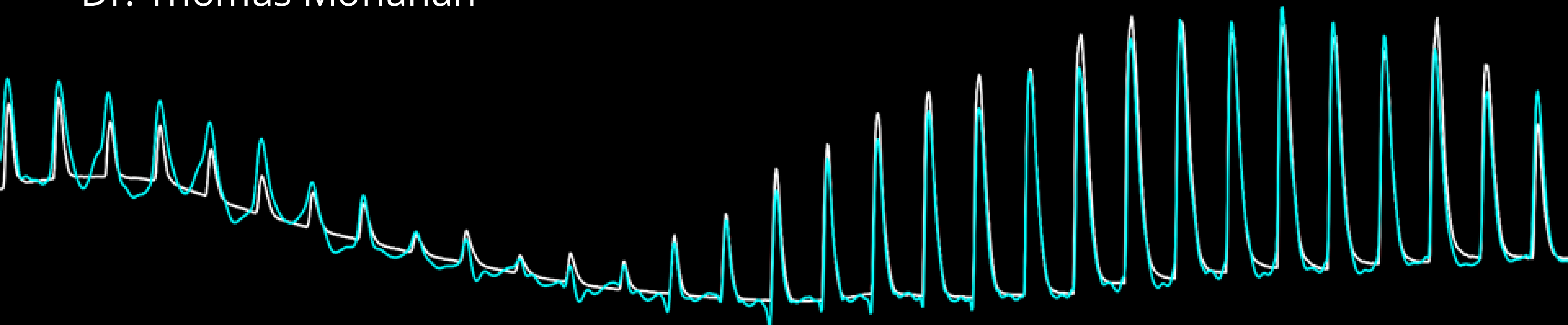
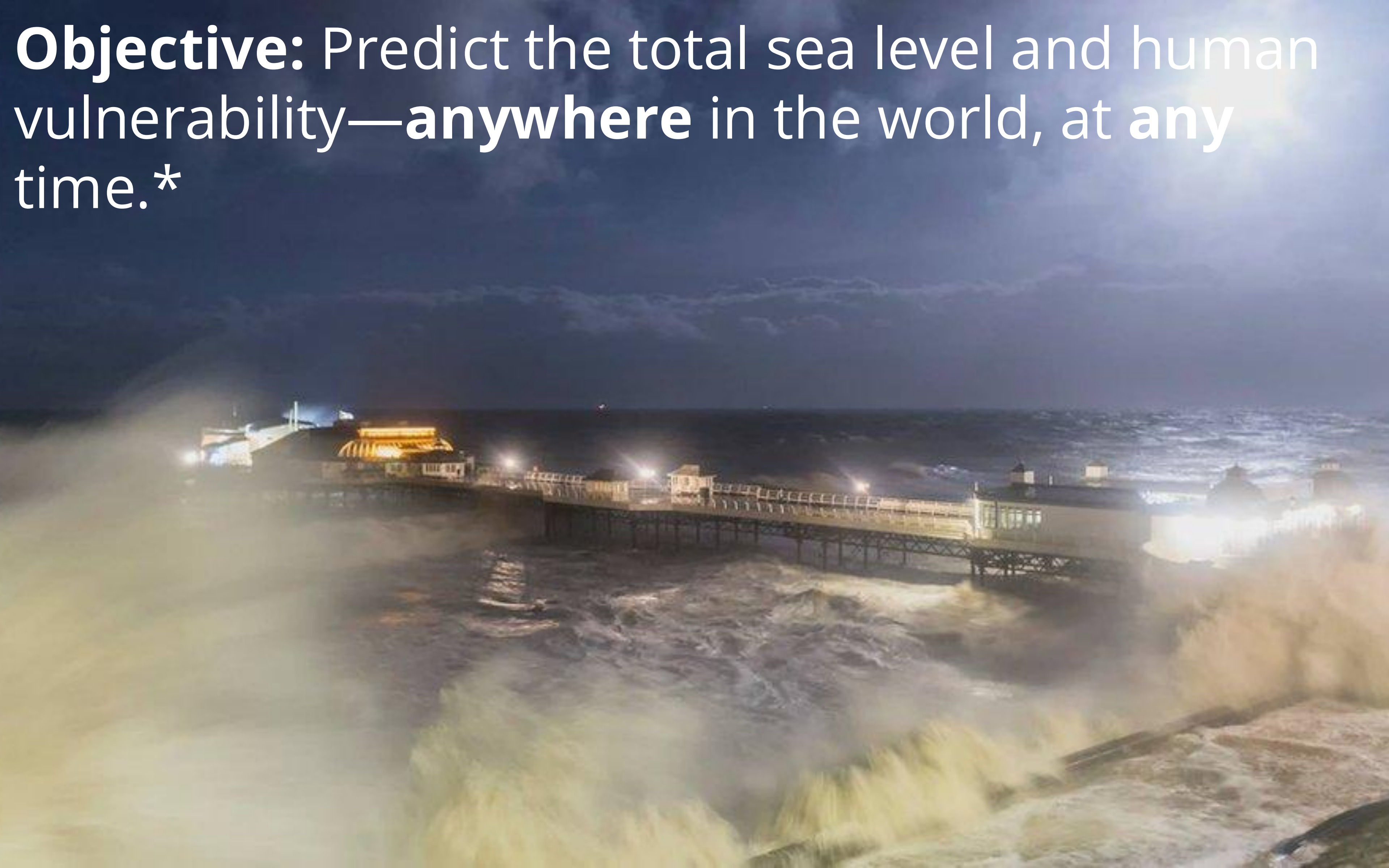
Spatiotemporal tidal prediction and analysis using physics-informed ML

Dr. Thomas Monahan



NEW COLLEGE
UNIVERSITY OF OXFORD
National Oceanography Centre
Ministry of Infrastructure and Water Management
Eric and Wendy Schmidt
AI in Science Postdoctoral Fellowship,
a program of SCHMIDT FUTURES
DEPARTMENT OF ENGINEERING SCIENCE
UNIVERSITY OF OXFORD

Objective: Predict the total sea level and human vulnerability—**anywhere** in the world, at **any** time.*

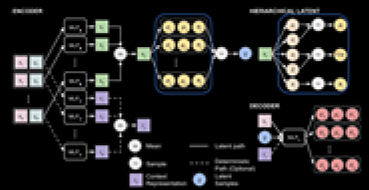


Tides from satellites

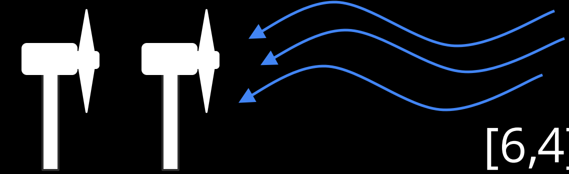
[1]

Probabilistic ML

[3]



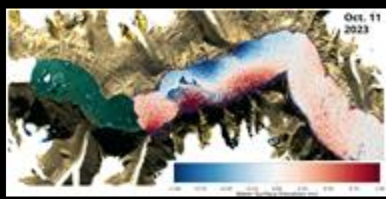
Tidal Currents



[6,4]

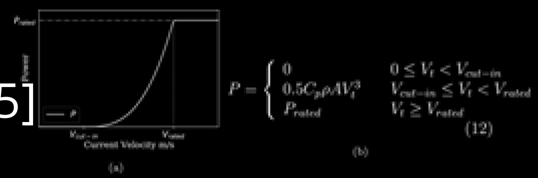
Seiches from space!

[2]



Tidal energy prediction

[5]



Storm Surge

[7]

Harmonic Analysis | Response Method | (And More!!)

Manuscripts:

[1] T. Monahan et al. Tidal corrections from and for SWOT using a spatially coherent variational Bayesian harmonic analysis **(JGR: Oceans)**

[2] T. Monahan et al. Observations of the seiche that shook the world **(Nature Communications)**

[3] T. Monahan et al. Mixture Density Neural Processes **(In preparation ProcRSoc A)**

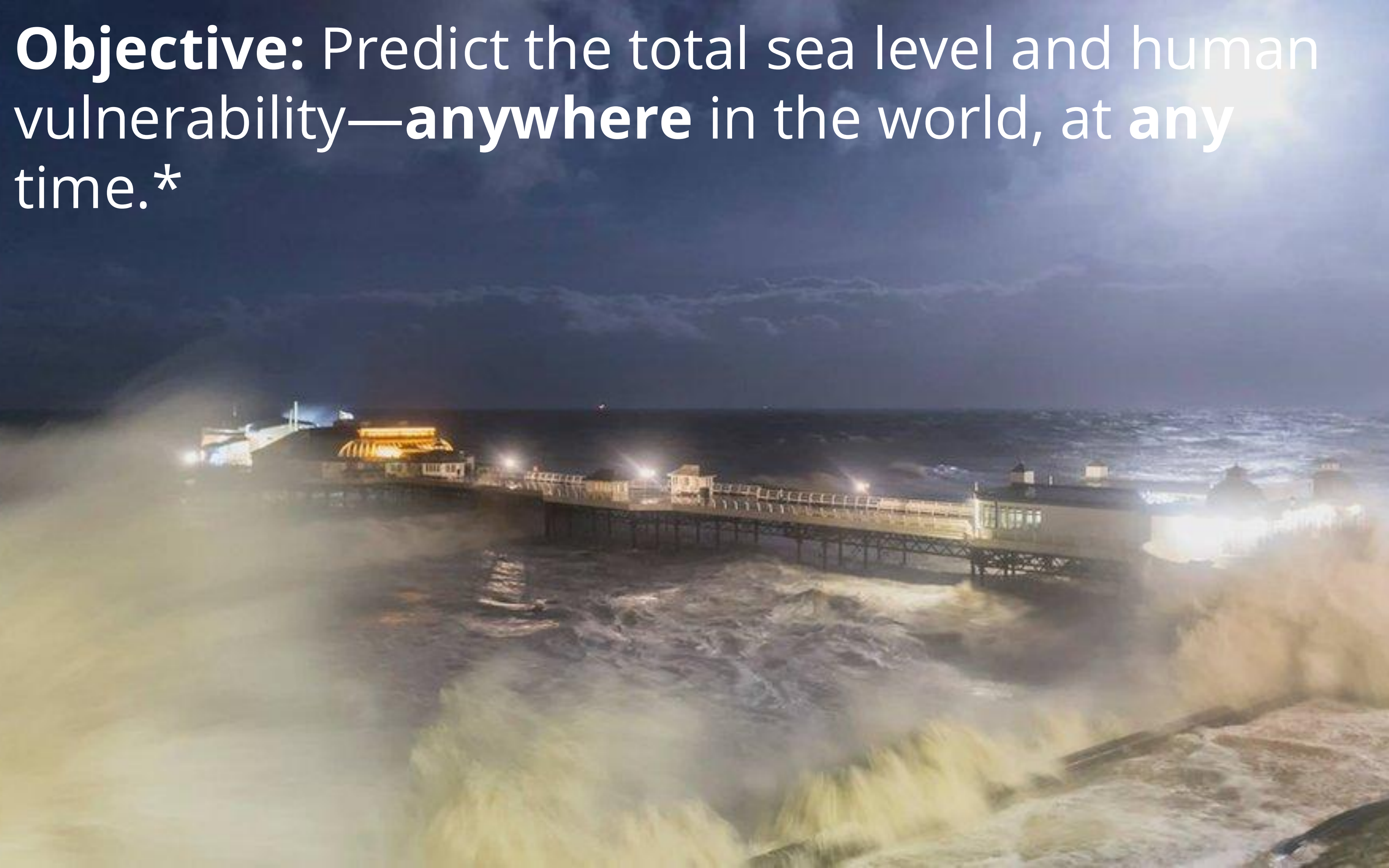
[4] T. Monahan et al. Prediction of tidal currents in the Inner Sound of the Pentland Firth using RTide **(EWTEC)**

[5] Monahan, T et al. A hybrid model for short-term online tidal energy forecasting **(Applied Ocean Research)**

[6] Monahan, T et al. Response-based prediction of tidal currents **(Under review JGR: Oceans)**

[7] Monahan, T., Roberts S. Mixture Density Neural Processes **(Under review Neurips)**

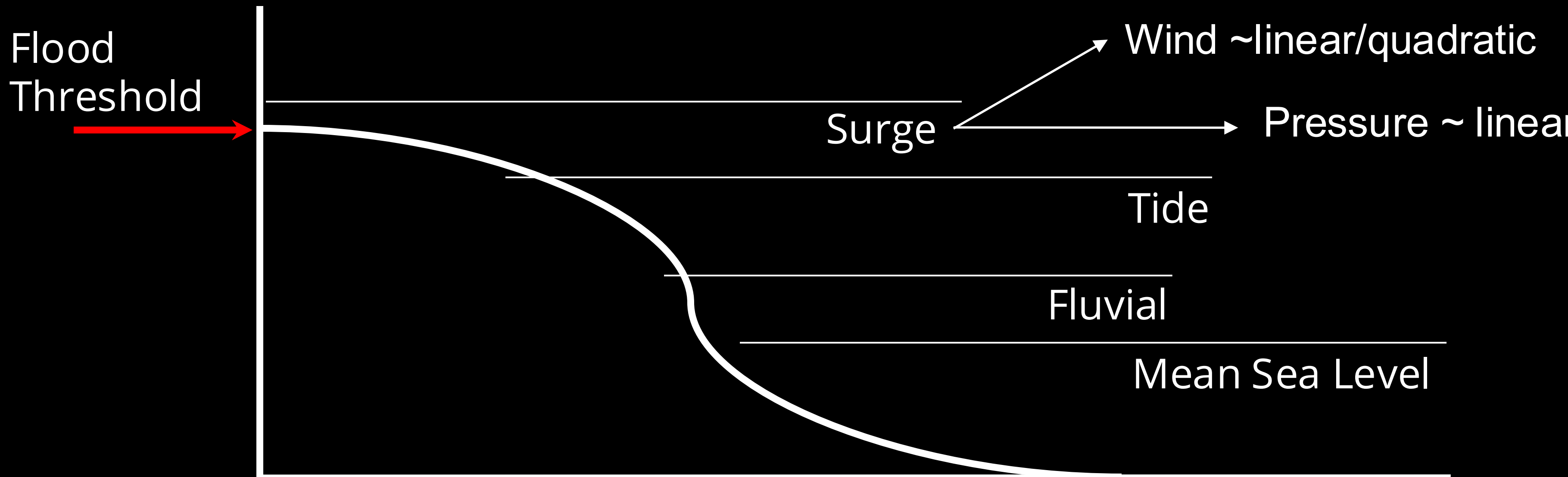
Objective: Predict the total sea level and human vulnerability—**anywhere** in the world, at **any** time.*



Tidal processes are compound events.

Basic Model:

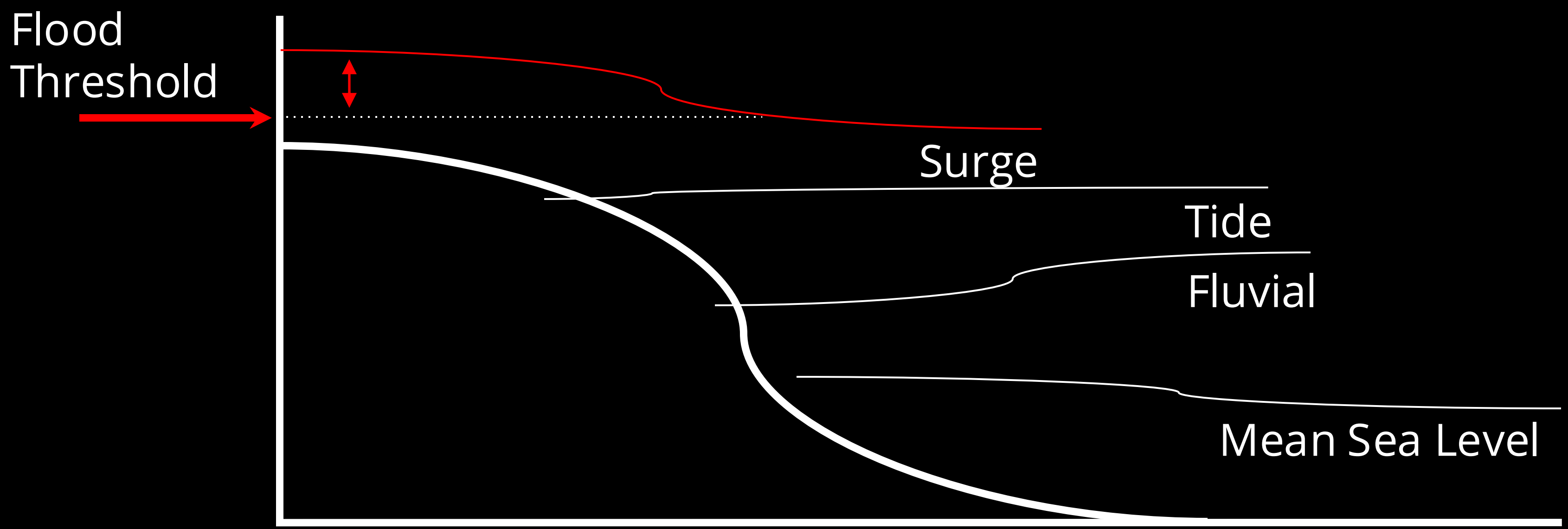
Sea-Level = MSL + Fluvial + Tide + Surge



Tidal processes are <u>compound events.</u>

Correct Model:

Sea-Level = g(MSL, Fluvial, Tide, Surge)



