



# Feedback control of liquid metal coating

*19<sup>th</sup> ERCOFTAC DaVinci Competition*

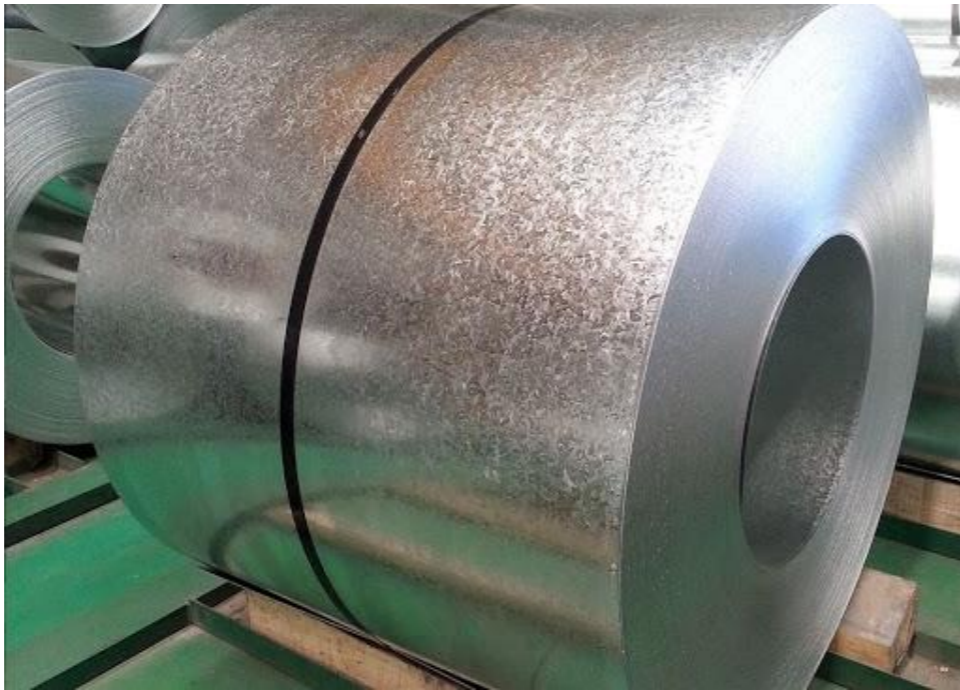
Fabio Pino

Supervisors: Miguel A. Mendez<sup>1</sup>, Benoit Scheid<sup>2</sup>

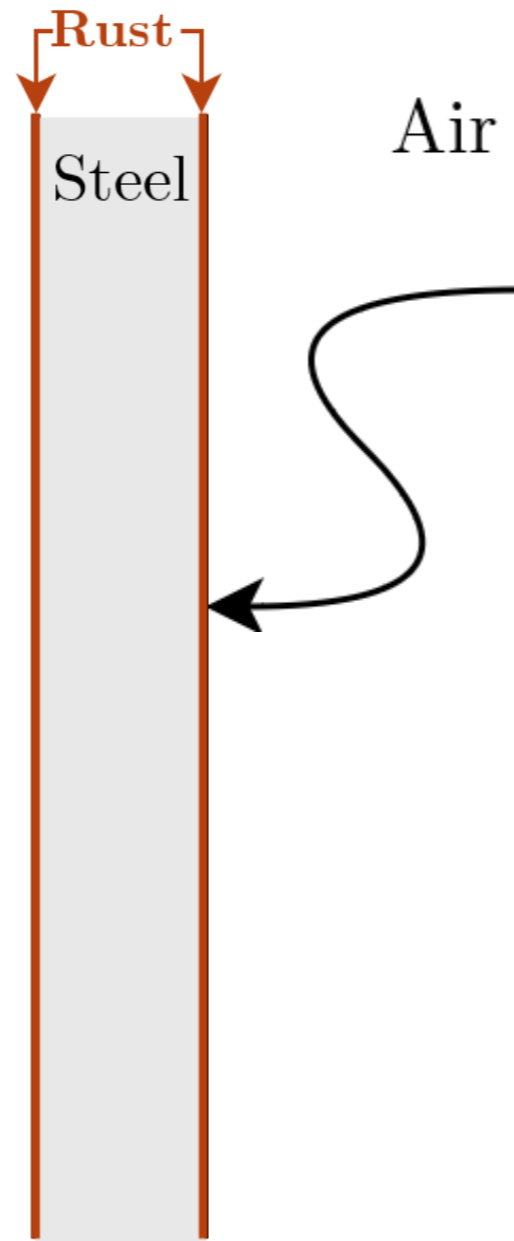
<sup>1</sup>Environmental and Applied Fluid Dynamics Department, von Karman Institute for Fluid Dynamics

