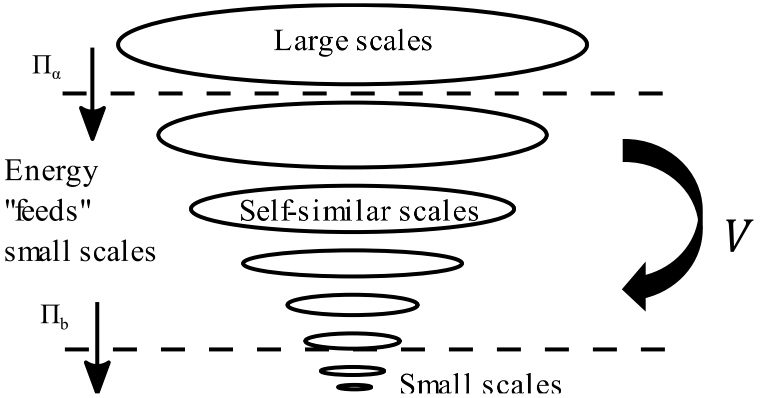
With κ= kL

A constraint to the cascade

Steiros (2022) PRE



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Can we estimate V? ➔ $\frac{dk}{dt} = C_v \Pi^{1/3} k^{5/3}$

Pao (1965)

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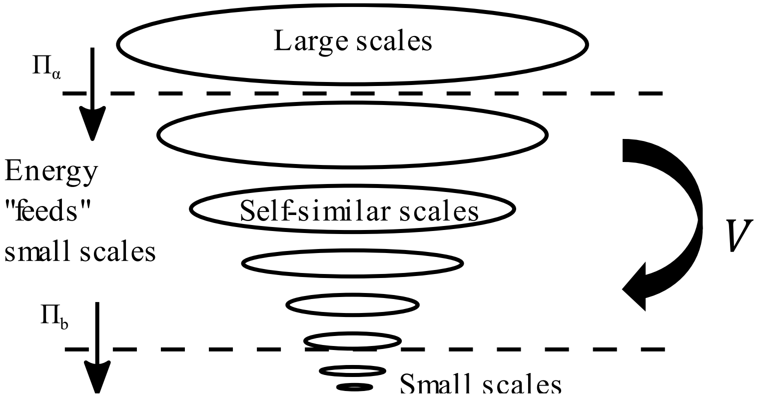
Steiros (2022) PRE



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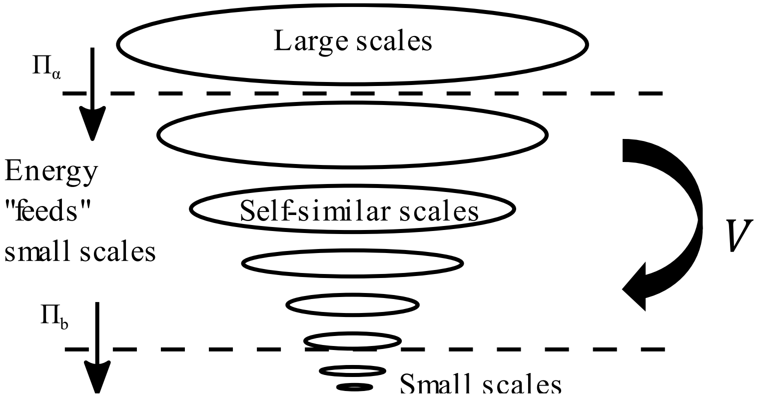

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Can we estimate V? ➔ $$\frac{dk}{dt} = C_v \Pi^{1/3} k^{5/3} \quad (2)$$

Pao (1965)

$$\Pi = g(\kappa)\epsilon(t) \quad (3)$$

A constraint to the cascade

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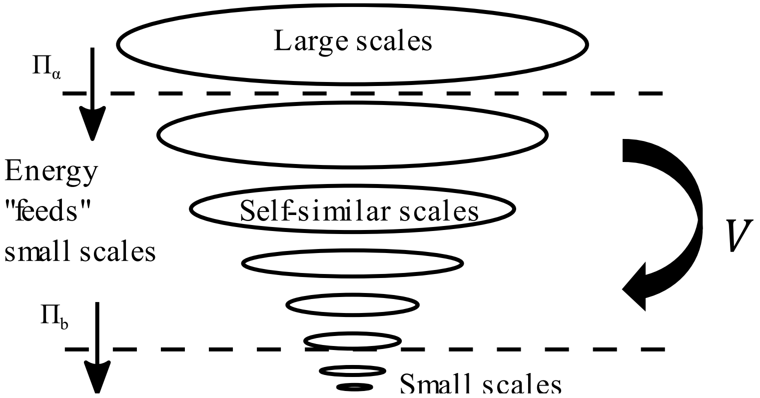

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With κ= kL

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Pao (1965)

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$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = -\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}}\frac{g'(\kappa)}{g^{2/3}(\kappa)}\kappa^{5/3} = -C_0$$