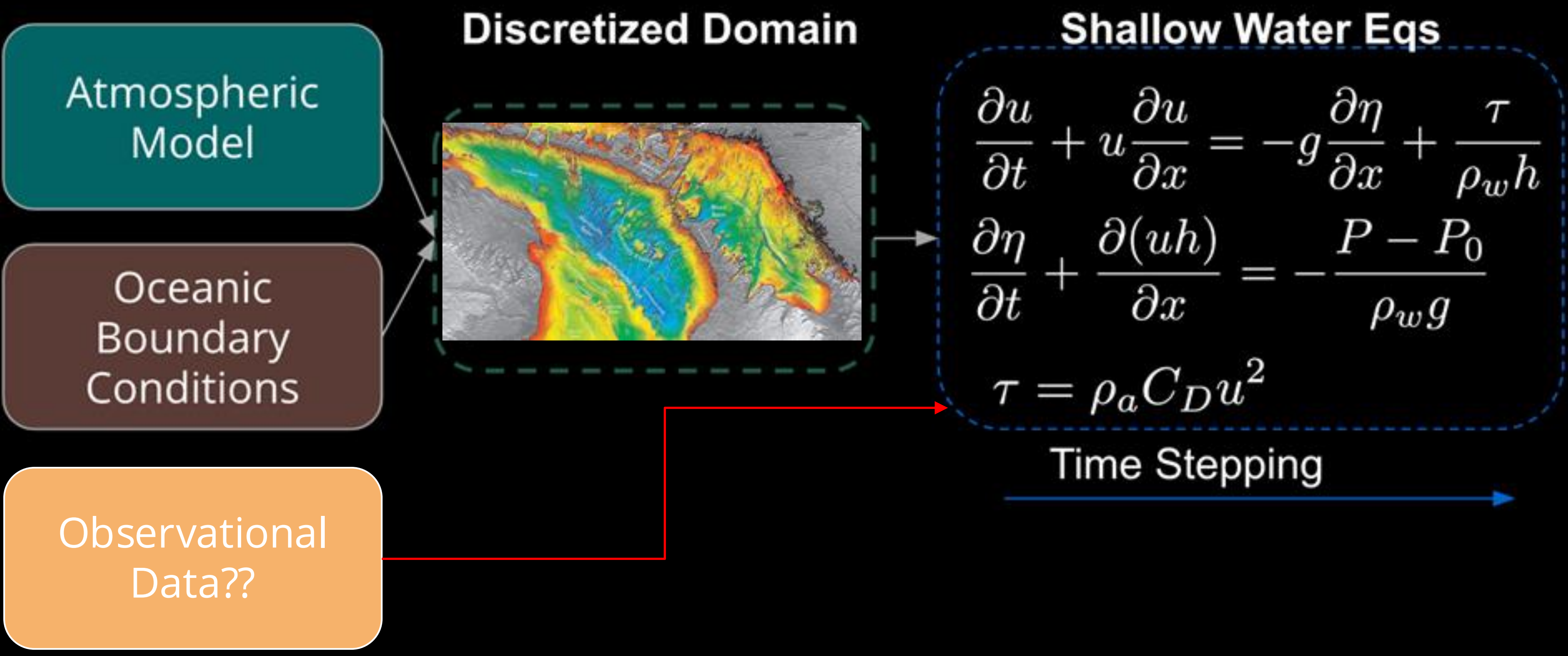
Tidal processes are shallow water waves

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} - f\bar{v} = -g\frac{\partial}{\partial x}(\zeta - \zeta_g' - \zeta_p') + \frac{1}{\rho h}\left(\tau_x^w - \tau_x^b\right)$$

$$\frac{\partial \bar{v}}{\partial t} + \frac{\partial \bar{u}\bar{v}}{\partial x} + \frac{\partial \bar{v}^2}{\partial y} + f\bar{u} = -g\frac{\partial}{\partial y}(\zeta - \zeta_g' - \zeta_p') + \frac{1}{\rho h}\left(\tau_y^w - \tau_y^b\right)$$

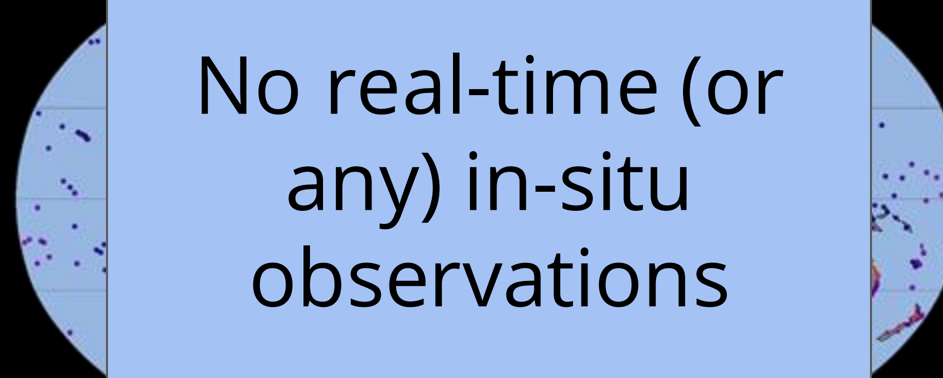
$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = -\frac{1}{h}\frac{\partial \zeta}{\partial t}.$$

Numerical modeling



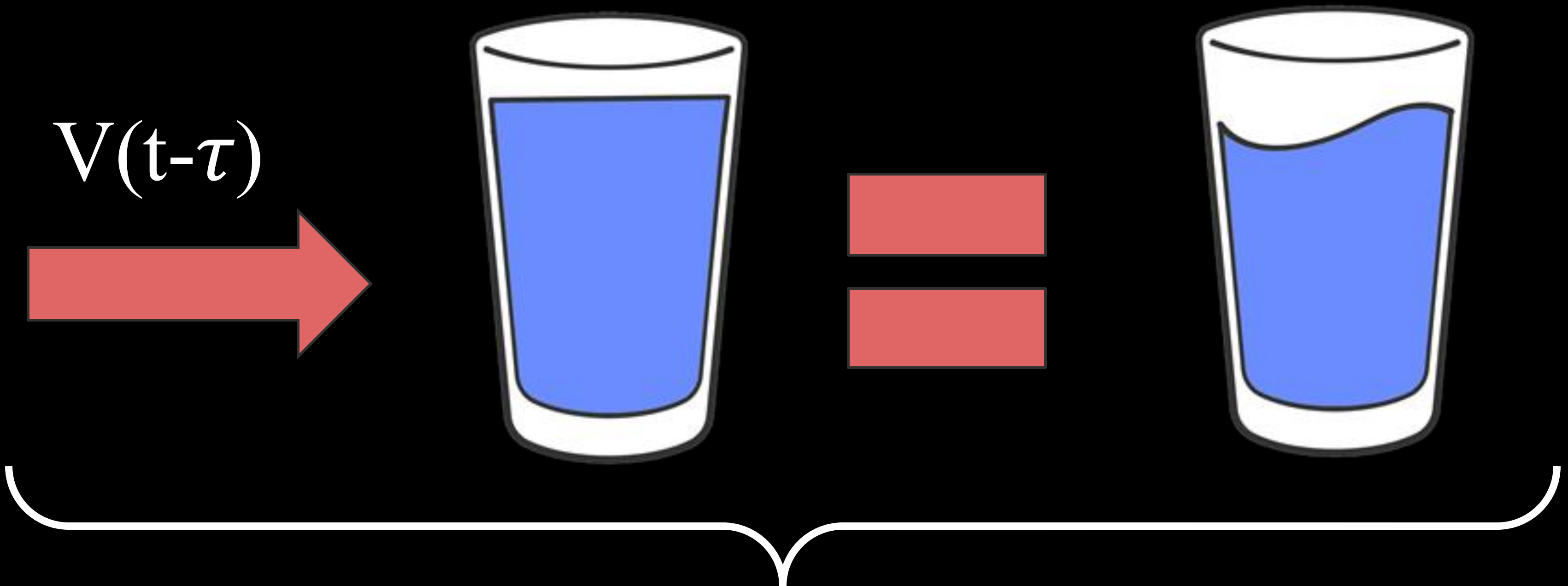
Problems to overcome

Poor Bathymetry



No real-time (or any) in-situ observations

Rethinking sea-level prediction



Time invariant & weakly nonlinear

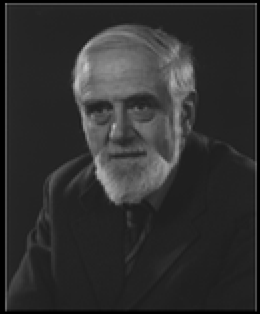
Impulse-response theory

TIDAL SPECTROSCOPY AND PREDICTION

By W. H. MUNK† and D. E. CARTWRIGHT‡

Institute of Geophysics and Planetary Physics, University of California, La Jolla

(*Communicated by Sir Edward Bullard, F.R.S.—Received* 21 *June* 1965)



$$\hat{\zeta} = \sum_{\tau} wV(t-\tau)$$

Problems to overcome:

Poor Bathymetry

No real-time (or any) in-situ observations

Impulse-response theory

TIDAL SPECTROSCOPY AND PREDICTION

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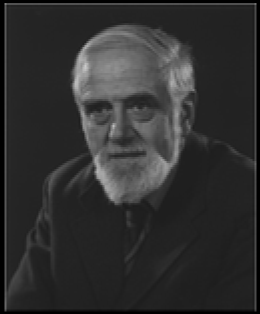


$$\hat{\zeta} = \sum_{\tau} wV(t - \tau)$$

Problems overcame*:

Poor Bathymetry

No real-time (or any) in-situ observations

Nonlinear responses

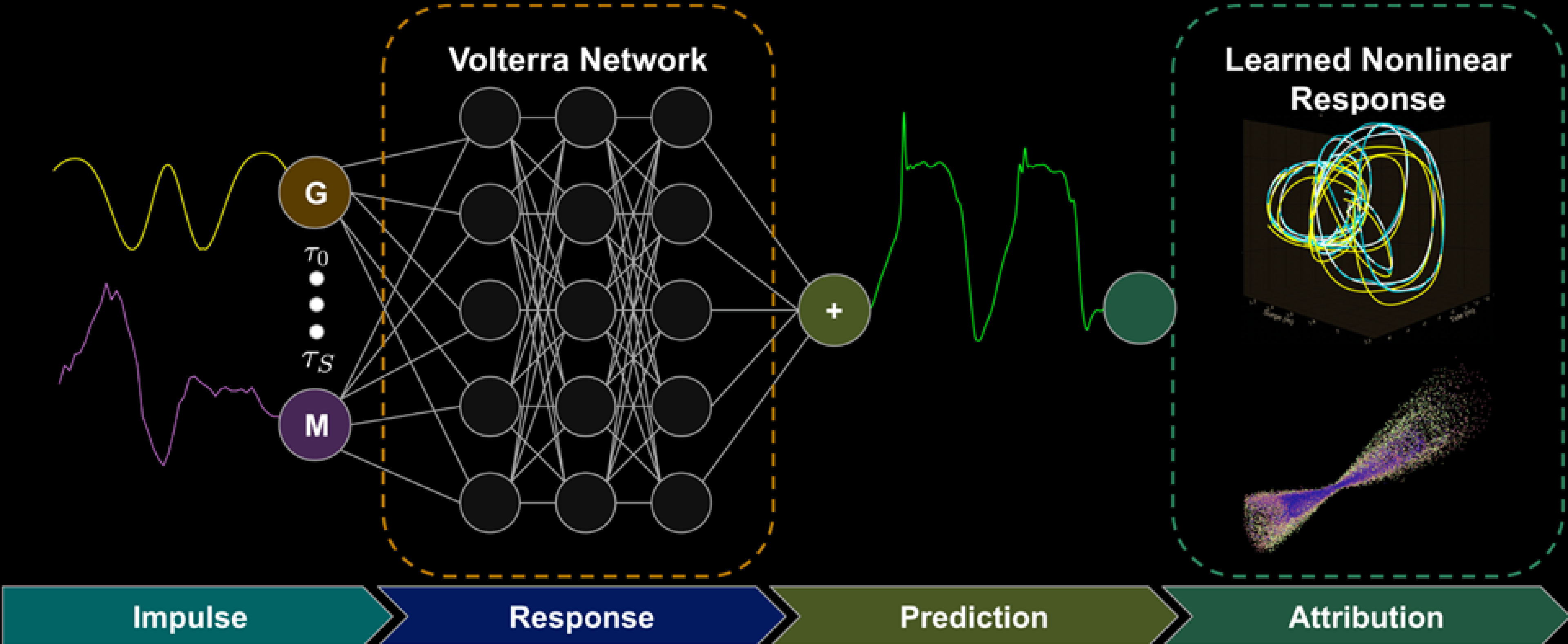
$$x^{th}\text{order response} = \sum_{i} \cdots \sum_{x} \sum_{s} \cdots \sum_{s'} w(i, \ldots, x, s, \ldots, s')(c(t-\tau_s))(\ldots)(\ldots)(c(t-\tau_{s'}))$$

Volterra Series!

Higher order interactions = exponentially more terms

RTide

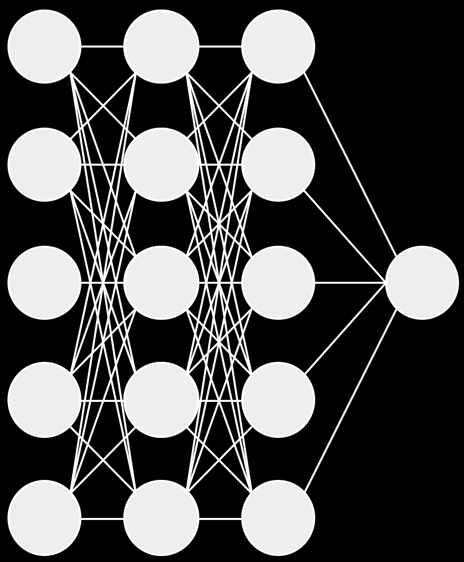
$$V_G, V_M \longrightarrow f(V_G, V_M) \longrightarrow \hat{\zeta}(t) \approx g = p_0 + \sum_{|F|} p_i^T$$



[1] Monahan et al. RTide: Automating the tidal response method (JGR: Machine learning and computation)

$$\hat{\zeta}(t) = \sum_{m,n} \sum_{s} [u_n^m(s) a_n^m(t - \tau_s) + v_n^m(s) b_n^m(t - \tau_s)]$$

$$x^{th}\text{order response} = \sum_{i} \cdots \sum_{x} \sum_{s} \cdots \sum_{s'} w(i, \ldots, x, s, \ldots, s')(c(t - \tau_s))(\ldots)(\ldots)(c(t - \tau_{s'}))$$

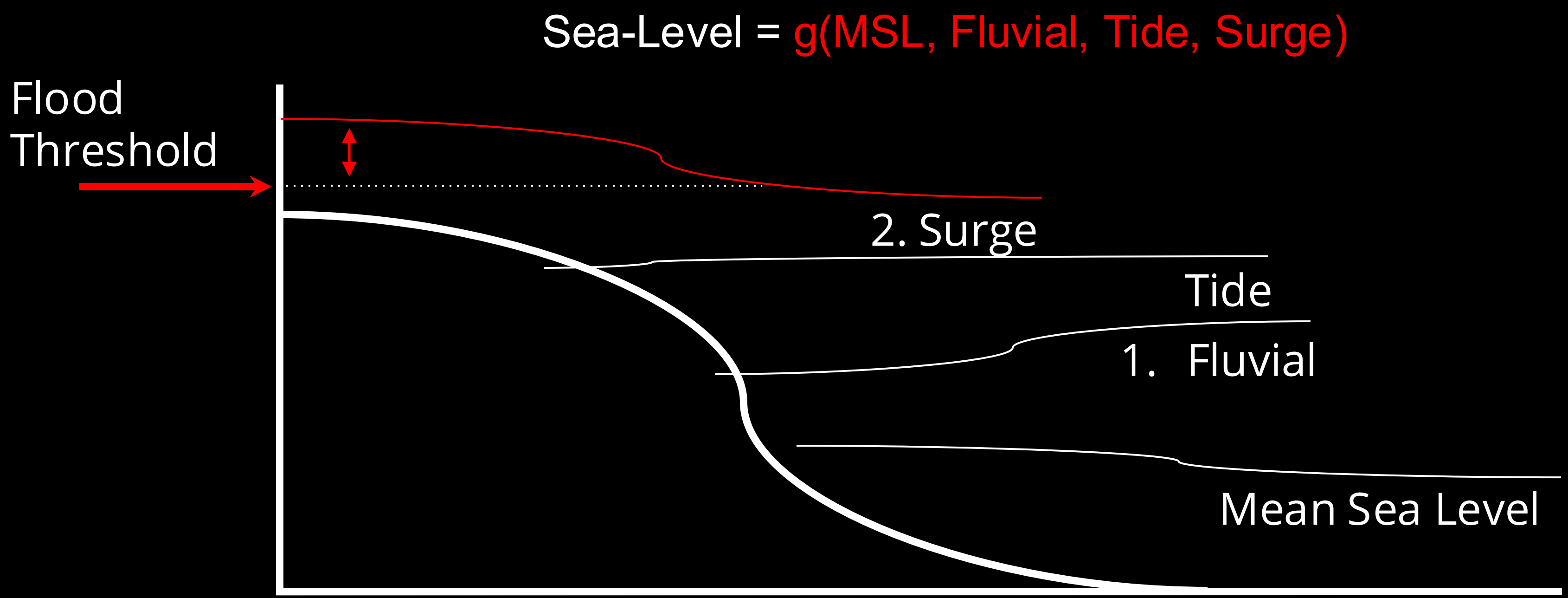


Volterra Network ≅ Volterra Series!