<sup>2</sup>Transfers, Interfaces and Processes (TIPs) laboratory, Université libre de Bruxelles

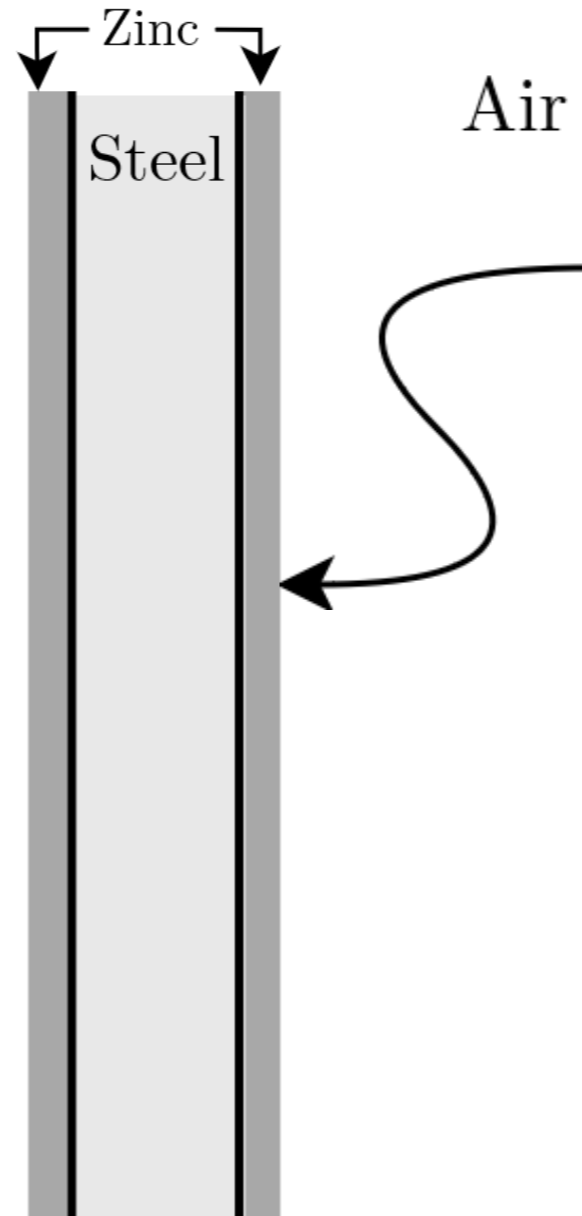
October 10<sup>th</sup>, 2024