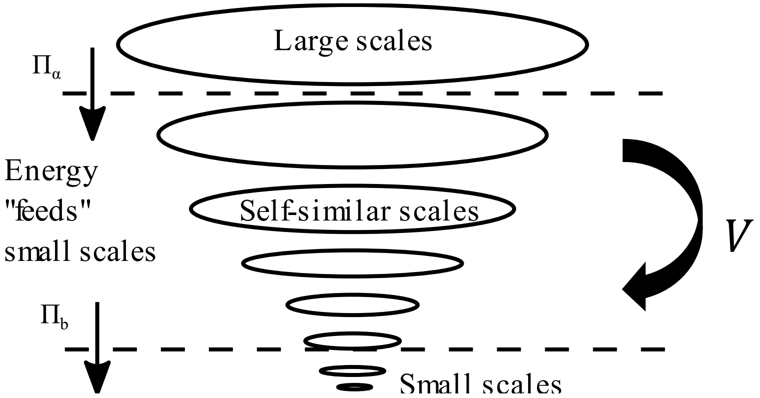
Function of time only

Function of normalized wavenumber only

Constant

A constraint to the cascade

Steiros (2022) PRE



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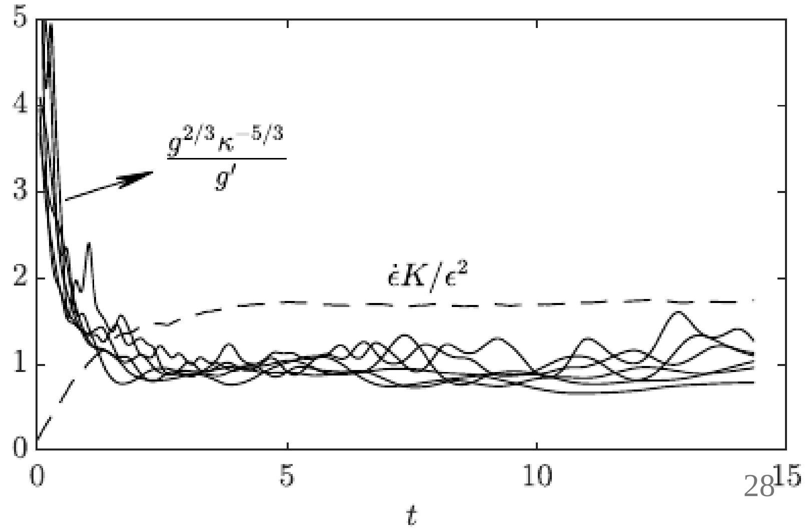
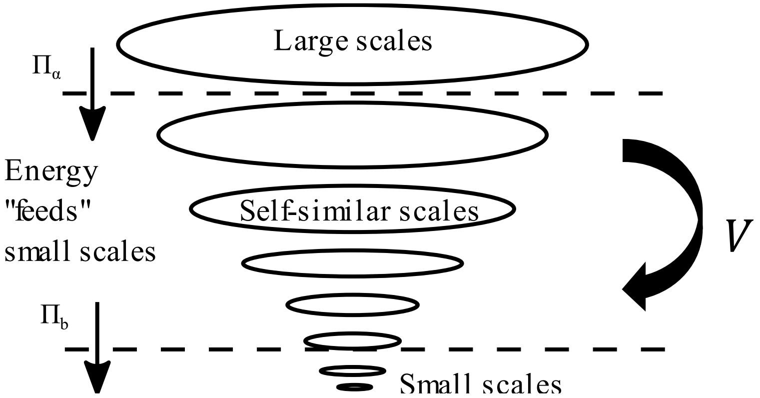


Forcing stops at t = 0

A constraint to the cascade

Steiros (2022) PRE



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$$\frac{D\Pi}{Dt} = \frac{\partial \Pi}{\partial t} + V\frac{\partial \Pi}{\partial \kappa} = 0 \quad (1)$$

With κ= kL

Can we estimate V? ➔ $$\frac{dk}{dt} = C_v \Pi^{1/3} k^{5/3} \quad (2)$$

Pao (1965)

$$\Pi = g(\kappa)\epsilon(t) \quad (3)$$

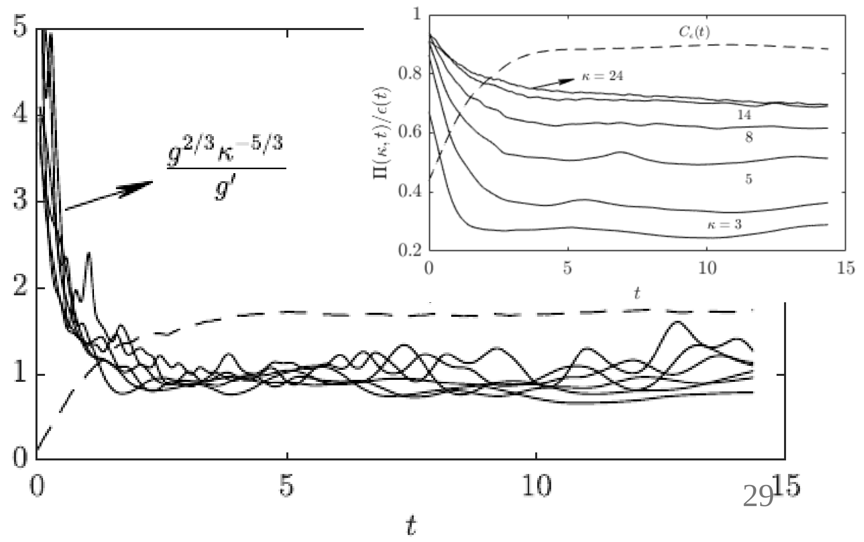
$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = -\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}}\frac{g'(\kappa)}{g^{2/3}(\kappa)}\kappa^{5/3} = -C_0$$