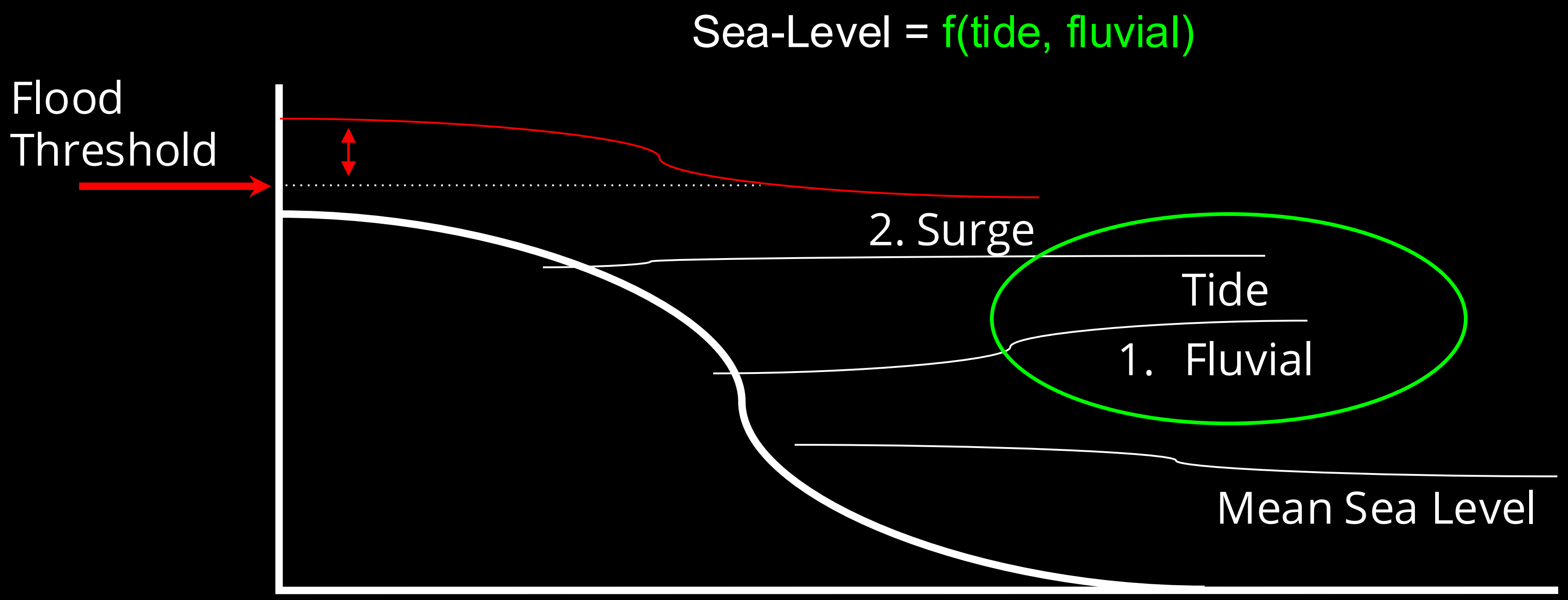
$$h_i[k_1, \ldots, k_i] = \sum_{j=1}^{L} w_{0j} \left(\frac{\varphi^{(i)}(b_j)}{i!} \right) w_{jk_1} \ldots w_{jk_i}$$

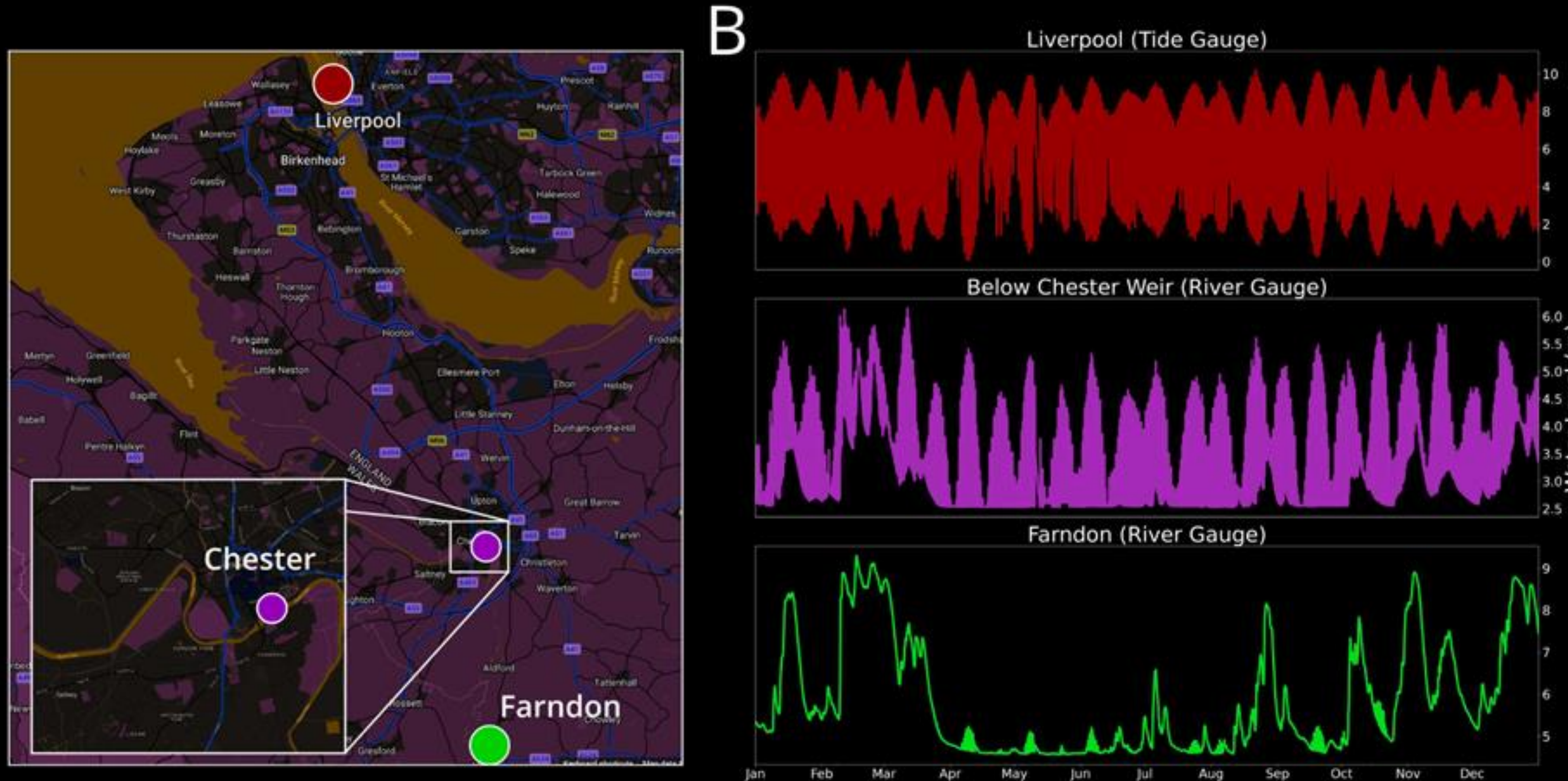
f(gravitational + other forcing)

Step one towards global forecasting

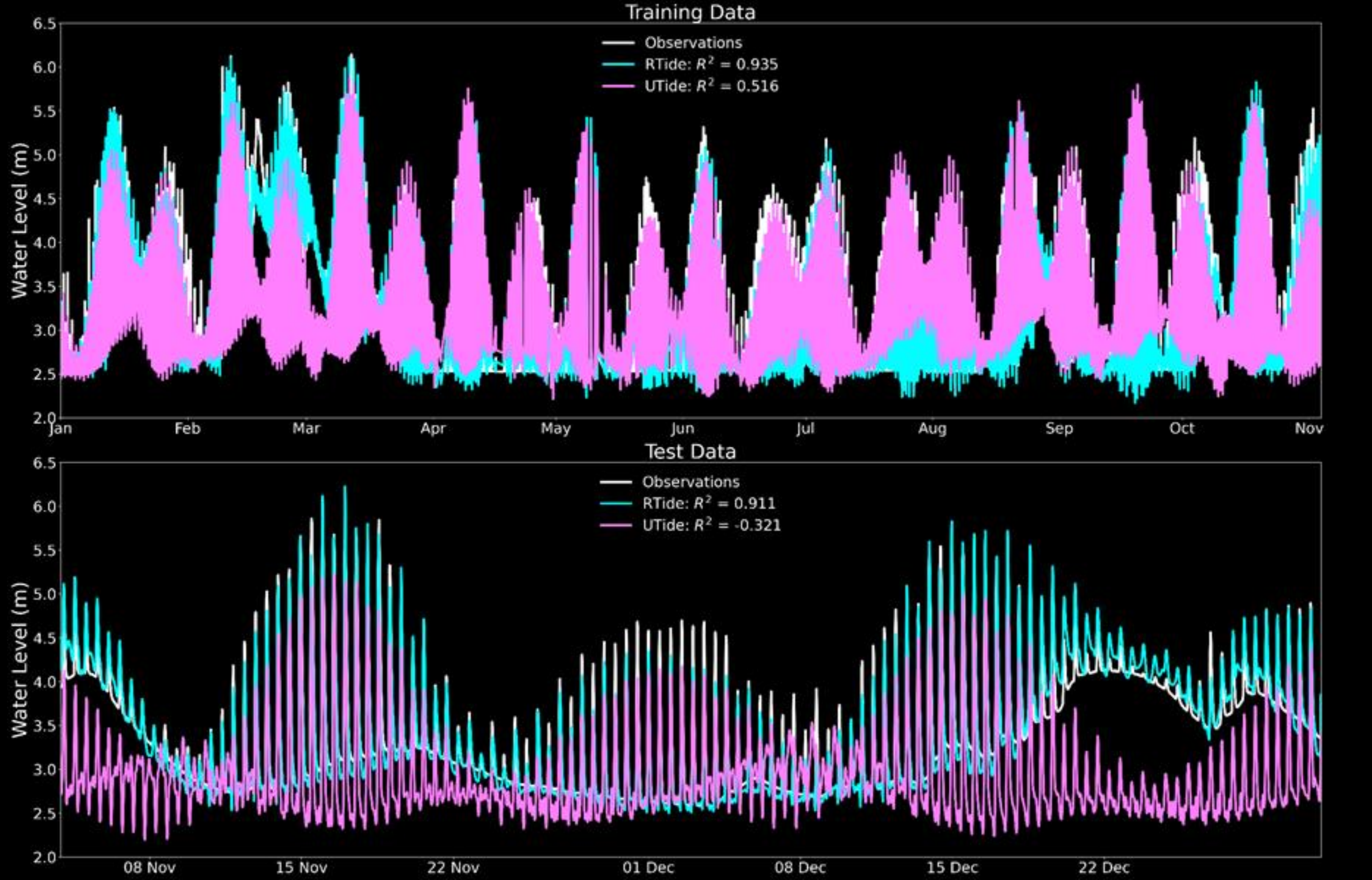


Step one towards global forecasting



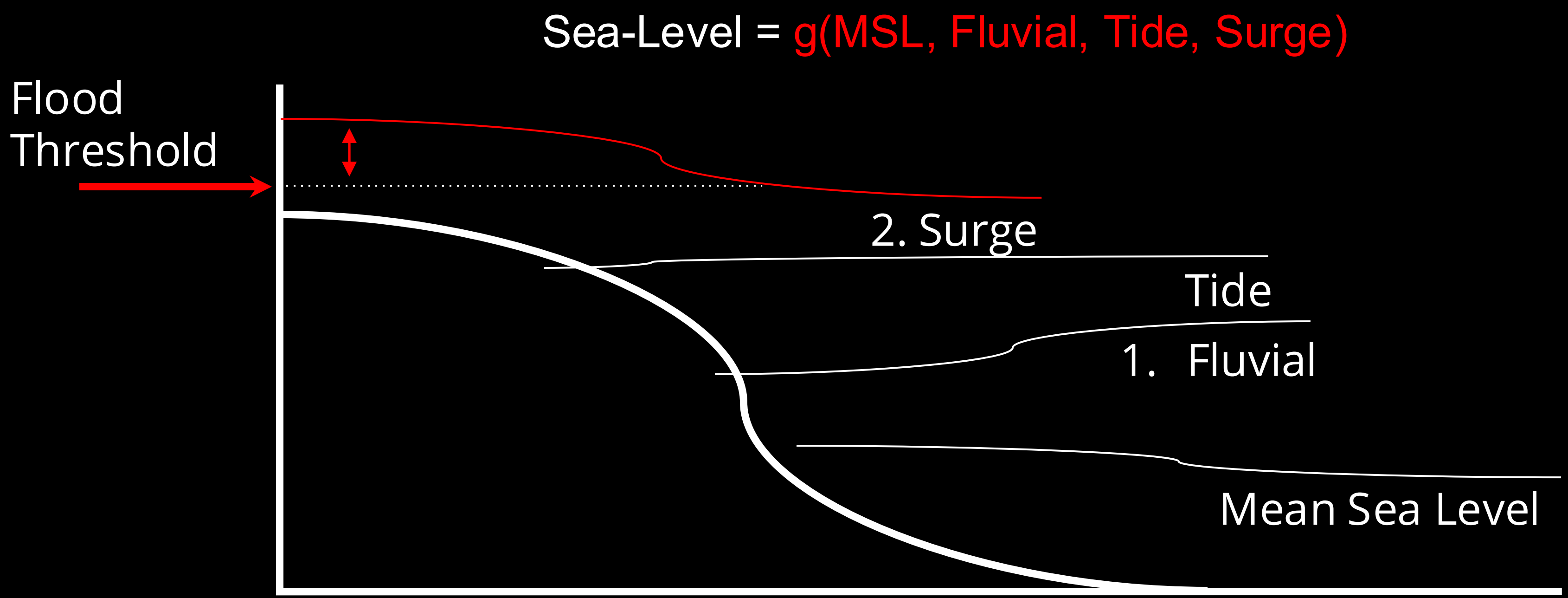


f(gravitational + fluvial)

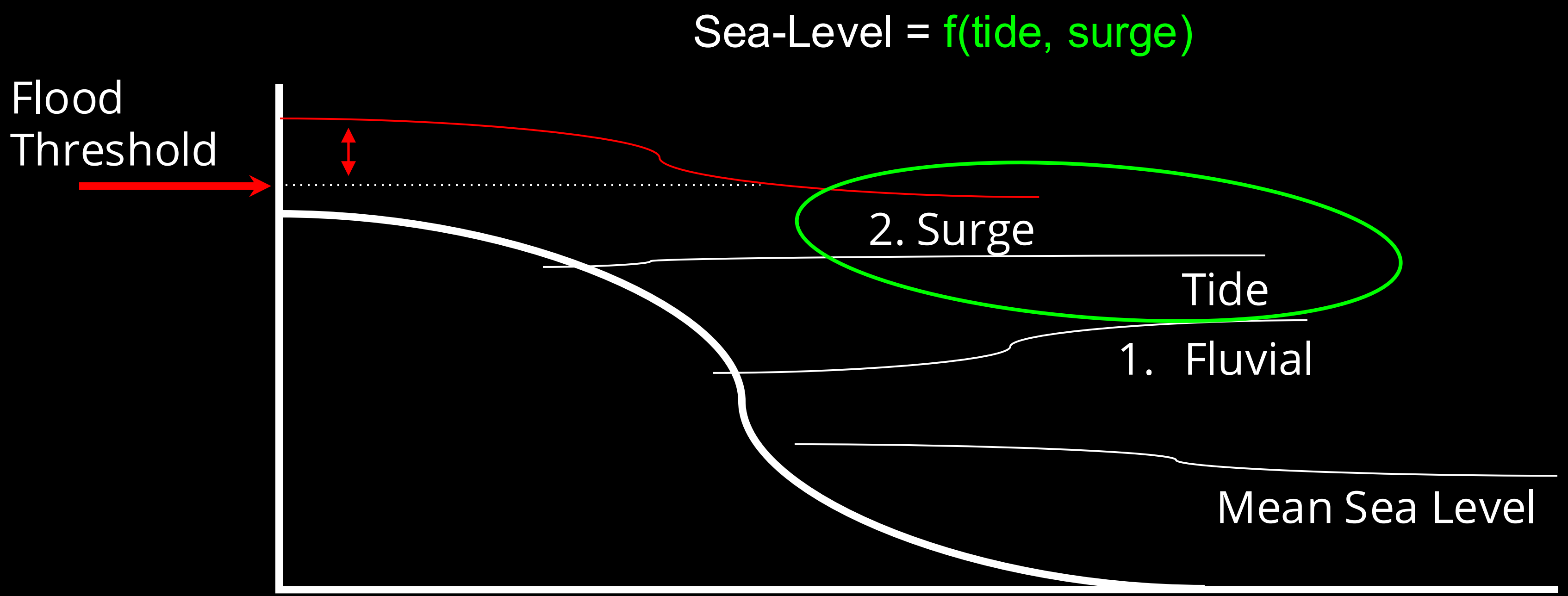


Training Data
Observations
RTide: R^2 = 0.935
UTide: R^2 = 0.516
Water Level (m)
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Test Data
Observations
RTide: R^2 = 0.911
UTide: R^2 = -0.321
Water Level (m)
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
08 Nov
15 Nov
22 Nov
01 Dec
08 Dec
15 Dec
22 Dec