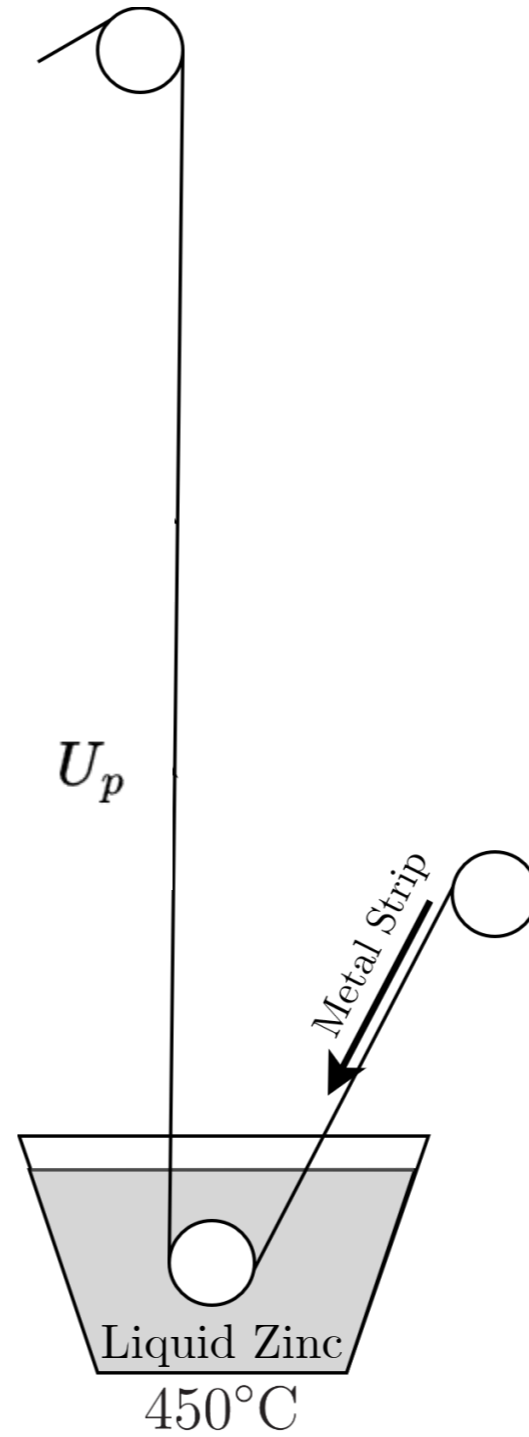
# Hot-Dip Galvanization



# Hot-Dip Galvanization



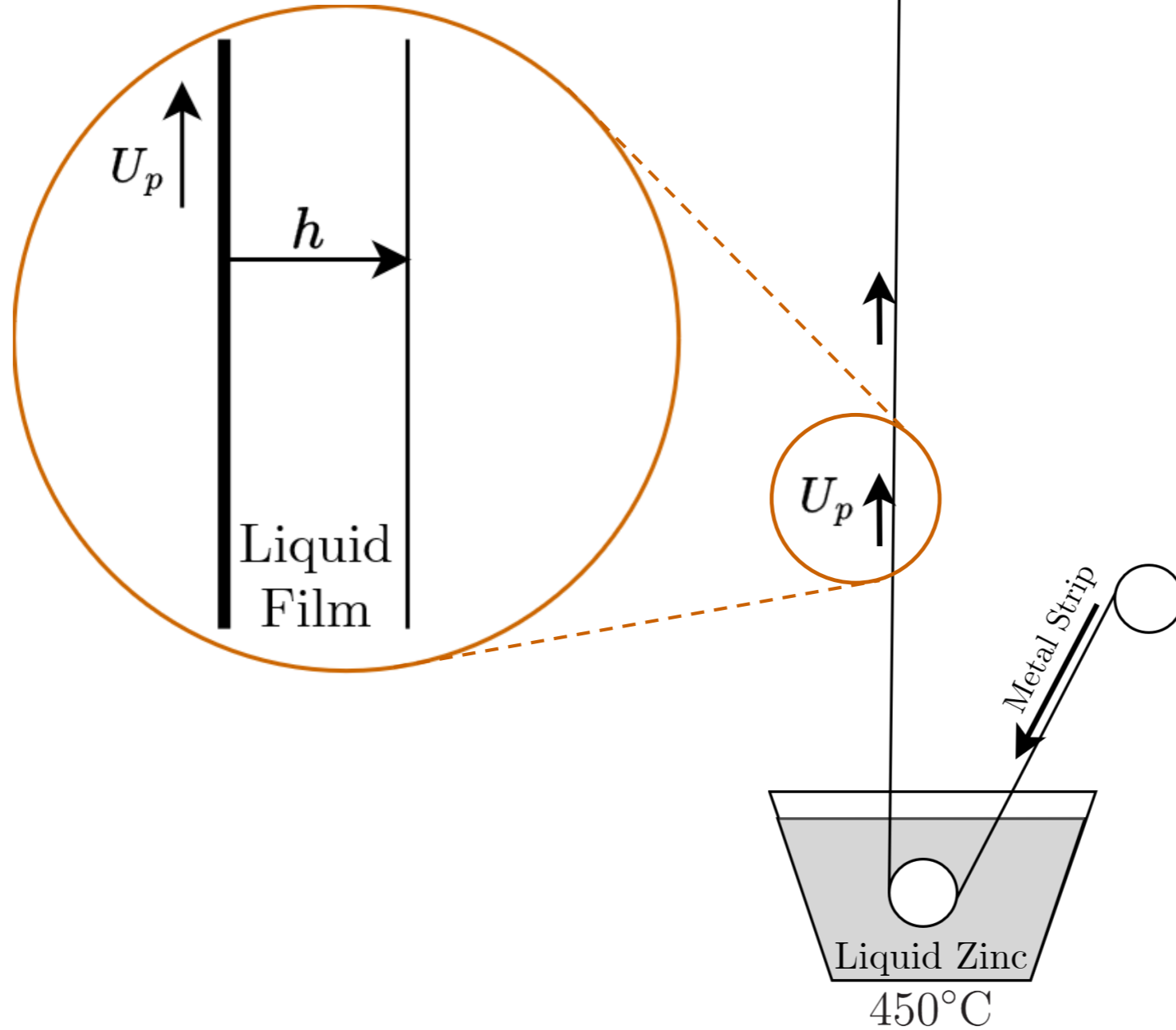
# Hot-Dip Galvanization