Function of time only

Function of normalized wavenumber only

Constant

Validation



Forcing stops at t = 0

Result: -5/3 correction

Steiros (2022) PRE

$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = \boxed{-\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}} \quad \frac{g'(\kappa)}{g^{2/3}(\kappa)} \quad \kappa^{5/3} = -C_0}$$

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$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = \boxed{-\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}} \quad \frac{g'(\kappa)}{g^{2/3}(\kappa)} \quad \kappa^{5/3} = -C_0} \quad \Longrightarrow \quad g = \left[1 - \frac{C_0 C_\epsilon^{2/3}}{3C_v}\kappa^{-2/3}\right]^3$$

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Steiros (2022) PRE

$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = \boxed{-\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}}\frac{g'(\kappa)}{g^{2/3}(\kappa)}\kappa^{5/3} = -C_0}$$

$$\Rightarrow \quad g = \left[1 - \underbrace{\frac{C_0 C_\epsilon^{2/3}}{3C_v}}_{c}\kappa^{-2/3}\right]^3$$

$$\frac{E}{\epsilon^{2/3}L^{2/3}} \propto \kappa^{-5/3} g^{2/3}(\kappa)$$

$$\frac{E}{\epsilon^{2/3}L^{2/3}} \propto \kappa^{-5/3}\left(1 - c\kappa^{-2/3}\right)^2$$

With c = ½ for our DNS

Result: -5/3 correction

Steiros (2022) PRE

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→

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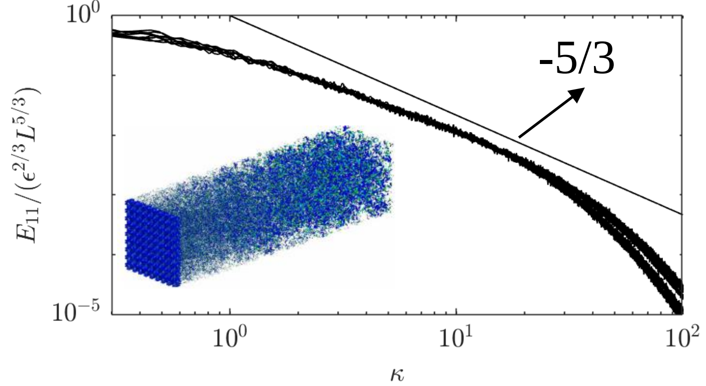
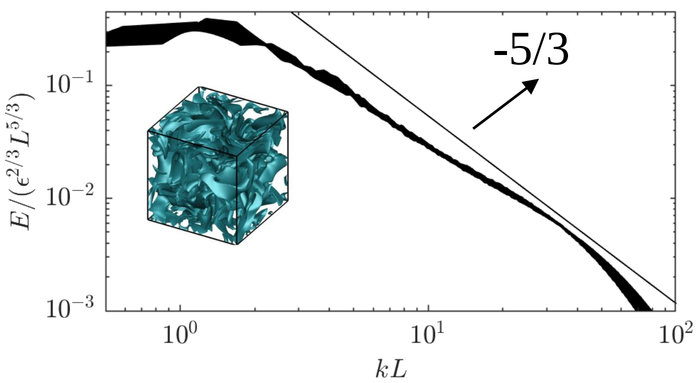
With c = ½ for our DNS

Validation

Grid turbulence (Hot wire)



Decaying turbulence (DNS)



Result: -5/3 correction

Steiros (2022) PRE

$$\frac{K}{\epsilon^2}\frac{d\epsilon}{dt} = \boxed{-\frac{3}{2}\frac{C_v}{C_\epsilon^{2/3}}\frac{g'(\kappa)}{g^{2/3}(\kappa)}\kappa^{5/3} = -C_0}$$

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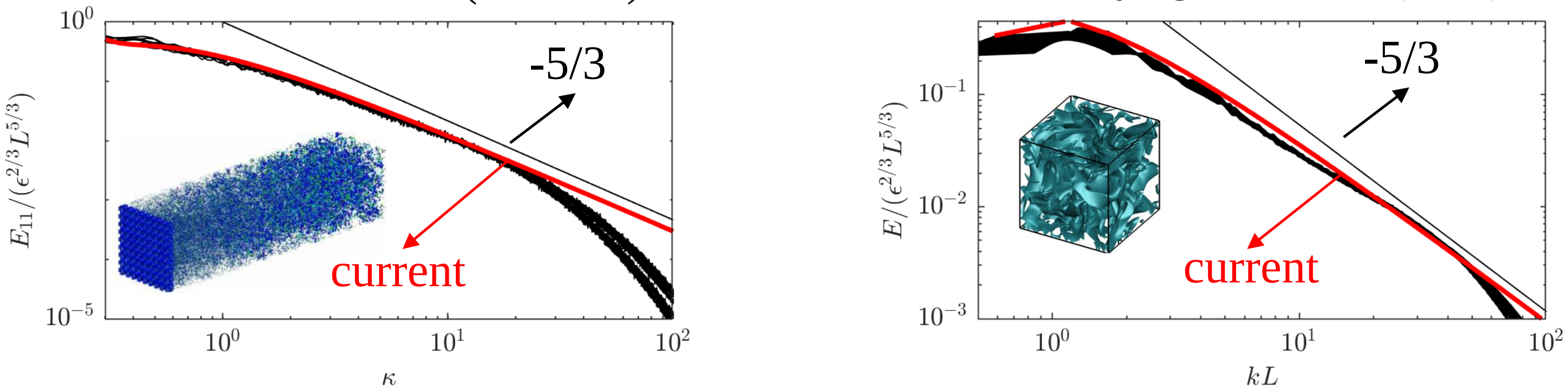
Decaying turbulence (DNS)