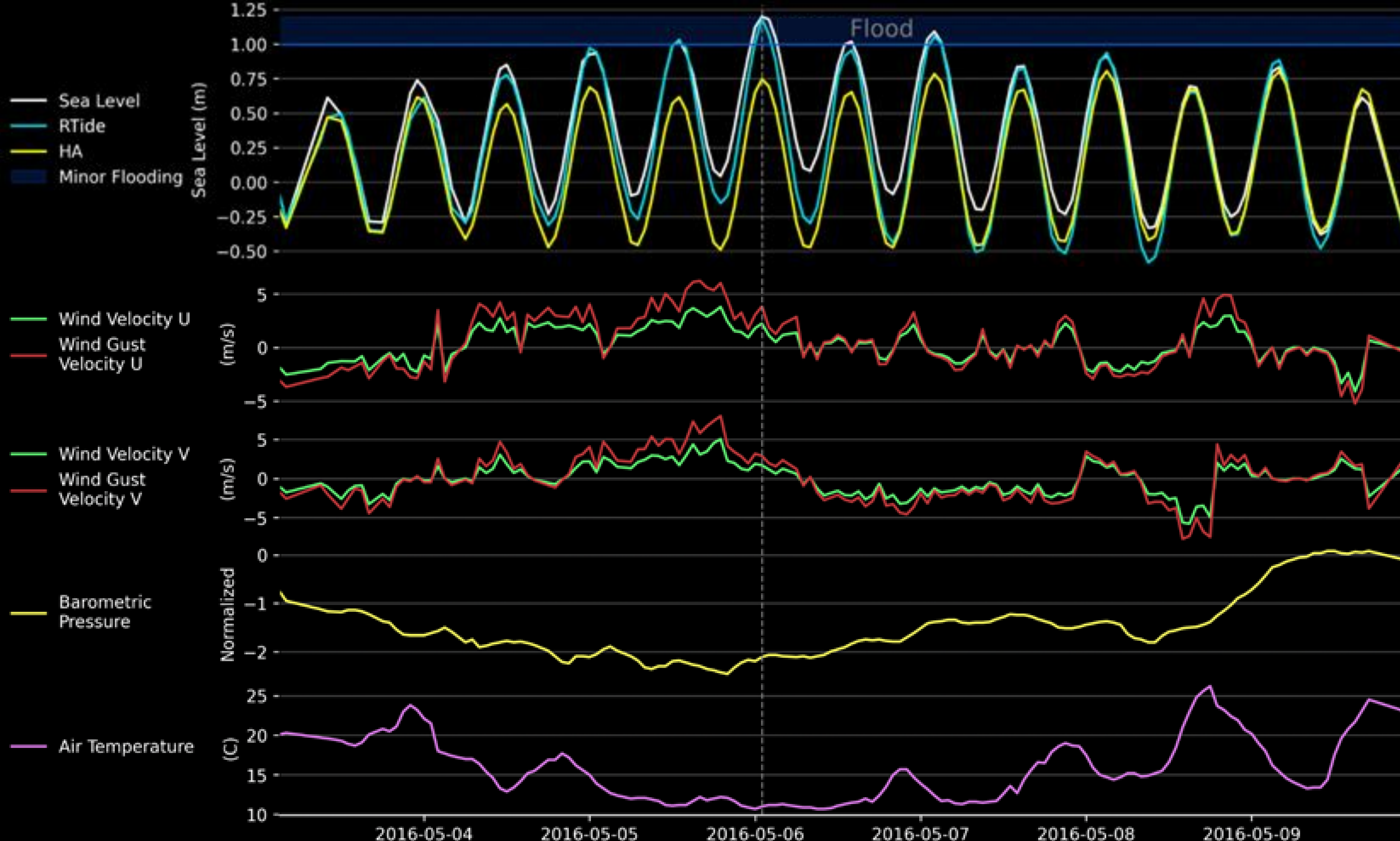
Step two towards global forecasting



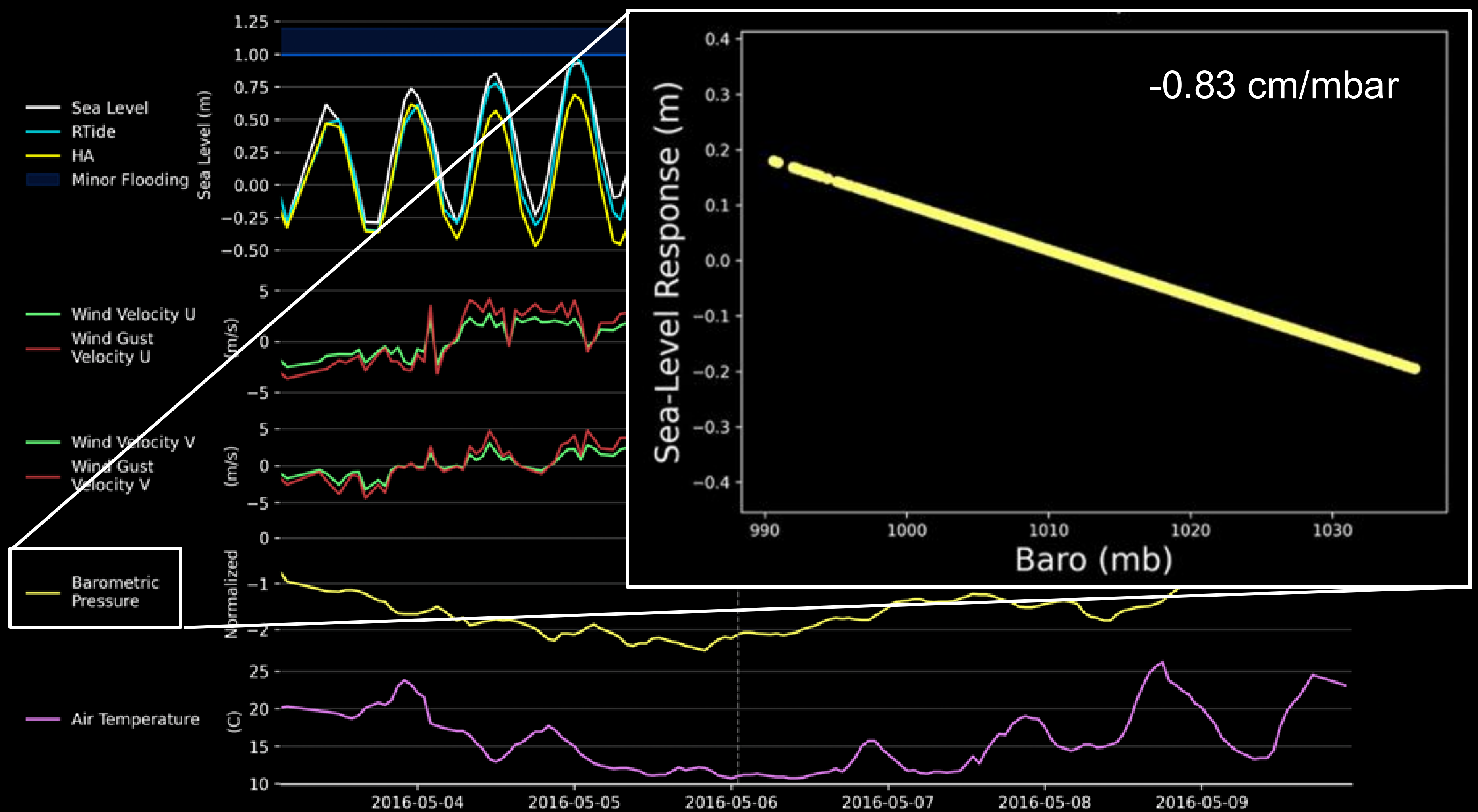
Step two towards global forecasting



f(gravitational + meteorological)

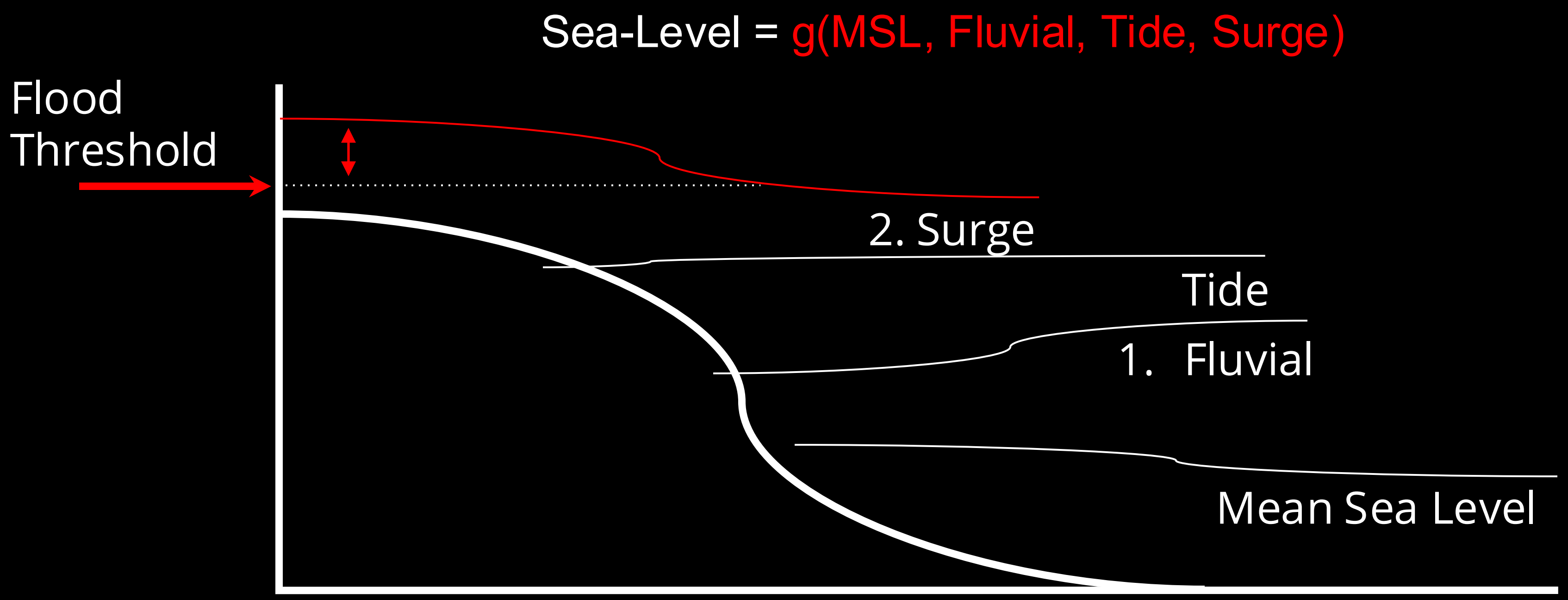


Flood
Sea Level
RTide
HA
Minor Flooding
Sea Level (m)
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
Wind Velocity U
Wind Gust Velocity U
(m/s)
5
0
−5
Wind Velocity V
Wind Gust Velocity V
(m/s)
Barometric Pressure
Normalized
0
−1
−2
Air Temperature
(C)
25
20
15
10
2016-05-04
2016-05-05
2016-05-06
2016-05-07
2016-05-08
2016-05-09

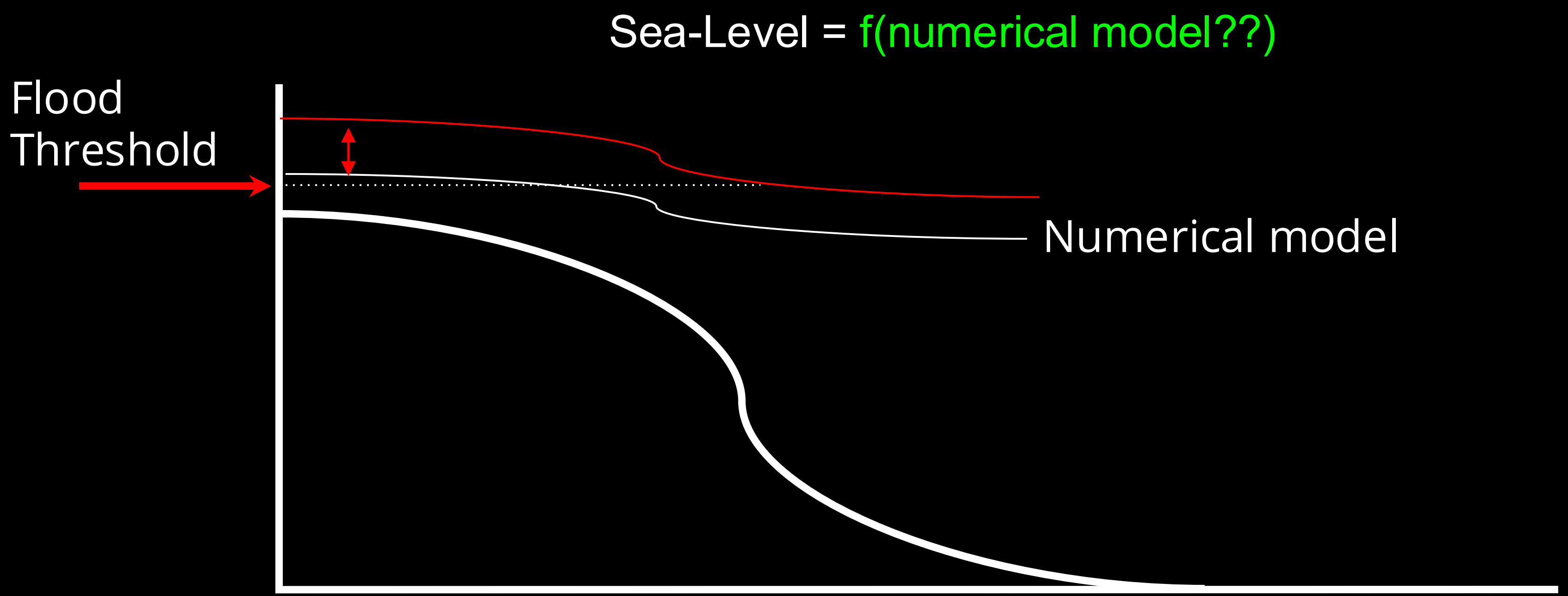


Sea Level
RTide
HA
Minor Flooding
Sea Level (m)
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
Wind Velocity U
Wind Gust Velocity U
(m/s)
Wind Velocity V
Wind Gust Velocity V
Barometric Pressure
Normalized
Air Temperature
(C)
2016-05-04
2016-05-05
2016-05-06
2016-05-07
2016-05-08
2016-05-09
-0.83 cm/mbar
Sea-Level Response (m)
Baro (mb)

Step three towards global forecasting



Step three towards global forecasting



Augmenting traditional forecasts

Numerical

Atmospheric Model

Oceanic Boundary Conditions

Discretized Domain

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial \eta}{\partial x} + \frac{\tau}{\rho_w h}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial (uh)}{\partial x} = -\frac{P - P_0}{\rho_w g}$$

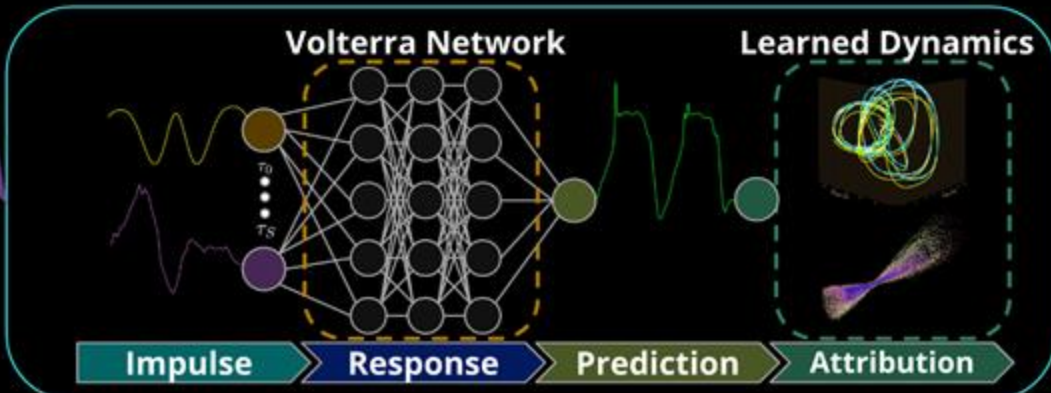
$$\tau = \rho_a C_D u^2$$

Time Stepping

- ✖ Resolution is limited by computational expense.
- ✖ Accuracy is dependent on boundary conditions, especially **bathymetry**.
- ✔ Great atmospheric surge
- ✖ Poor oceanic response.

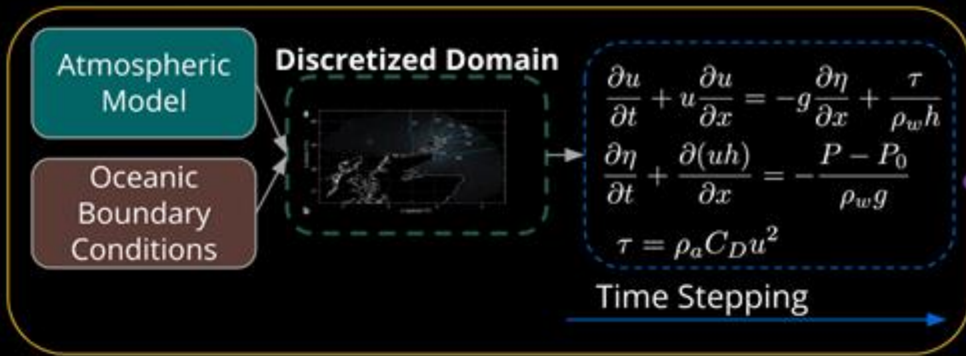
RTide

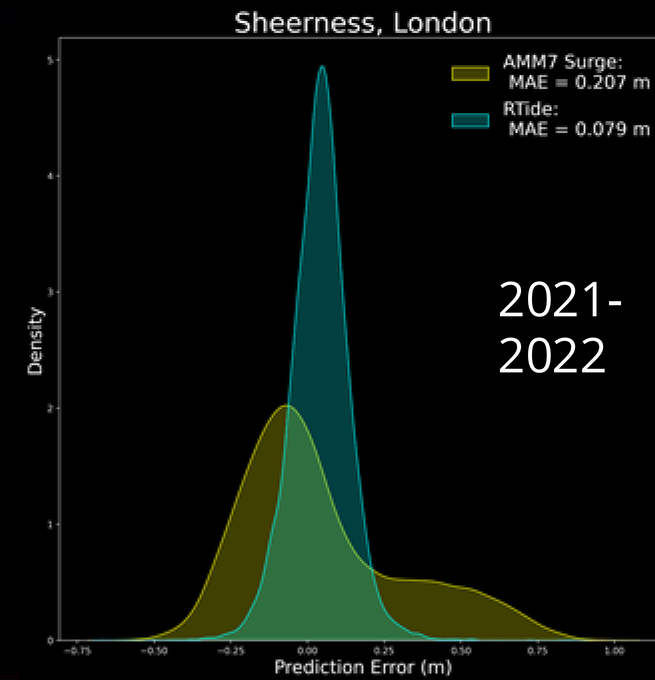
- ✔ Learns response directly from data —> no need for bathymetry!
- ✔ Can capture localized nonlinearities.
- ✔ Provides insights into dynamics.
- ✖ **But**, cannot account for externally generated surges.