# Derjaguin's solution

$$h \propto \sqrt{U_p}$$

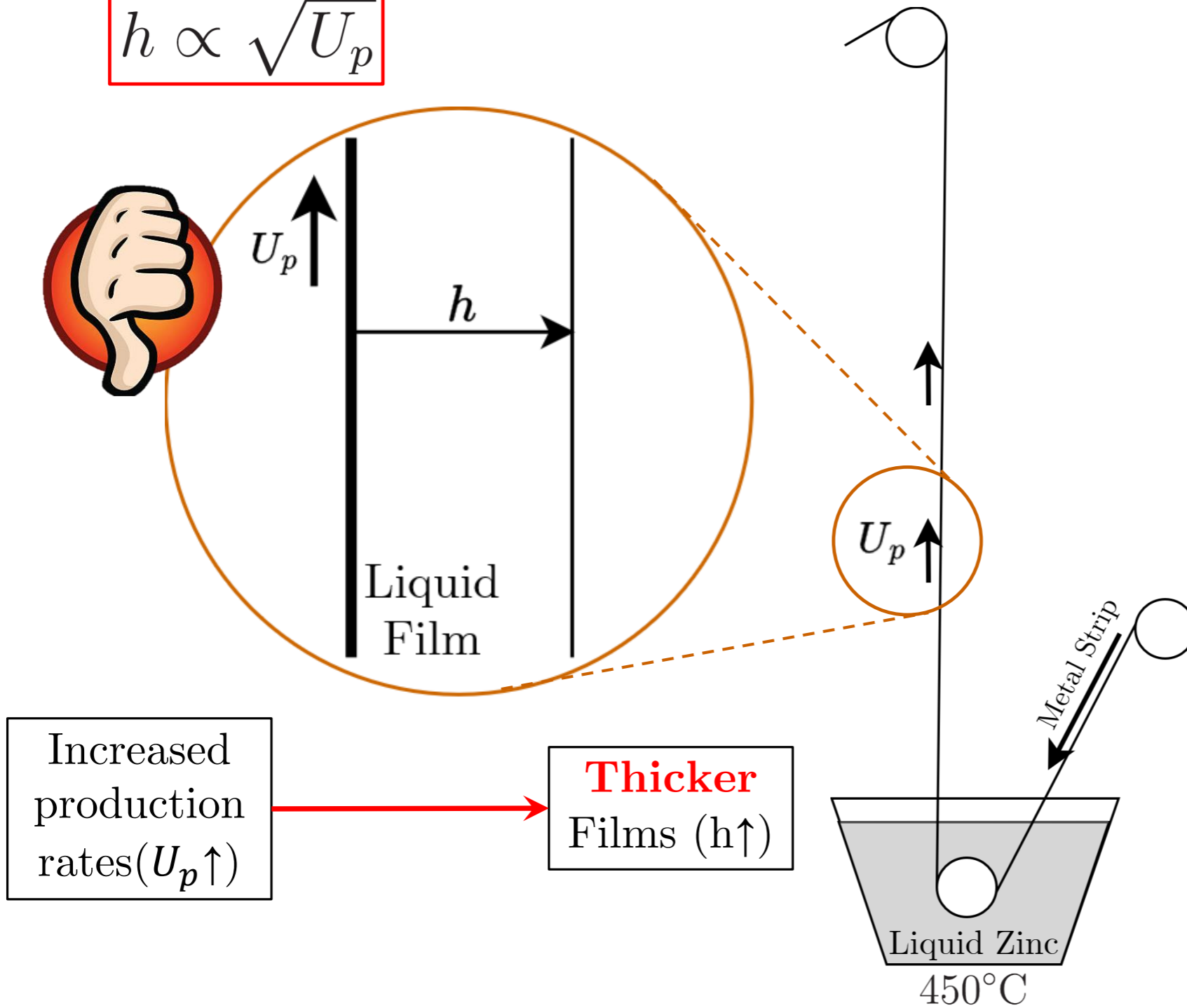
# Hot-Dip Galvanization



# Derjaguin's solution

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