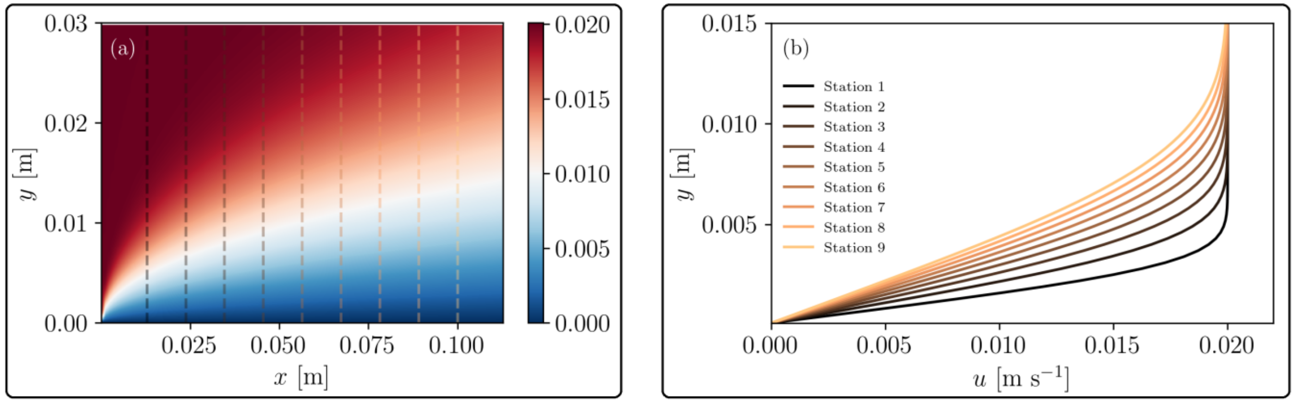


Outline

- Large-scale correction to K41 using self-similar dynamics
- Data-driven extraction of self-similarity

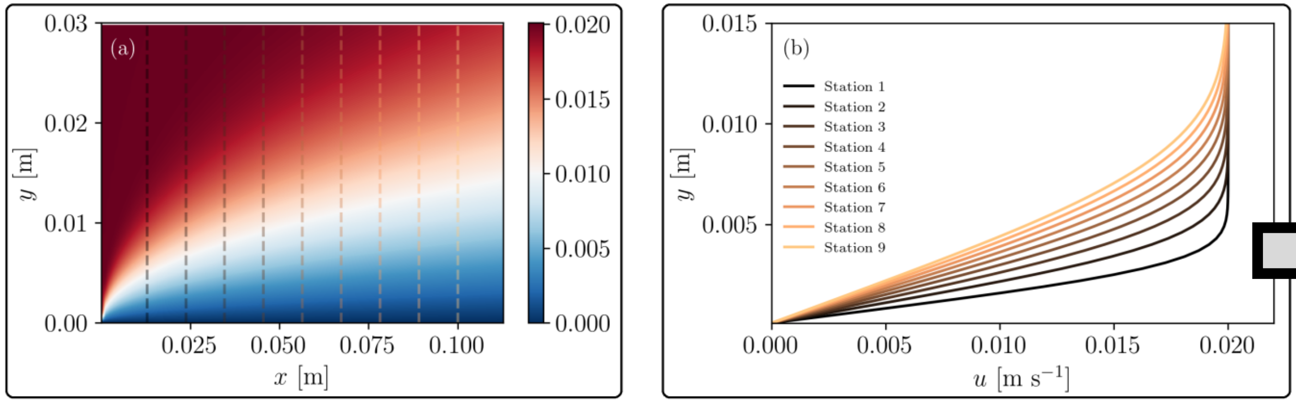
Data-driven identification of self-similarity