Augmenting traditional forecasts

Numerical





RTide

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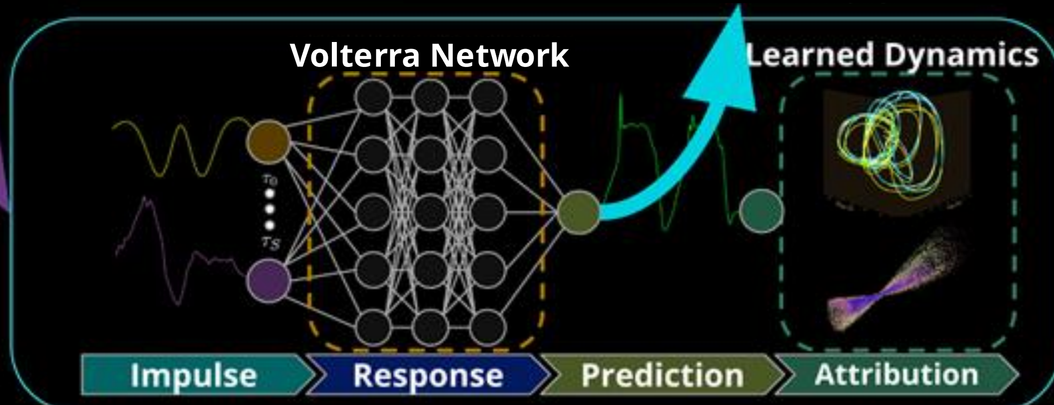
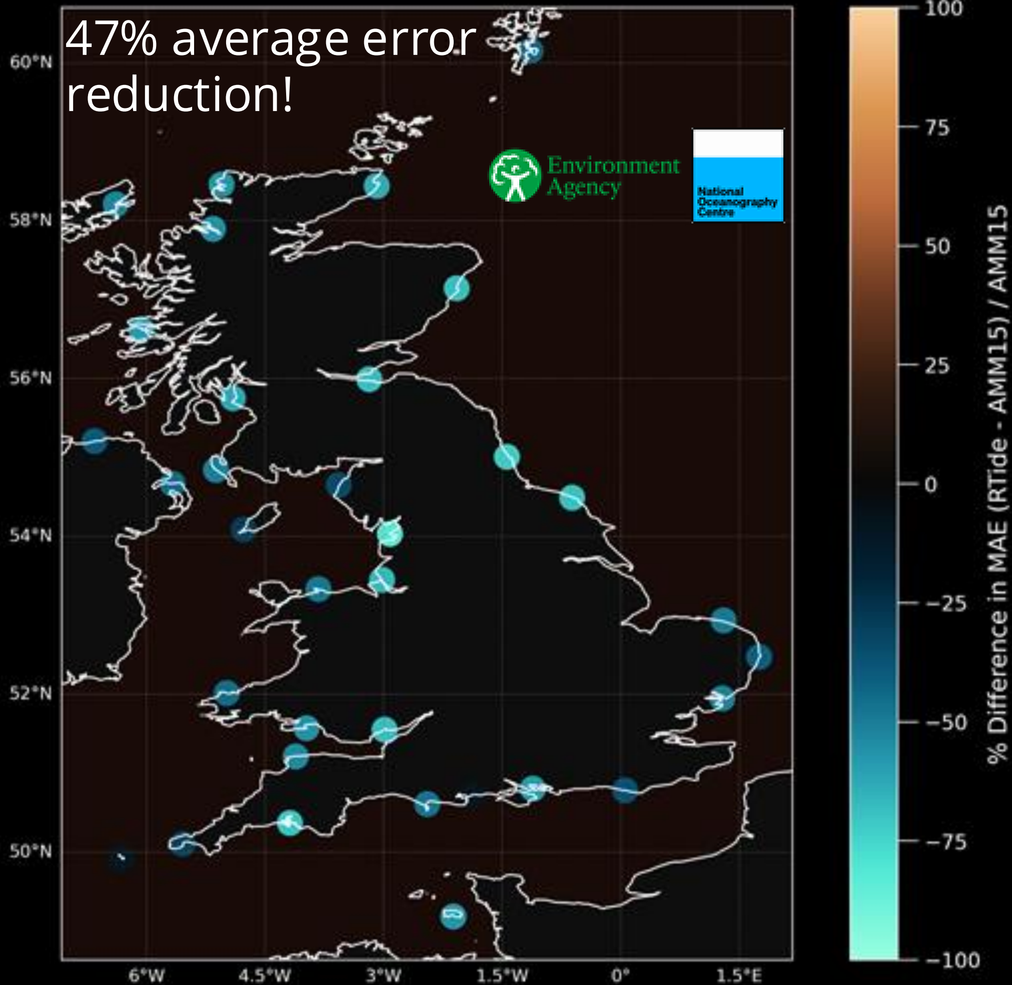


Augmenting traditional forecasts



Ministry of Infrastructure and Water Management

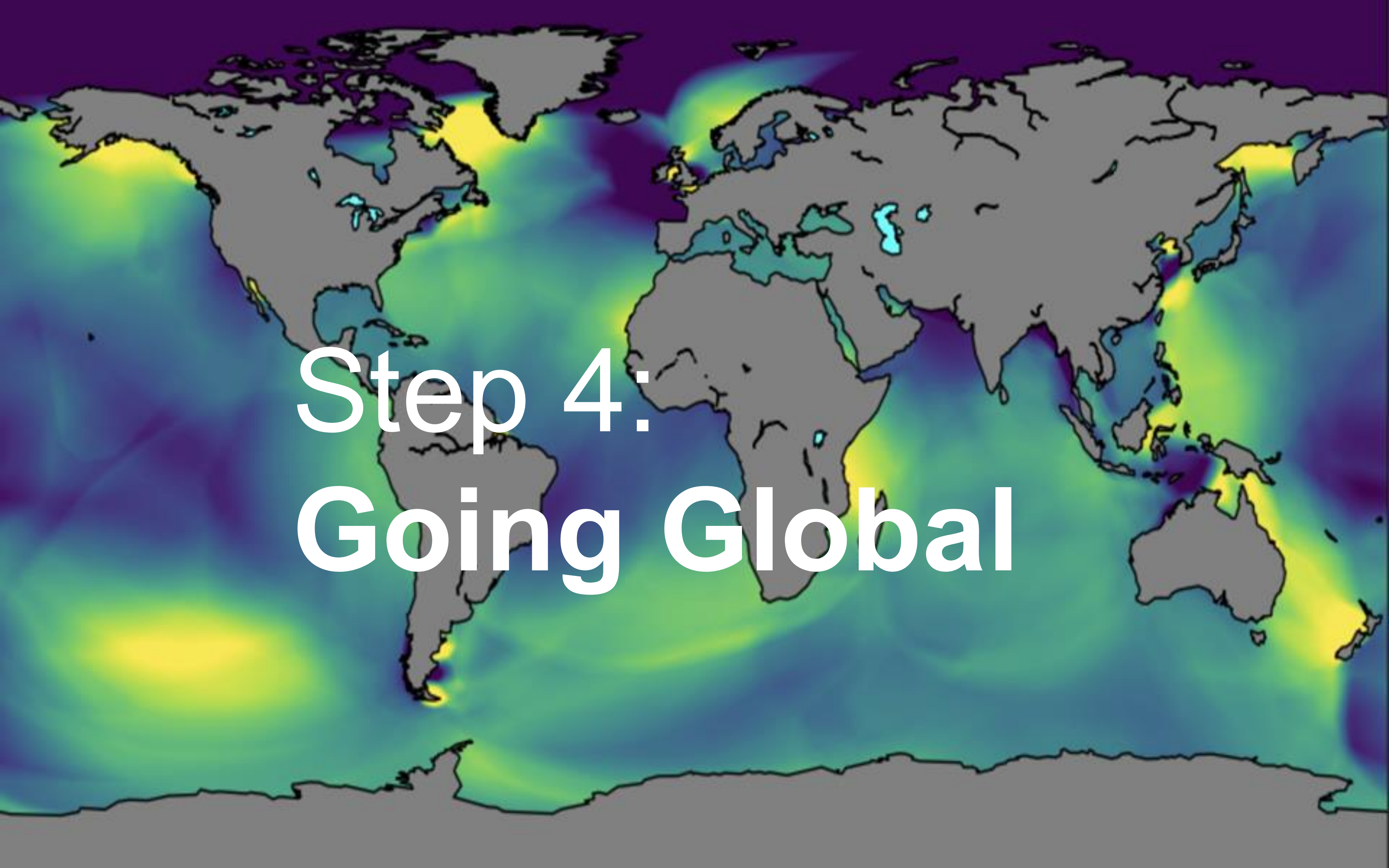
Training 2022-2023 and Forecast 2024

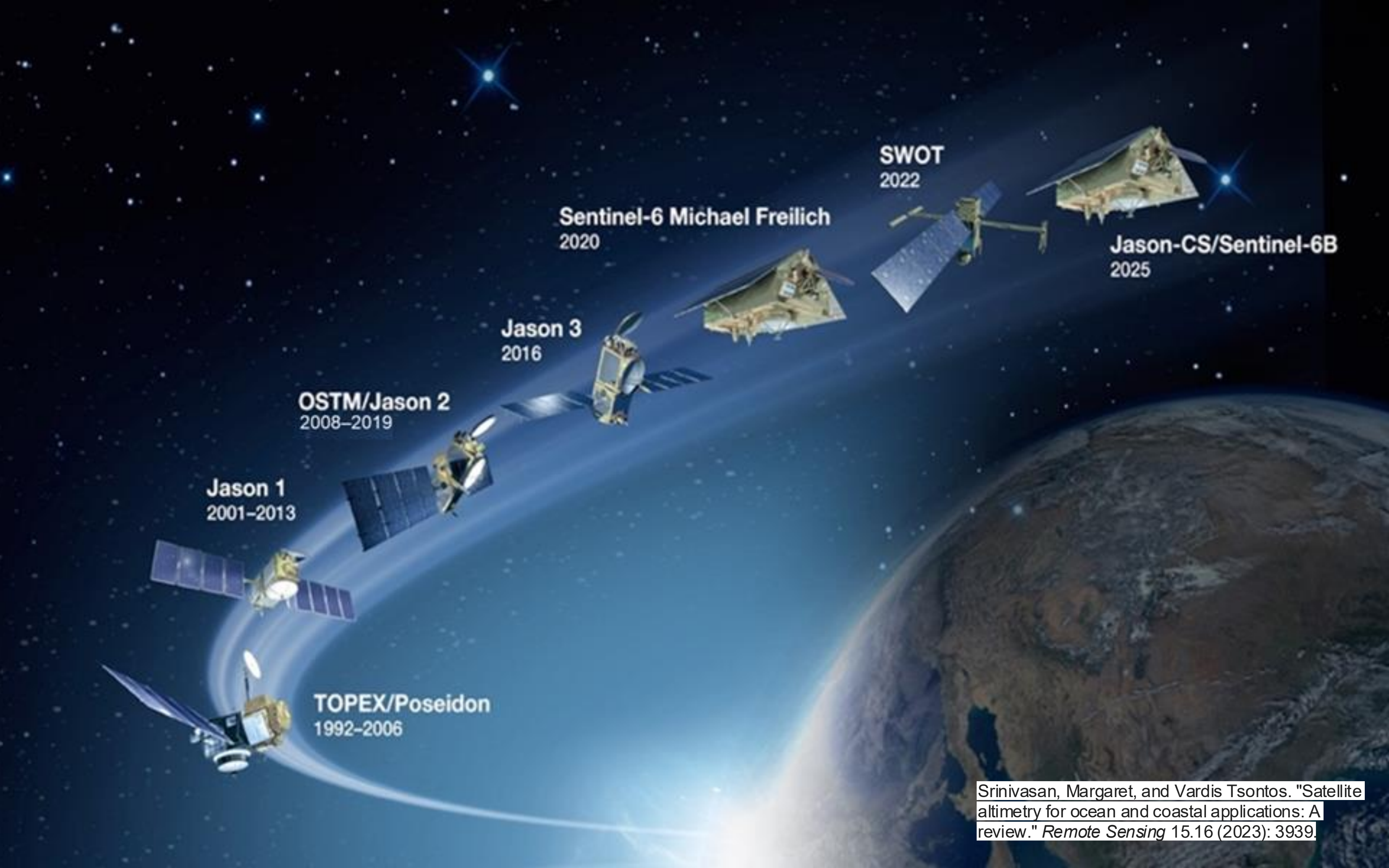
DCSM7 + RTide: 36% lower error

Model	MAE	Brier Score (99th percentile)
AMM15	.155m	.0009
AMM15 + RTide	.063m	.0002

$$\mathrm{BS} = \frac{1}{N}\sum (f_i - o_i)^2$$

Step 4:
Going Global





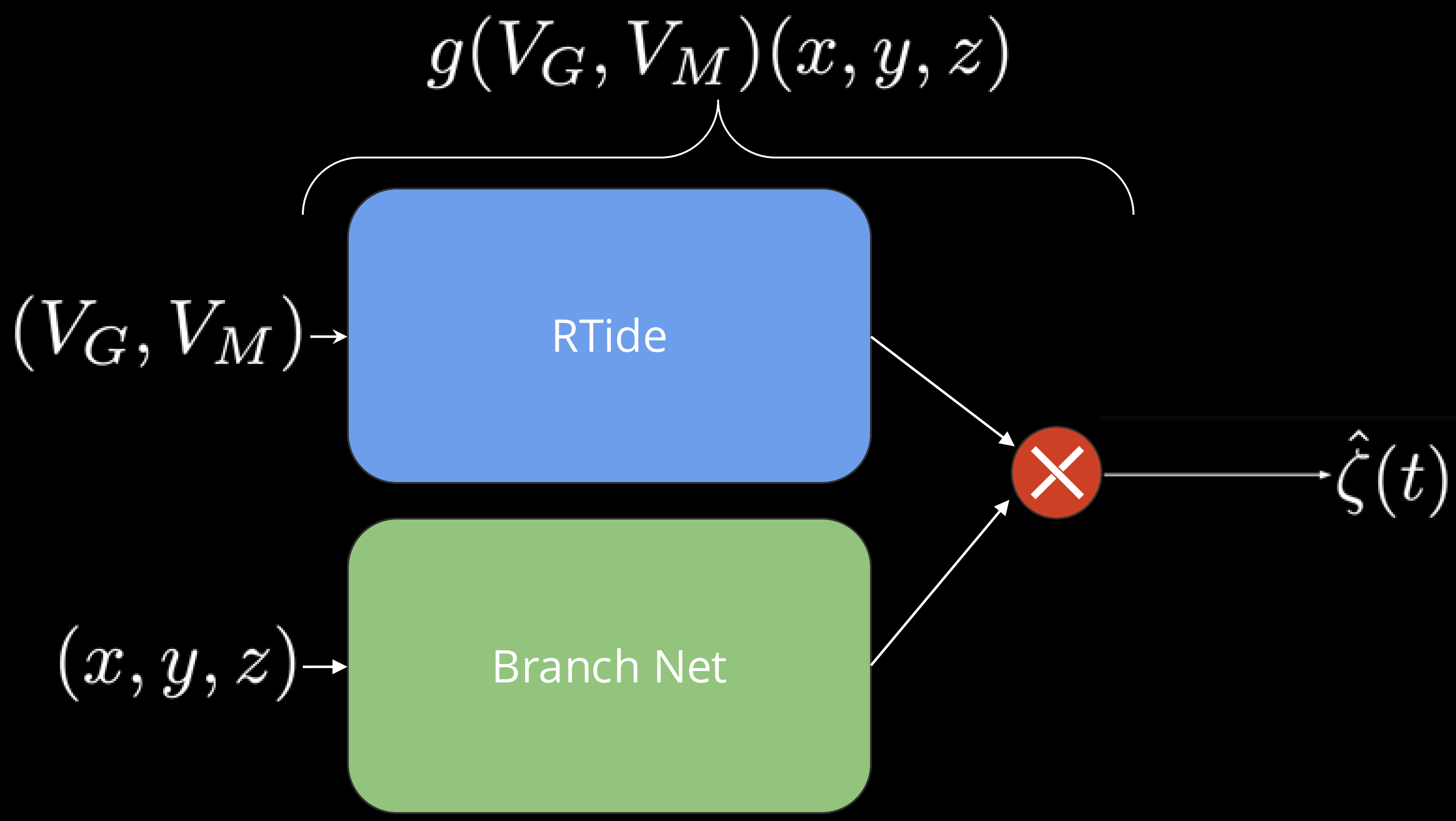
Srinivasan, Margaret, and Vardis Tsontos. "Satellite altimetry for ocean and coastal applications: A review." *Remote Sensing* 15.16 (2023): 3939.

Single Location:

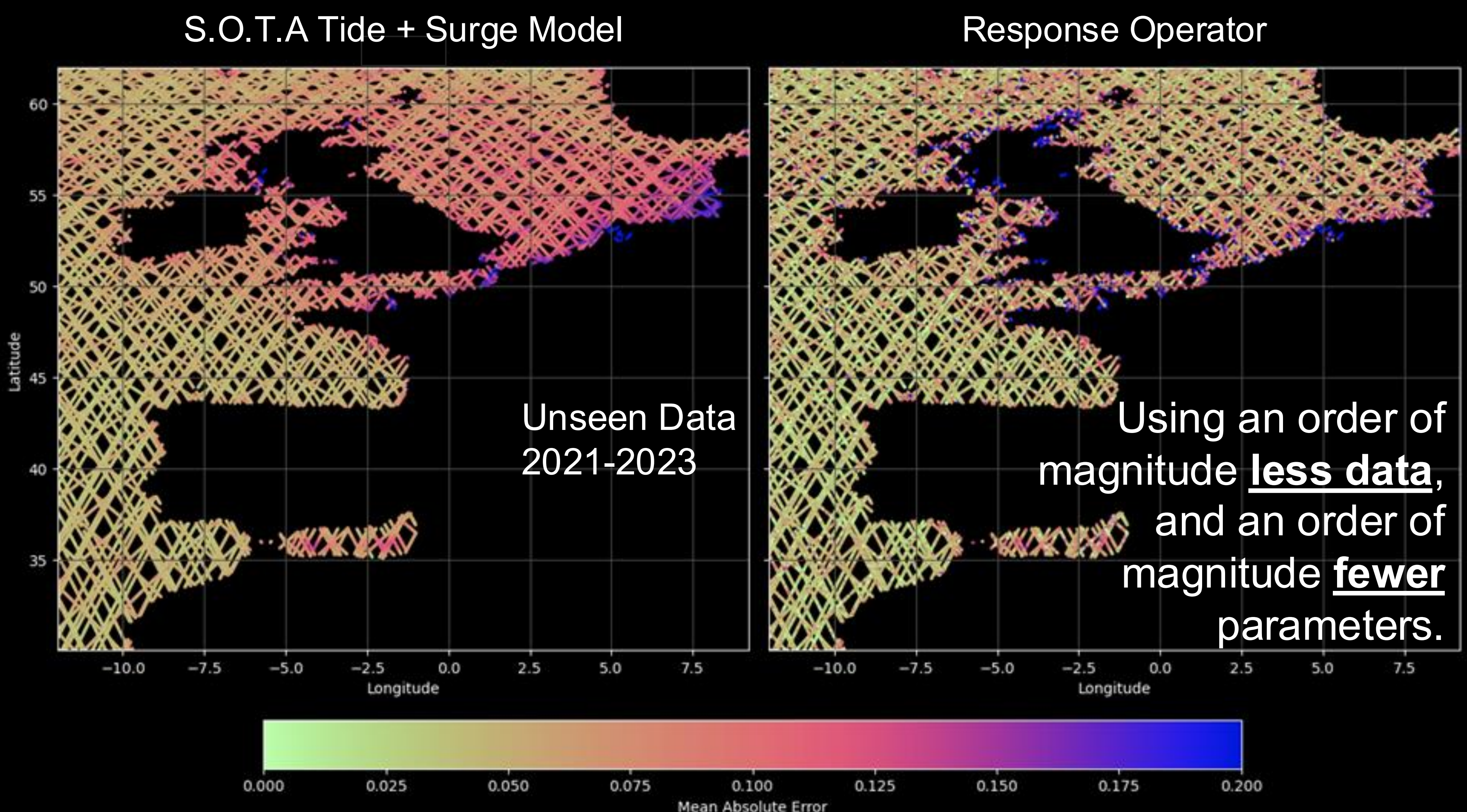
$$f(V_G, V_M) \longrightarrow \hat{\zeta}(t)$$

Multiple Locations:

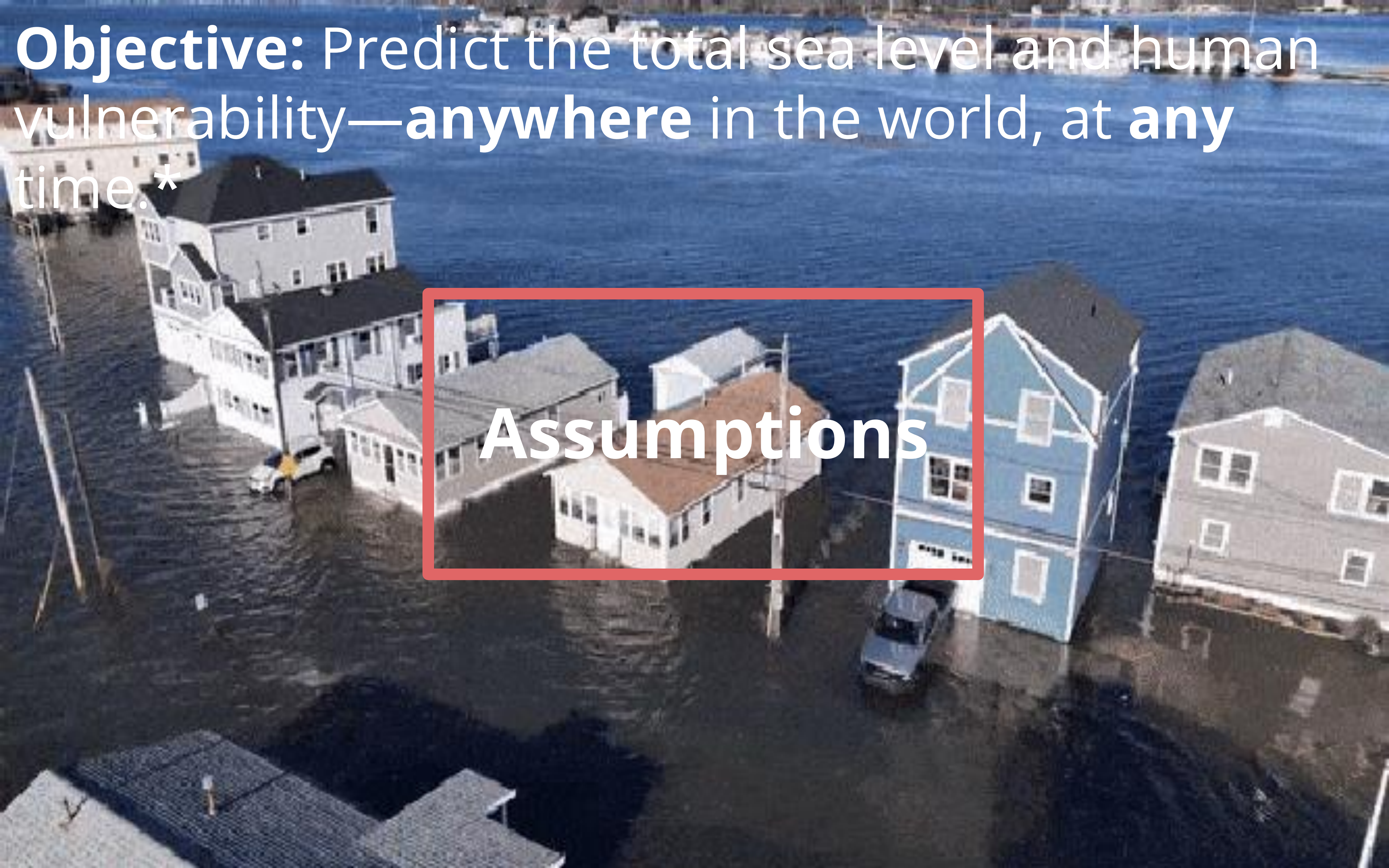
$$g(V_G, V_M)(x, y, z) \longrightarrow \hat{\zeta}(t)$$



Lu, Lu, et al. "Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators." *Nature machine intelligence* 3.3 (2021): 218-229.

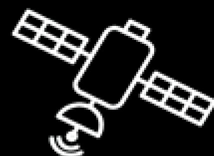

S.O.T.A Tide + Surge Model
Response Operator
Unseen Data
2021-2023
Using an order of
magnitude less data,
and an order of
magnitude fewer
parameters.
Latitude
Longitude
60
55
50
45
40
35
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
Mean Absolute Error

Objective: Predict the total sea level and human vulnerability—**anywhere** in the world, at **any** time.*

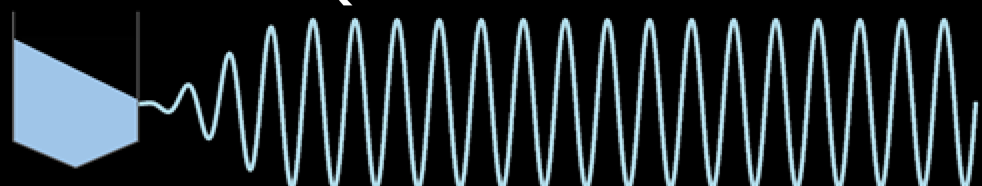


Assumptions

Special thanks to supervisors and collaborators:
Thomas Adcock, Tianning Tang, Stephen Roberts, and Jeff Polton



Questions?



thomas.monahan@eng.ox.ac.uk

NEW COLLEGE UNIVERSITY OF OXFORD
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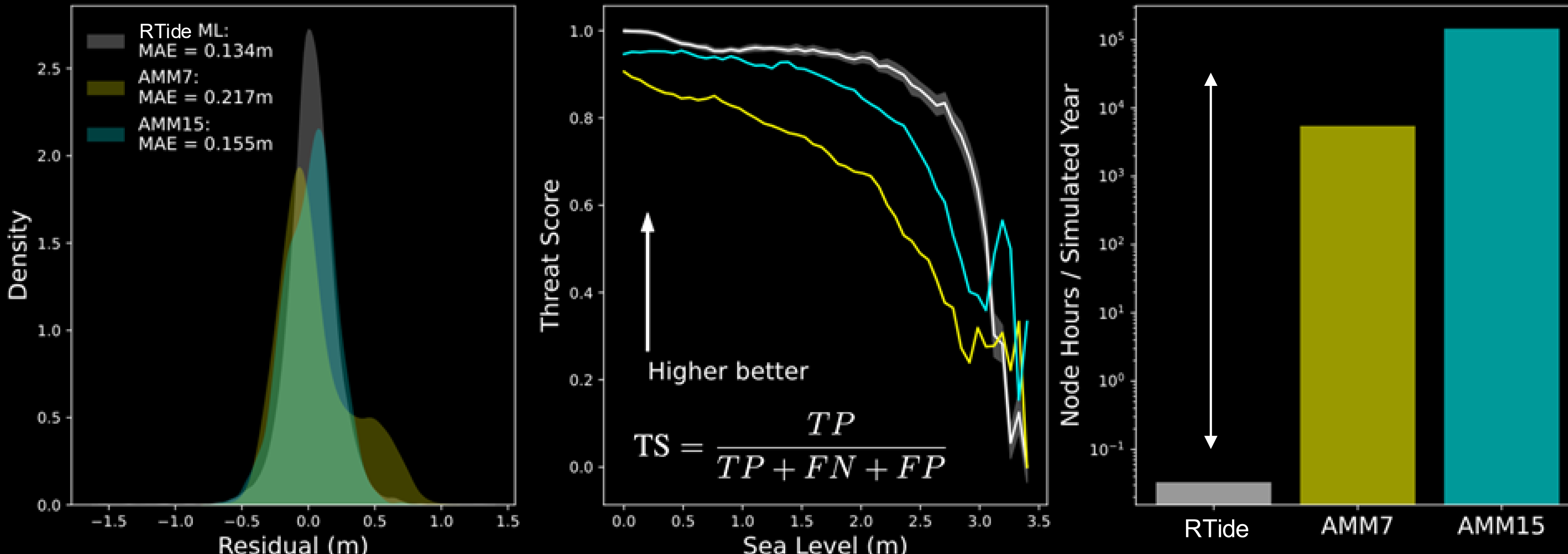
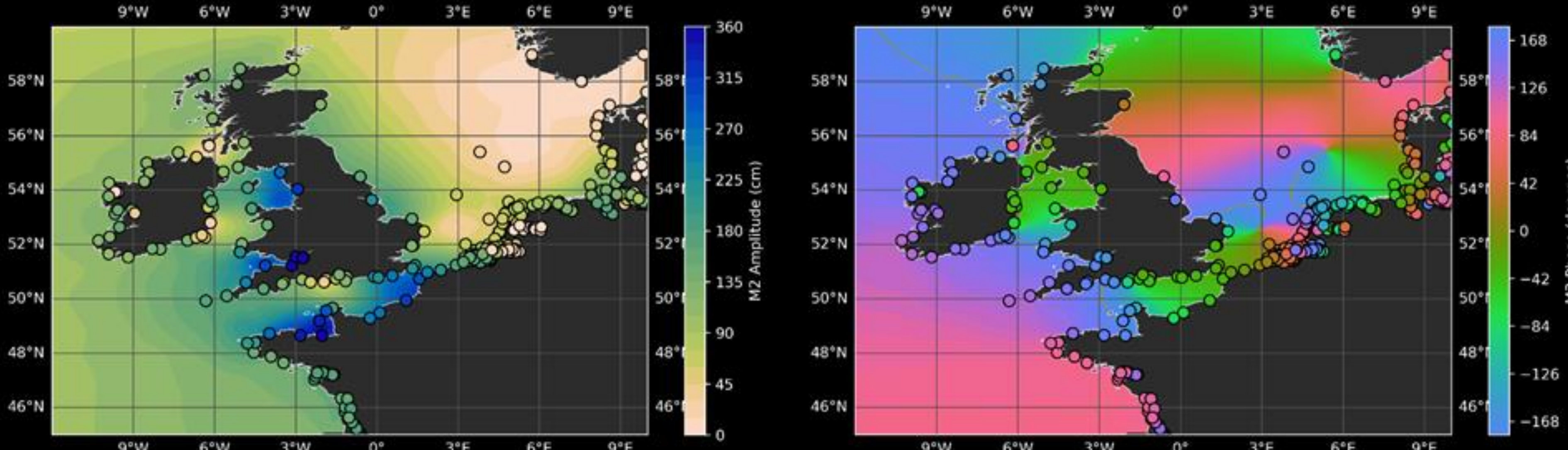
Eric and Wendy Schmidt AI in Science Postdoctoral Fellowship, a program of SCHMIDT FUTURES
DEPARTMENT OF ENGINEERING SCIENCE
UNIVERSITY OF OXFORD

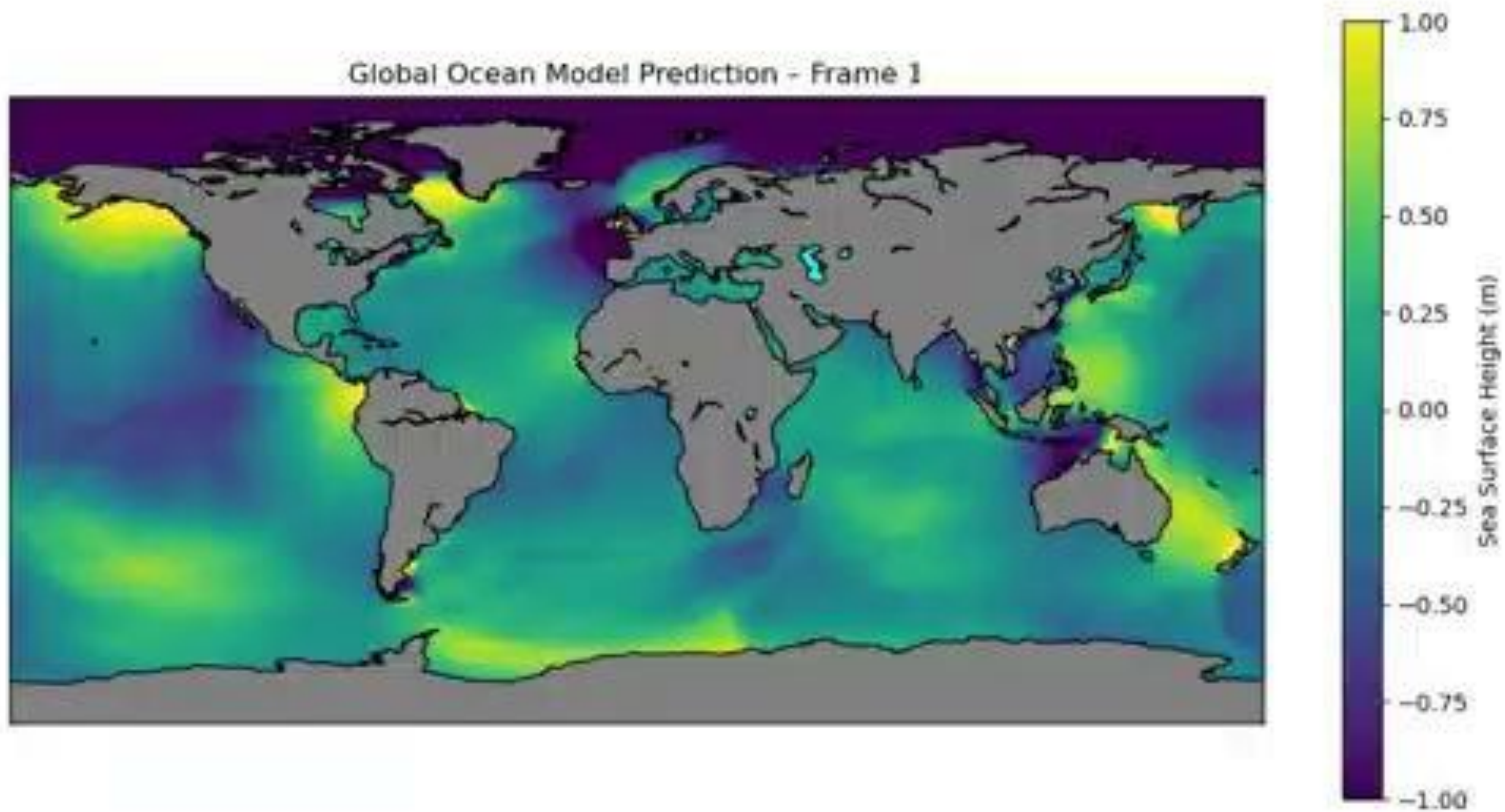
Do we need the numerical model at all?

>4 million times more efficient!