Bempedelis, Magri and Steiros ArXiv 2024



Data-driven identification of self-similarity

Bempedelis, Magri and Steiros ArXiv 2024



Optimization to find the factors that collapse the profiles

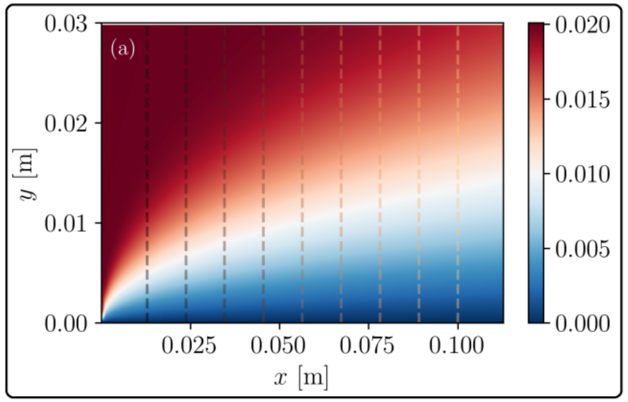
Step 1: Search for similarity variables

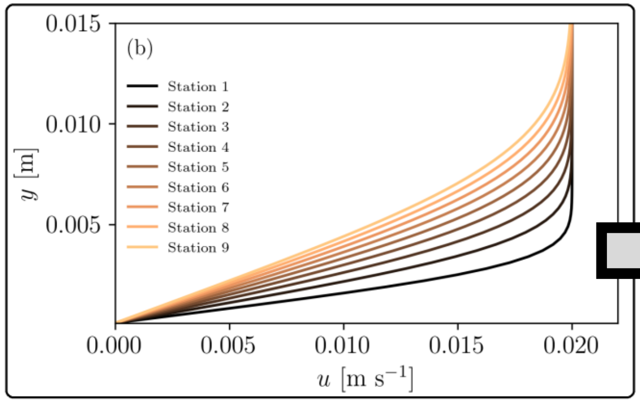
$$\tilde{y} = \alpha y \cancel{+ \gamma}, \quad \tilde{u} = \beta u \cancel{+ \delta}$$

$$\underset{\alpha(x),\, \beta(x)}{\arg\min}\ \frac{1}{2} \sum_{i=1}^{n_x} \sum_{j=1}^{n_x} \| \beta(x_i) u(\alpha(x_i) y, x_i) - \beta(x_j) u(\alpha(x_j) y, x_j) \|_2^2$$

Data-driven identification of self-similarity

Bempedelis, Magri and Steiros ArXiv 2024





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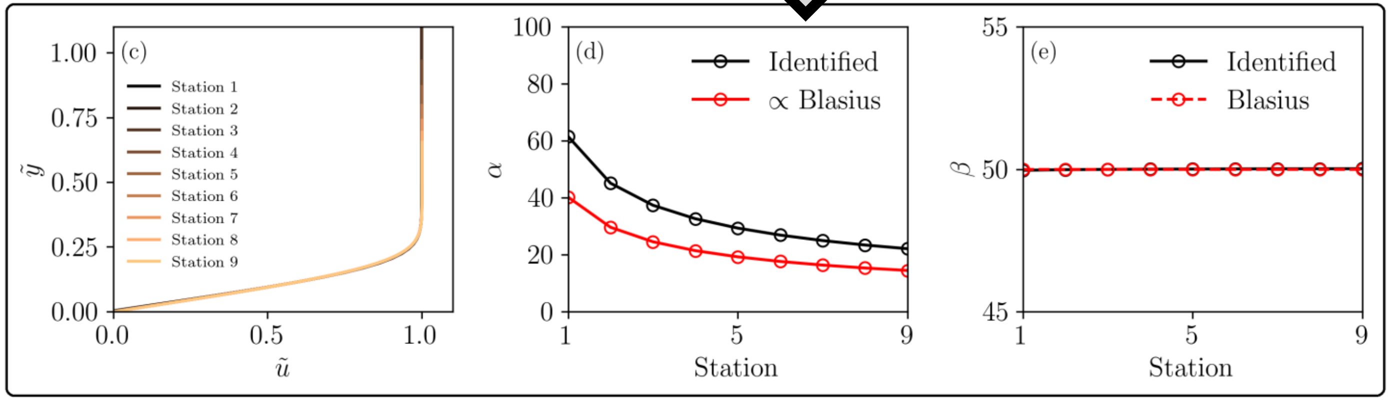

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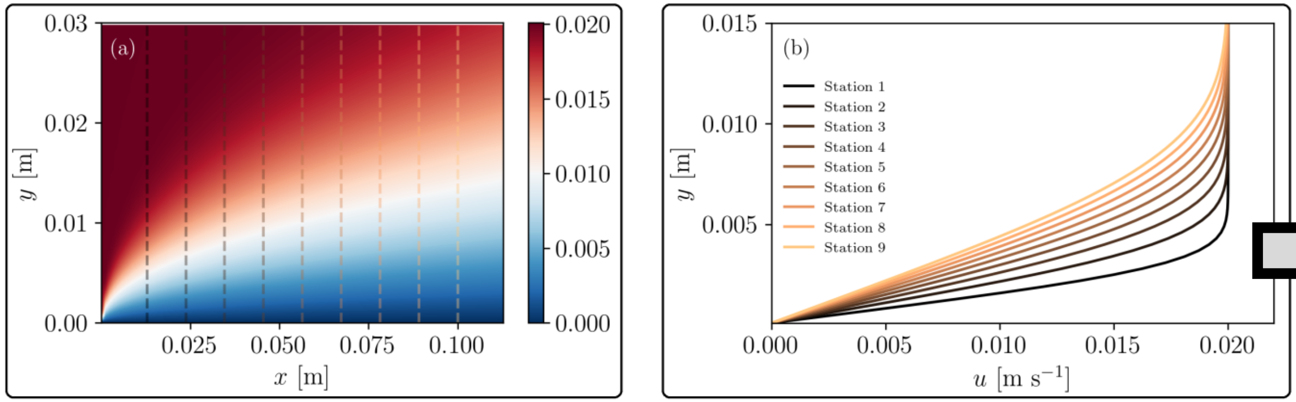
Bempedelis, Magri and Steiros ArXiv 2024



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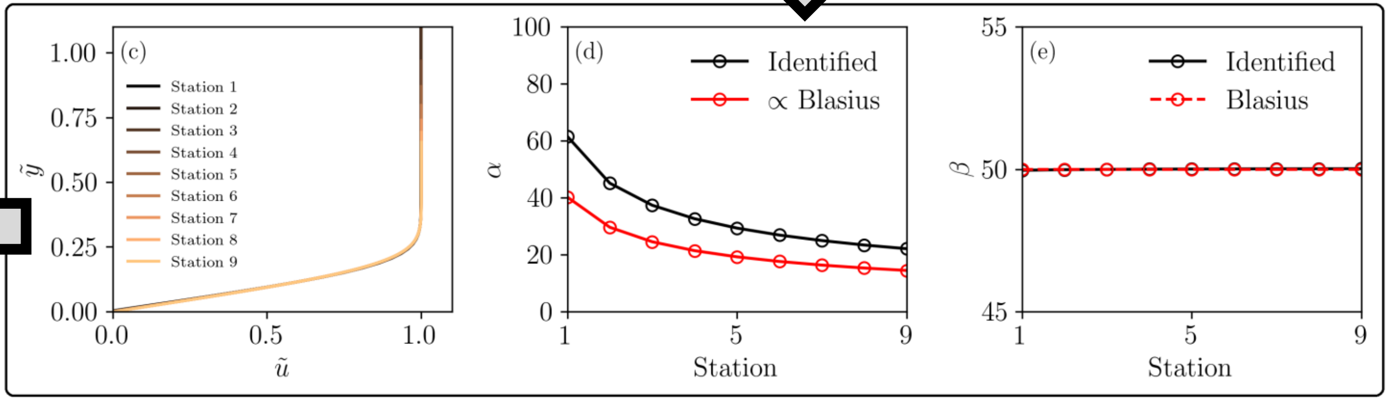


Symbolic Regression (PySR)

Step 2: Analytic form of the transformations

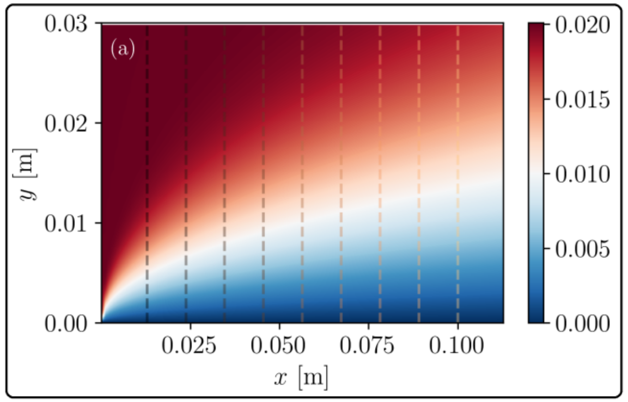
$$\alpha = \psi_1(U_\infty, \nu, x), \quad \beta = \psi_2(U_\infty, \nu, x)$$

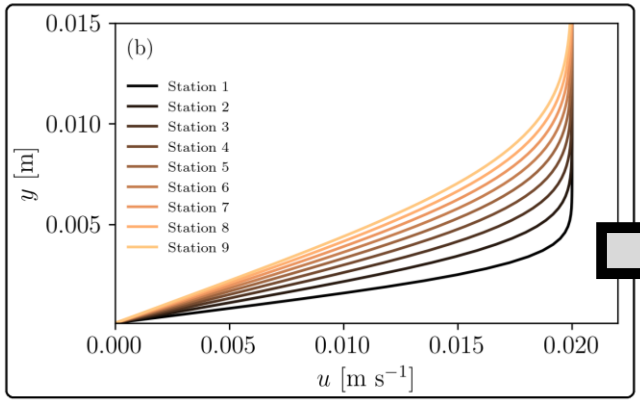
$$\underset{\psi_1}{\arg\min} \; \| \psi_1(U_\infty, \nu, x) - \alpha(x) \|_2^2 + w_D \| \, [\psi_1] - [\alpha] \, \|$$

$$\underset{\psi_2}{\arg\min} \; \| \psi_2(U_\infty, \nu, x) - \beta(x) \|_2^2 + w_D \| \, [\psi_2] - [\beta] \, \|$$

Data-driven identification of self-similarity

Bempedelis, Magri and Steiros ArXiv 2024





Optimization to find the factors that collapse the profiles

Step 1: Search for similarity variables

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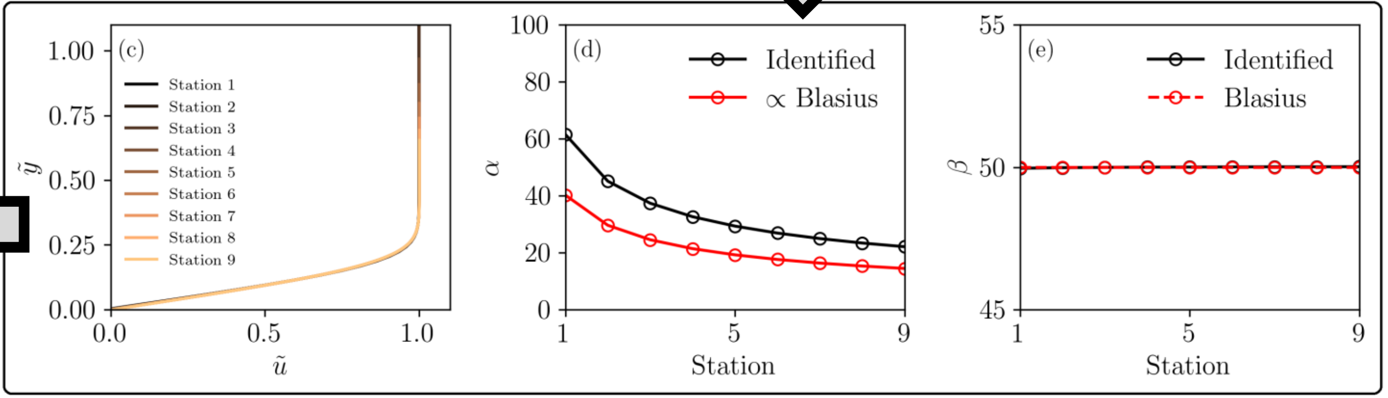