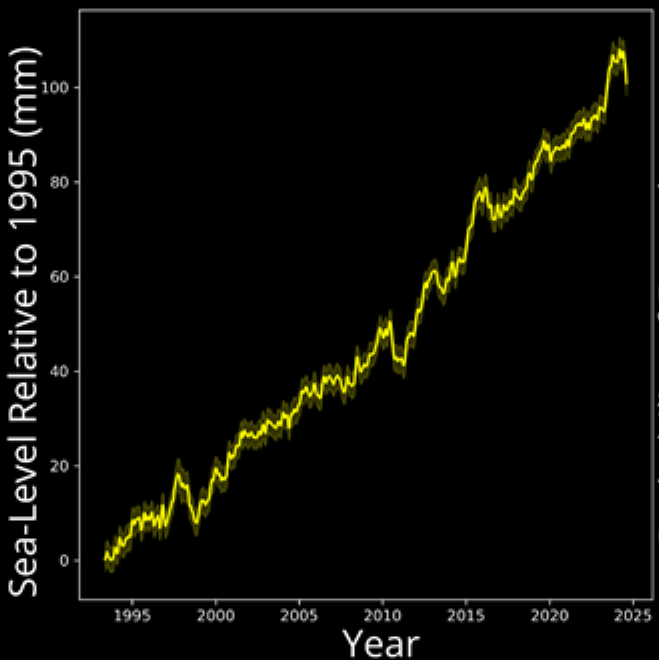


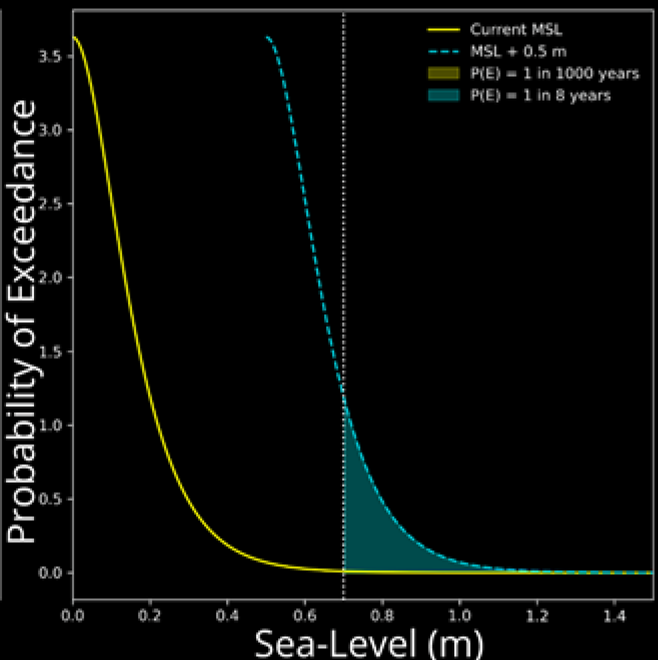

9°W
6°W
3°W
0°
3°E
6°E
9°E
58°N
56°N
54°N
52°N
50°N
48°N
46°N
M2 Amplitude (cm)
360
315
270
225
180
135
90
45
0
M2 Phase (degrees)
168
126
84
42
0
−42
−84
−126
−168

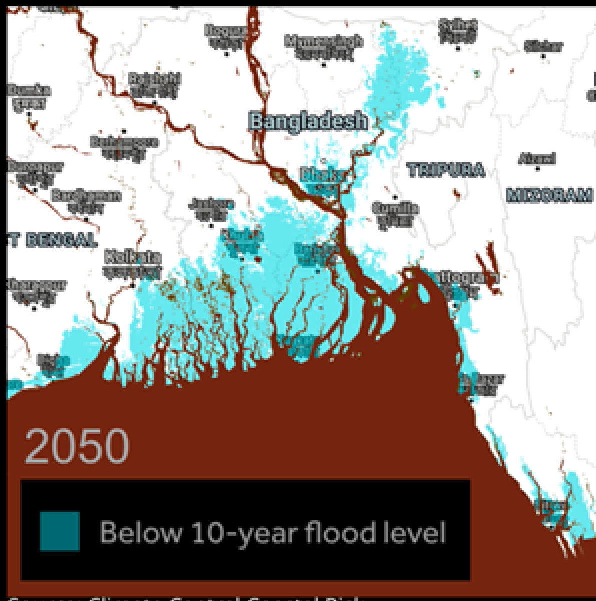
Going Global



Sea-level rise is dramatically reducing the return period of flooding events.



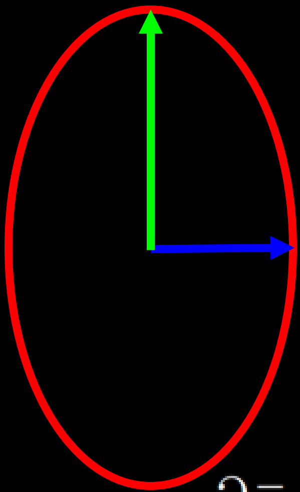




Source: Climate Central Coastal Risk


ATLANTIS
3

Tidal currents theory



$$u(t) = \sum_n A_n^u \cos(\omega_n t + \phi_n^u)$$

$$v(t) = \sum_n A_n^v \cos(\omega_n t + \phi_n^v)$$

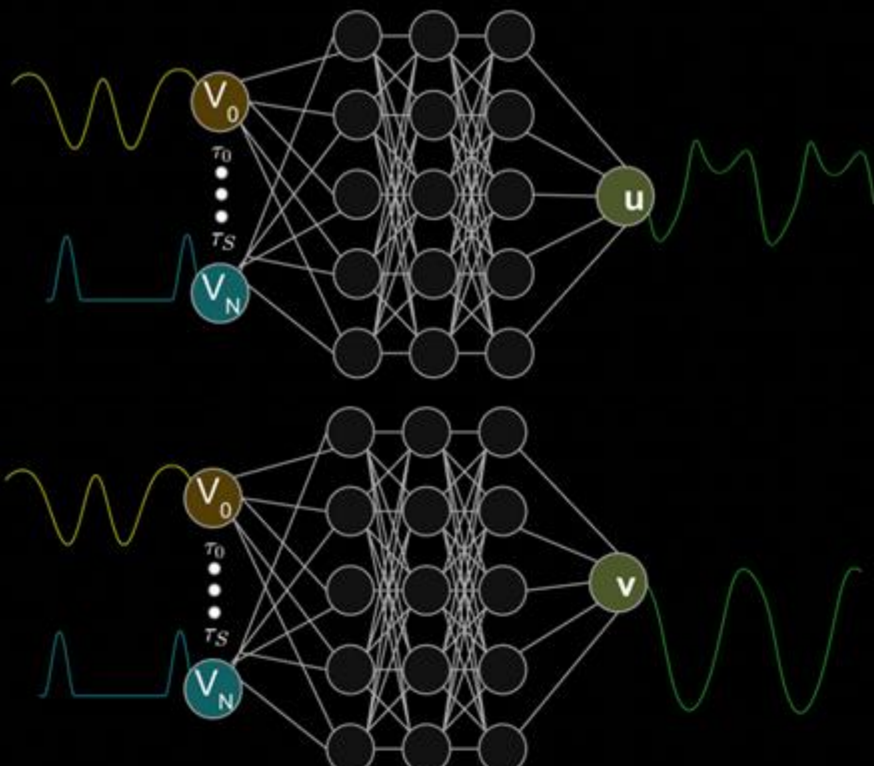
$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} - f\bar{v} = -g\frac{\partial}{\partial x}(\zeta - \zeta_g' - \zeta_p') + \frac{1}{\rho h}\left(\tau_x^w - \tau_x^b\right)$$

$$\frac{\partial \bar{v}}{\partial t} + \frac{\partial \bar{u}\bar{v}}{\partial x} + \frac{\partial \bar{v}^2}{\partial y} + f\bar{u} = -g\frac{\partial}{\partial y}(\zeta - \zeta_g' - \zeta_p') + \frac{1}{\rho h}\left(\tau_y^w - \tau_y^b\right)$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = -\frac{1}{h}\frac{\partial \zeta}{\partial t}.$$

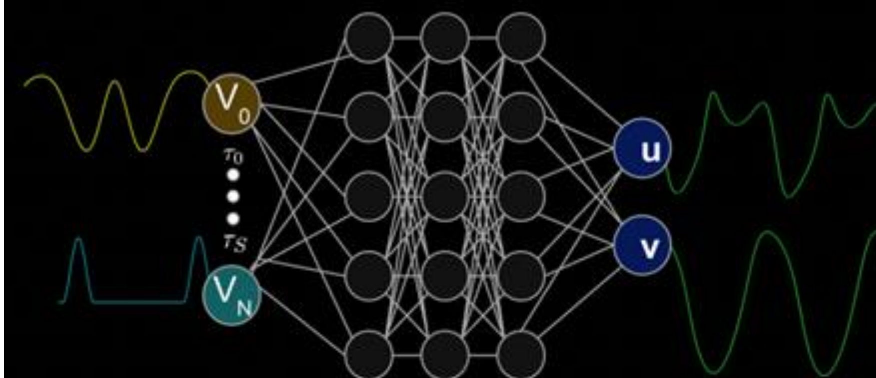
Uncoupled (standard)

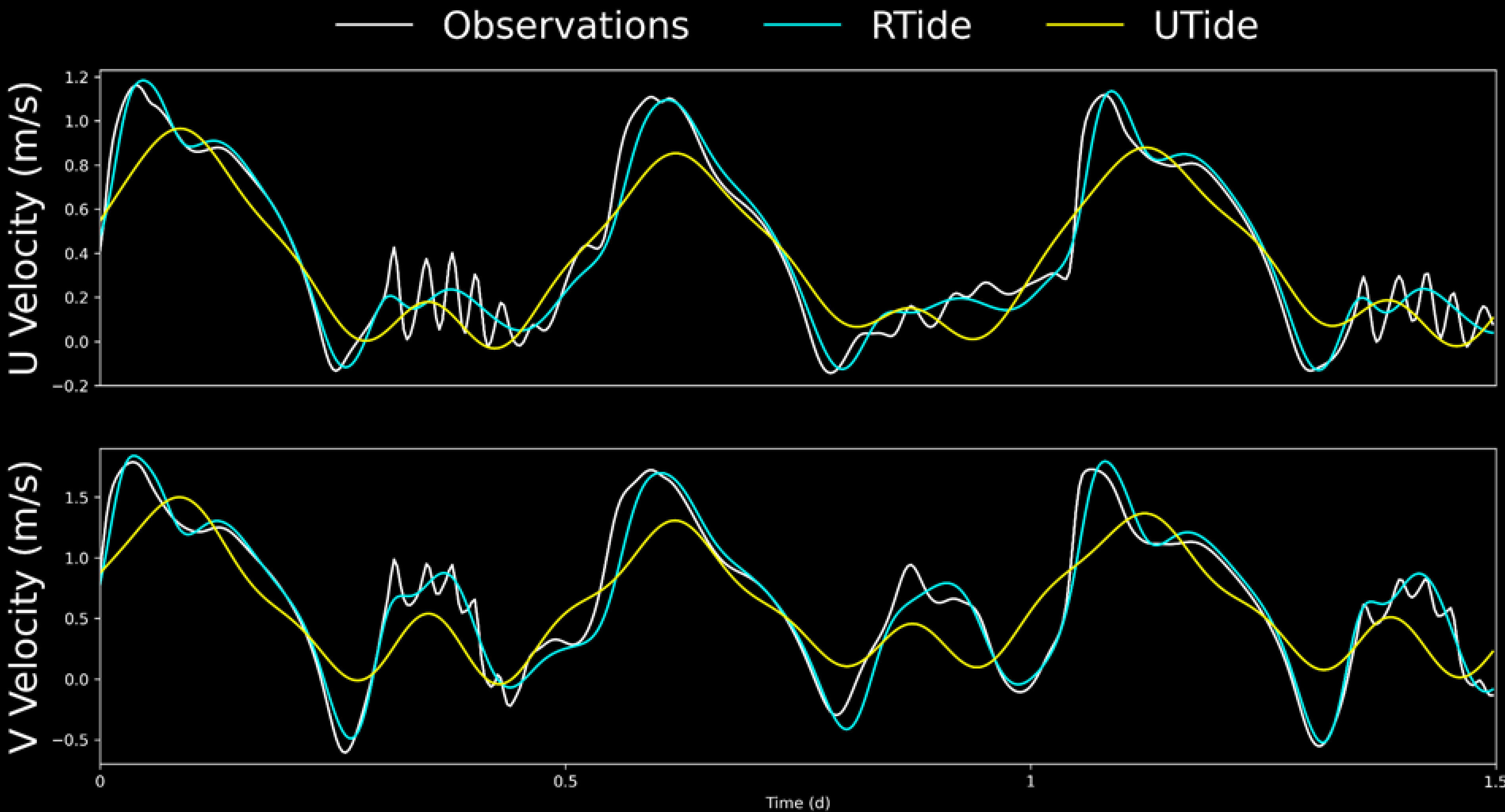
$$\hat{\beta}(t) = \begin{bmatrix} \hat{u}(t) \\ \hat{v}(t) \end{bmatrix} = \begin{bmatrix} f(V_0(t-\tau_0),\ldots,V_X(t-\tau_S)) \\ g(V_0(t-\tau_0),\ldots,V_X(t-\tau_S)) \end{bmatrix}$$



Coupled

$$\hat{\beta}(t) = \begin{bmatrix} \hat{u}(t) \\ \hat{v}(t) \end{bmatrix} = f(V_0(t-\tau_0),\ldots,V_X(t-\tau_S))$$



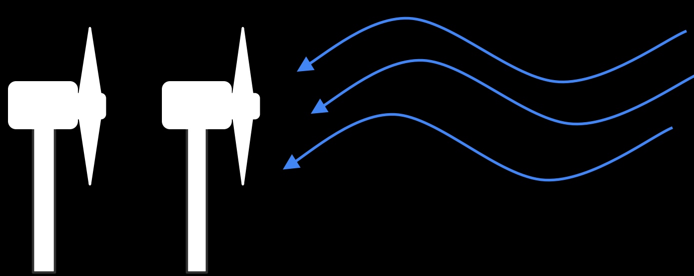


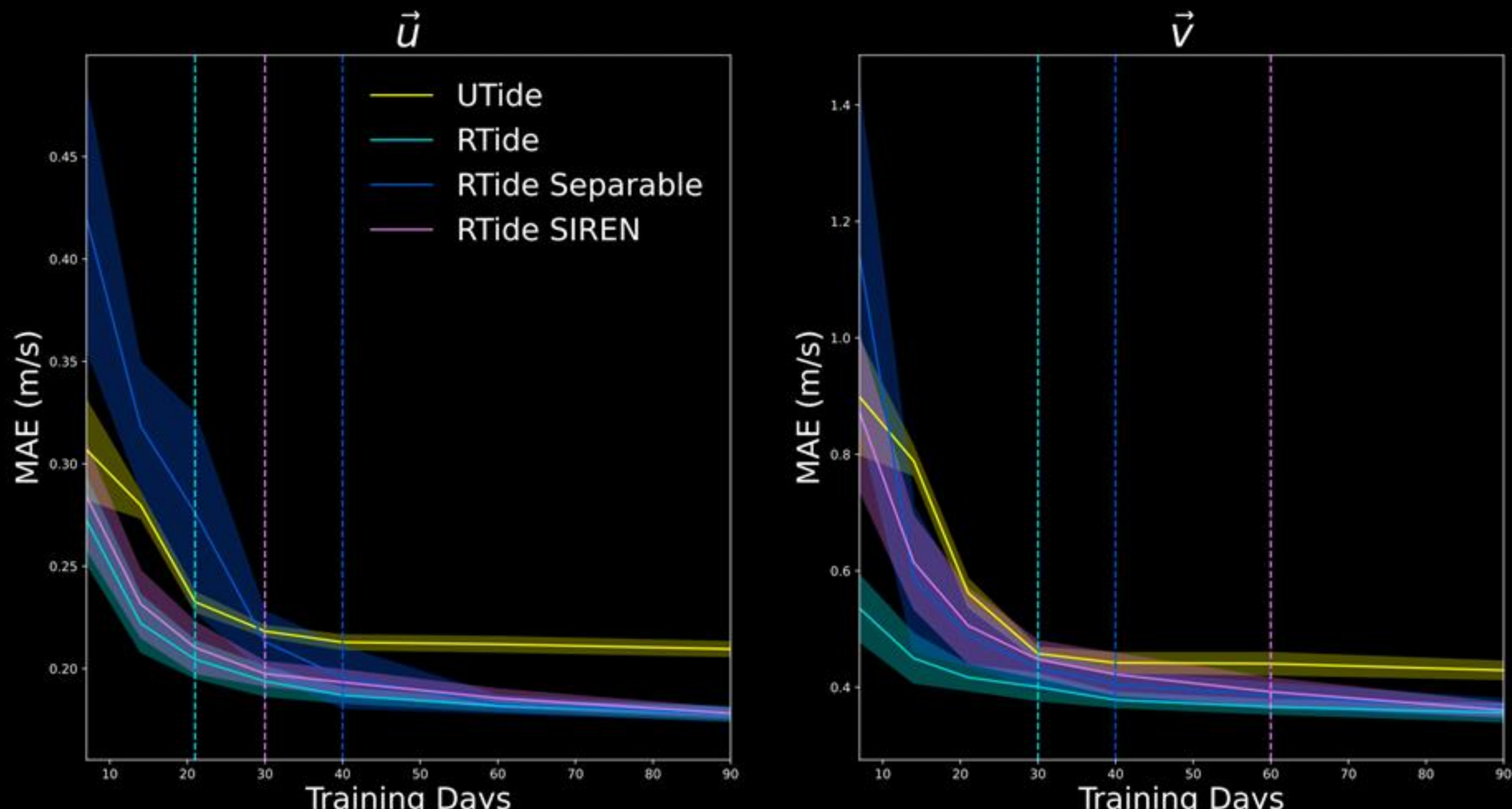
Observations
RTide
UTide
U Velocity (m/s)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
V Velocity (m/s)
1.5
1.0
0.5
0.0
−0.5
0
0.5
1
1.5
Time (d)

Predicted tidal turbine power percentage error over 180 day forecast on test-data.