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$$\underset{\psi_2}{\arg\min} \ \|\psi_2(U_\infty, \nu, x) - \beta(x)\|_2^2 + w_D \| \, [\psi_2] - [\beta] \, \|$$

$$\psi_1 = 5.28 U_\infty^{0.5018} \nu^{-0.5018} x^{-0.4981} \longrightarrow \tilde{y} \propto y U_\infty^{0.5018} \nu^{-0.5018} x^{-0.4981}$$

$$\psi_2 = 1.00 U_\infty^{-0.9995} \nu^{-0.0005} x^{0.0005} \longrightarrow \tilde{u} \propto u U_\infty^{-0.9995} \nu^{-0.0005} x^{0.0005}$$

Validation in four flow examples

Turbulent Wake

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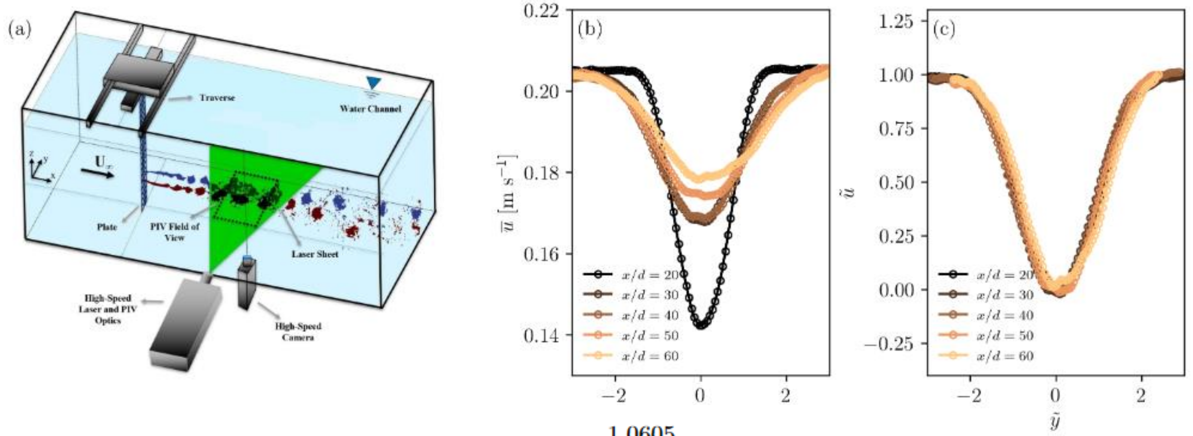
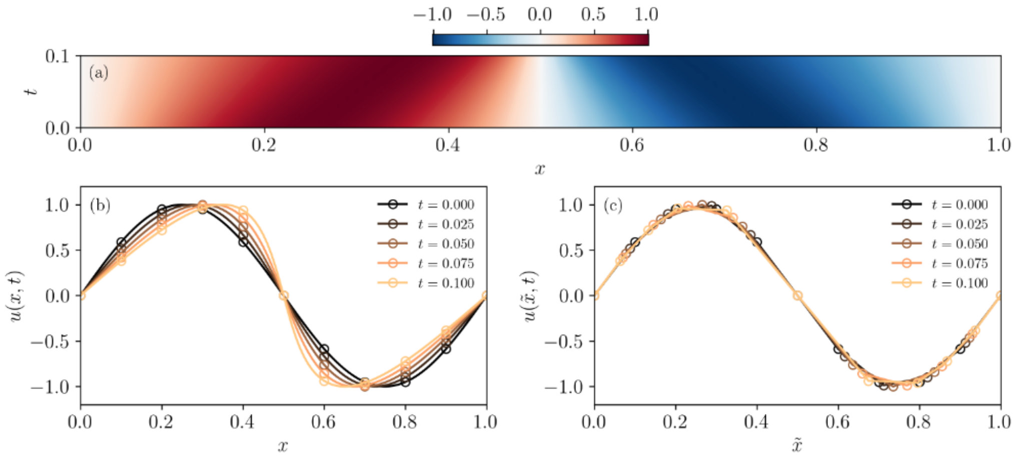
Burger's Equation