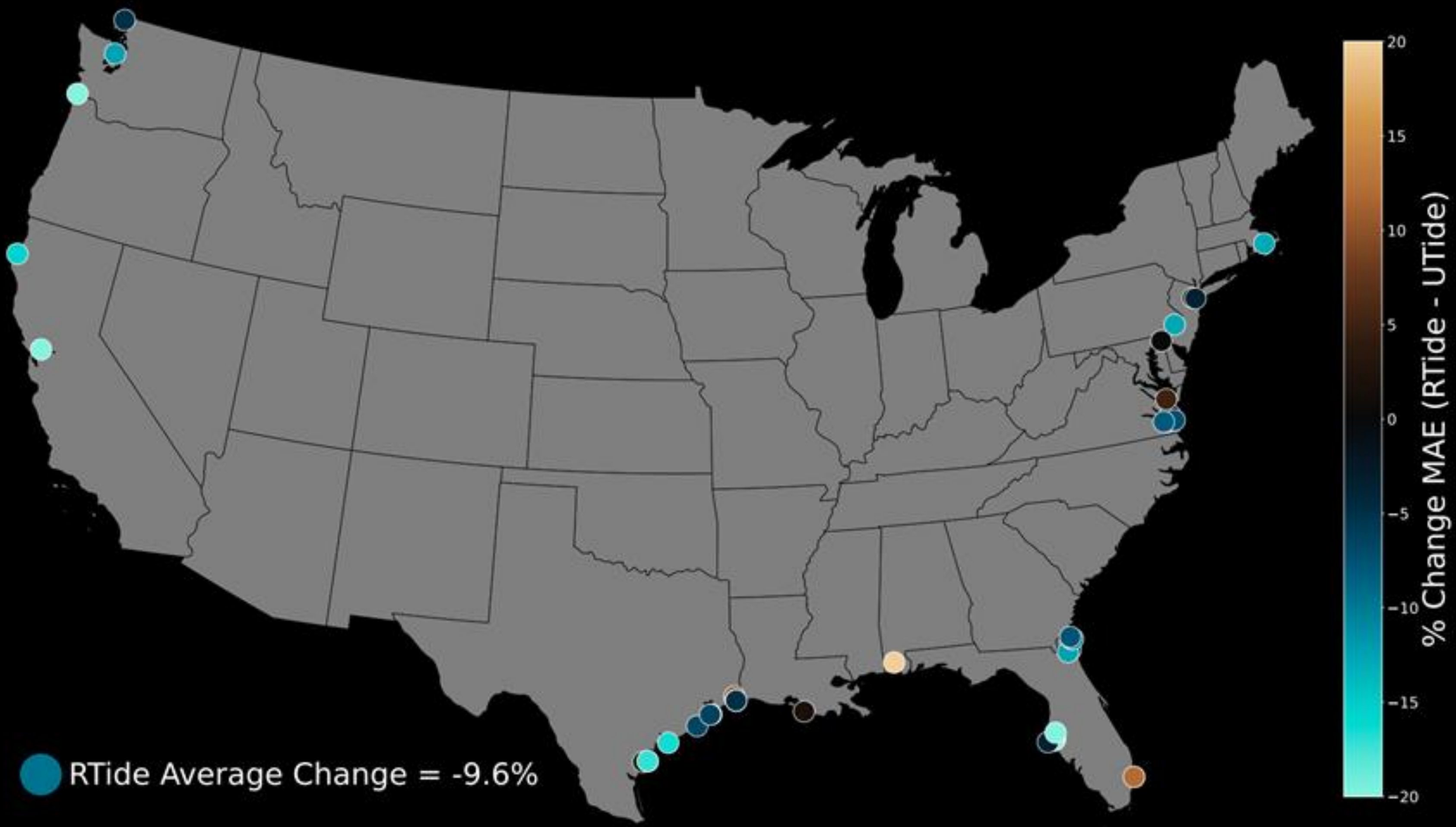
Model	P_{Error}
UTide	24.8%
Operational Numerical Model	22.4%
RTide	**12.3%**

Using an order of magnitude less data!

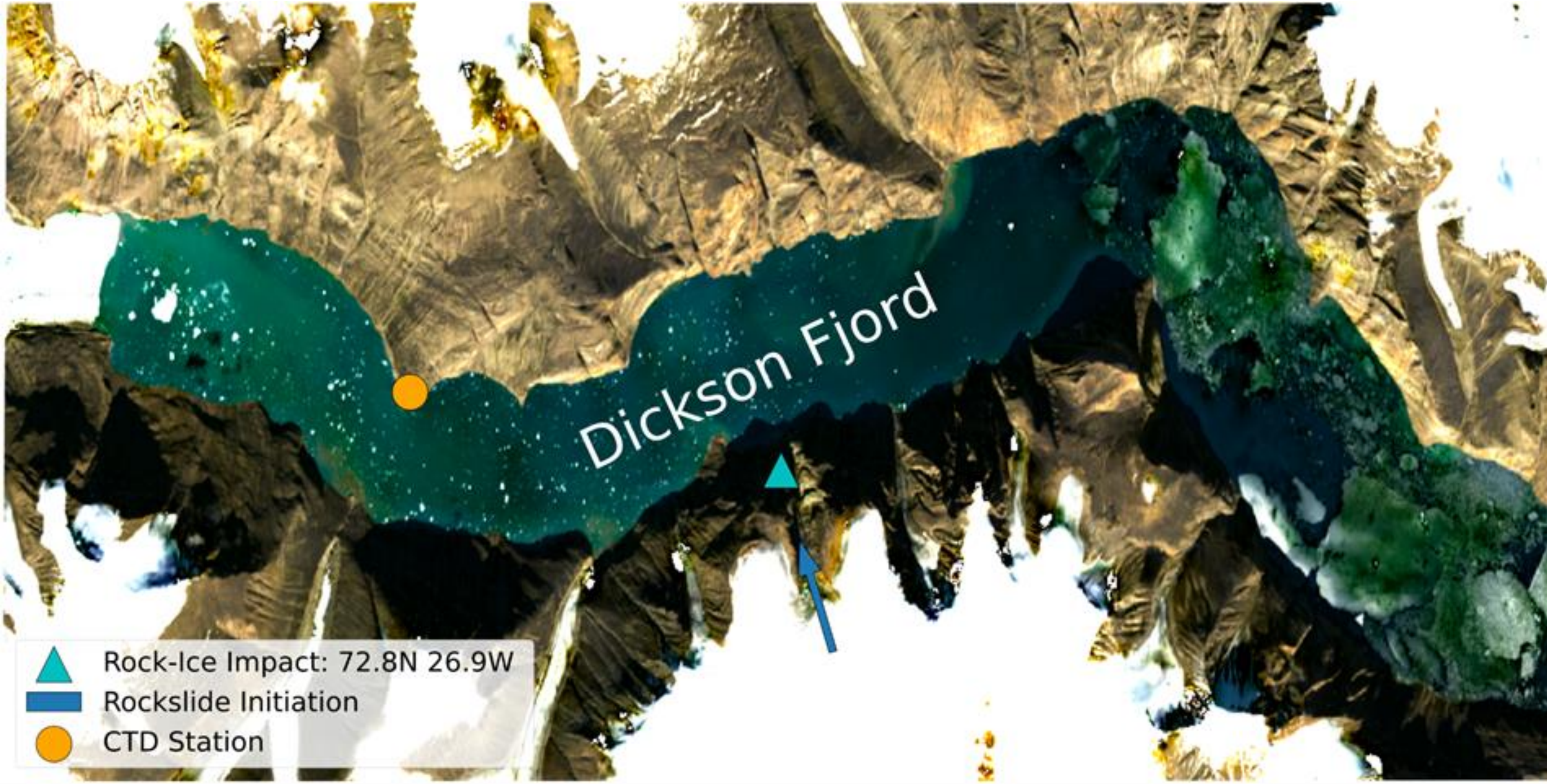




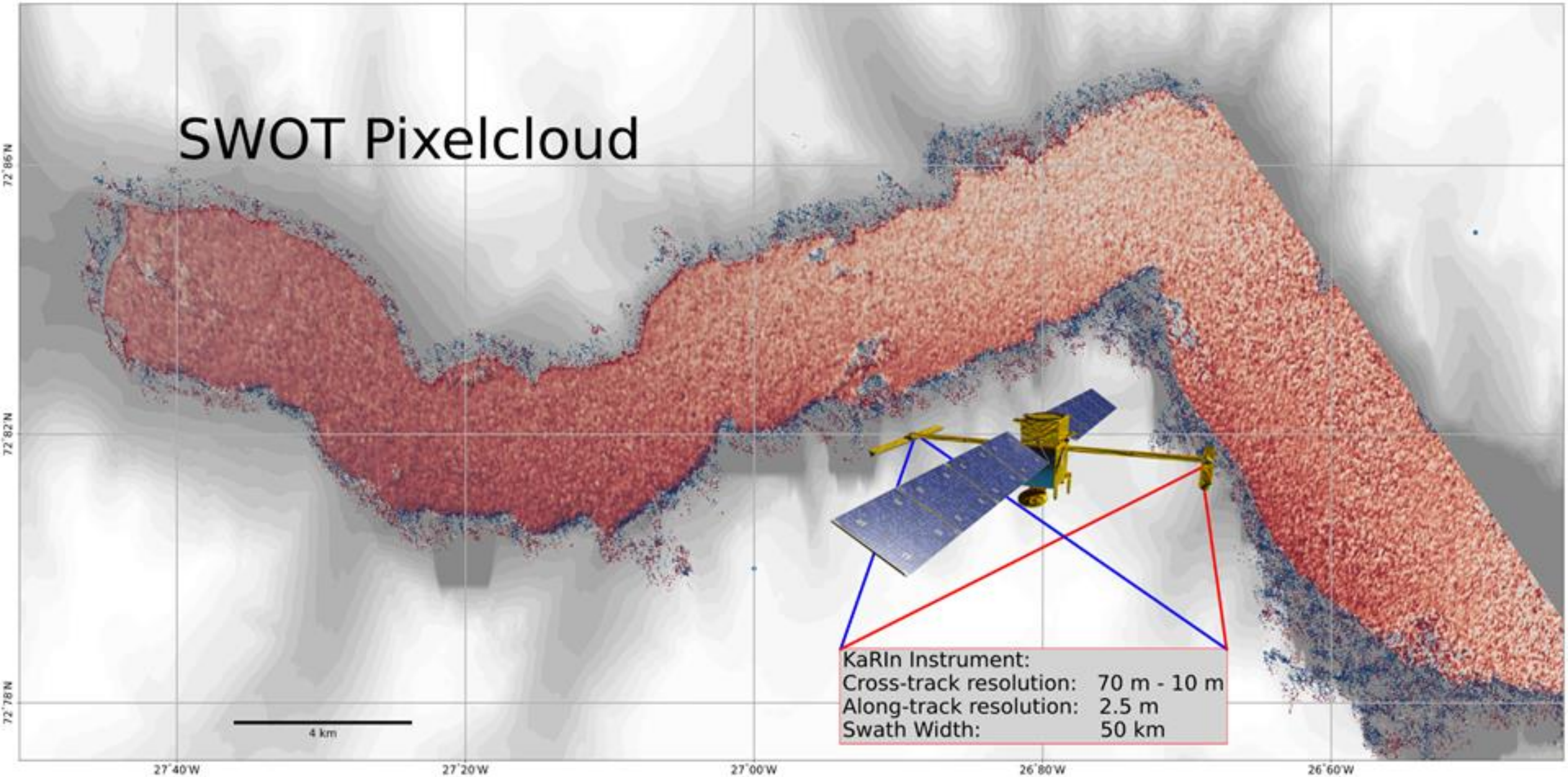
$\vec{u}$
$\vec{v}$
UTide
RTide
RTide Separable
RTide SIREN
MAE (m/s)
Training Days
0.20
0.25
0.30
0.35
0.40
0.45
0.4
0.6
0.8
1.0
1.2
1.4
10
20
30
40
50
60
70
80
90

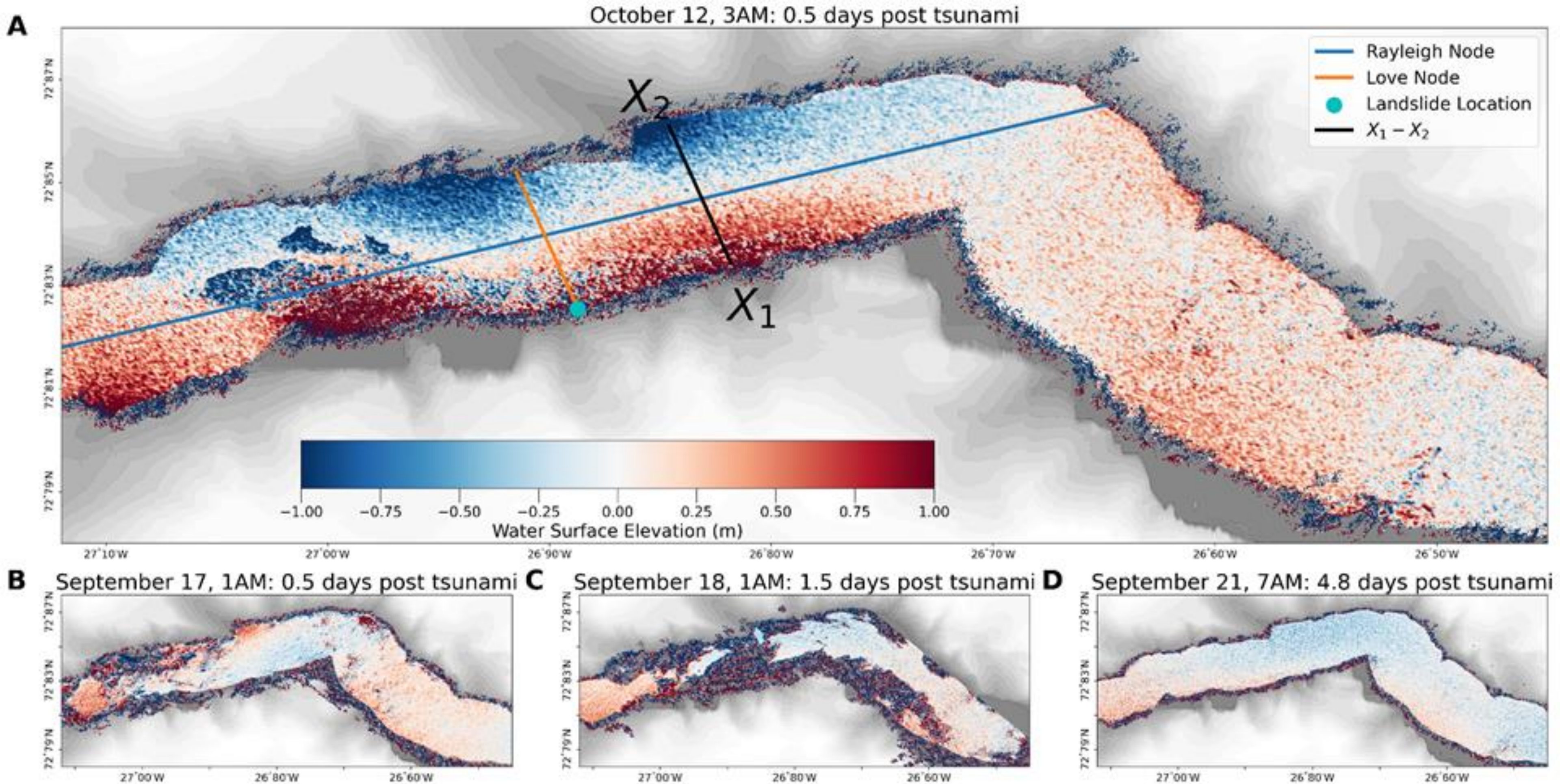


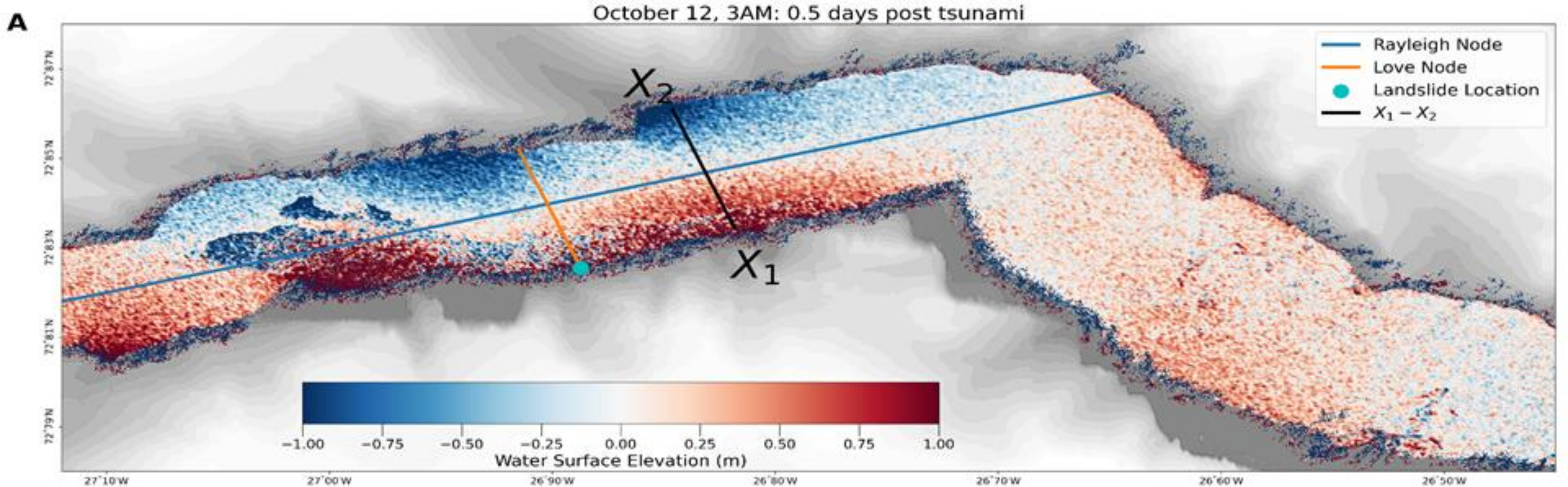
RTide Average Change = -9.6%
% Change MAE (RTide - UTide)
20
15
10
5
0
−5
−10
−15
−20



Dickson Fjord
Rock-Ice Impact: 72.8N 26.9W
Rockslide Initiation
CTD Station


SWOT Pixelcloud
KaRIn Instrument:
Cross-track resolution: 70 m - 10 m
Along-track resolution: 2.5 m
Swath Width: 50 km
4 km
27°40'W
27°20'W
27°00'W
26°80'W
26°60'W
72°86'N
72°82'N
72°78'N


A
October 12, 3AM: 0.5 days post tsunami
Rayleigh Node
Love Node
Landslide Location
$X_1 - X_2$
X_2
X_1
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
Water Surface Elevation (m)
27˚10W
27˚00W
26˚90W
26˚80W
26˚70W
26˚60W
26˚50W
72˚87N
72˚85N
72˚83N
72˚81N
72˚79N
B
September 17, 1AM: 0.5 days post tsunami
C
September 18, 1AM: 1.5 days post tsunami
D
September 21, 7AM: 4.8 days post tsunami



SWOT Vs Simulation