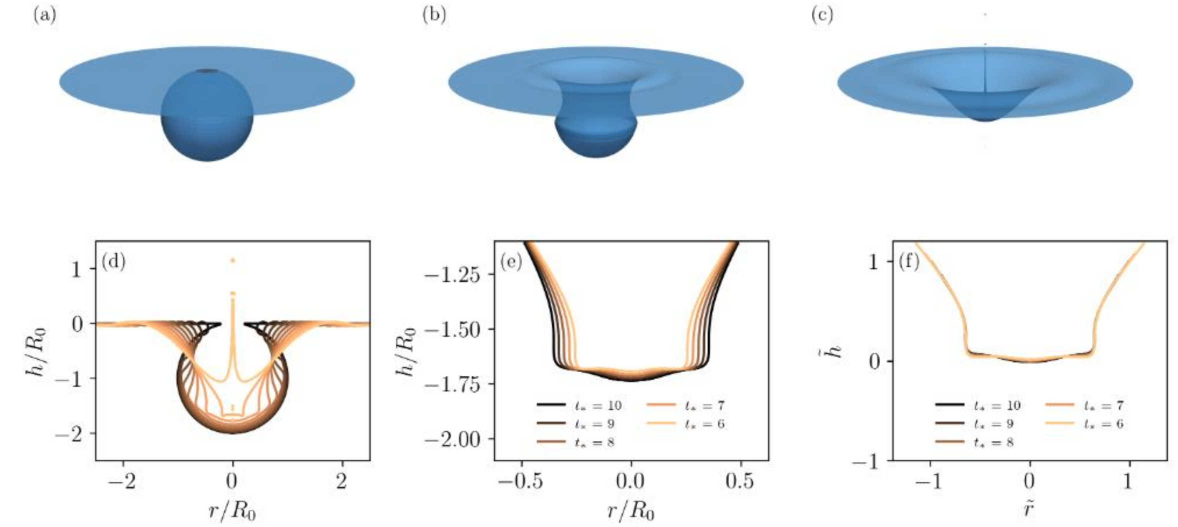


$$\tilde{y} = \alpha y = \frac{1.0605}{y_{1/2}(x)} y \qquad [11a]$$

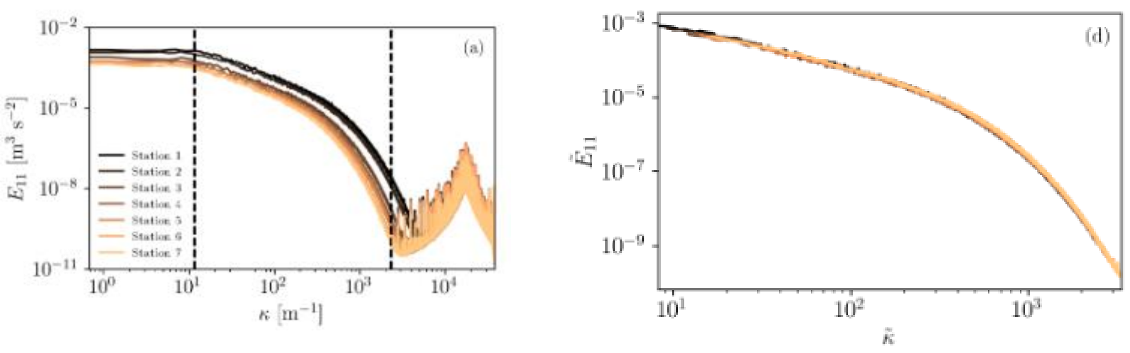
$$\tilde{u}(x, \tilde{y}) = \beta \bar{u} + \gamma = \frac{1}{U_\infty - \bar{u}_{\mathrm{cntr}}} \bar{u} - \frac{\bar{u}_{\mathrm{cntr}}}{U_\infty - \bar{u}_{\mathrm{cntr}}} = 1 - \tilde{\zeta} \qquad [11b]$$



$$\tilde{x} = x + a \qquad a = -ut$$



Bubble Bursting



Grid Turbulence

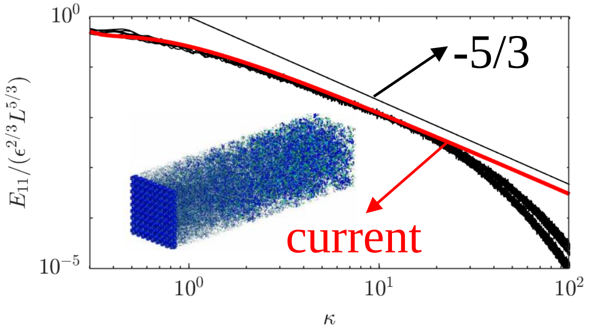
$$\tilde{E}_{11} \propto E_{11} \epsilon^{-1.126} L^{-0.930} \eta^{-1.196} k^{0.689}$$

Summary & Conclusions

Large-scale correction to K41

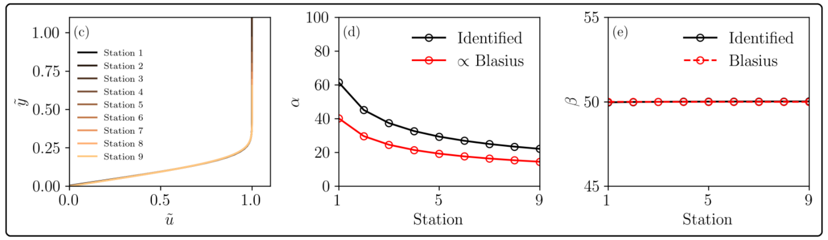
Self-similar Π

Conserved Π



Steiros (2022) PRE

Data-driven extraction of self-similarity



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Thank you