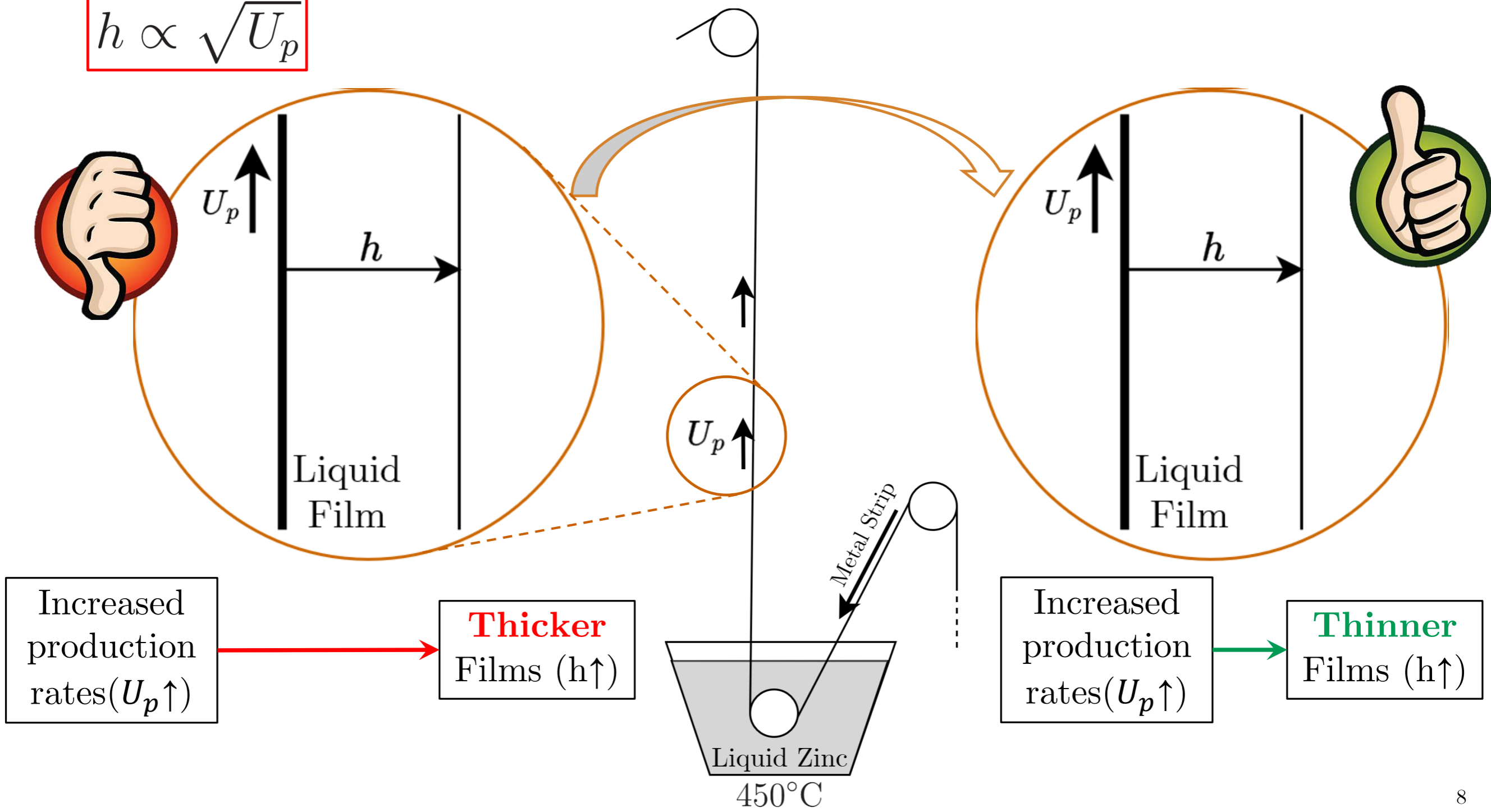
# Hot-Dip Galvanization



# Derjaguin's solution

$$h \propto \sqrt{U_p}$$

# Hot-Dip Galvanization

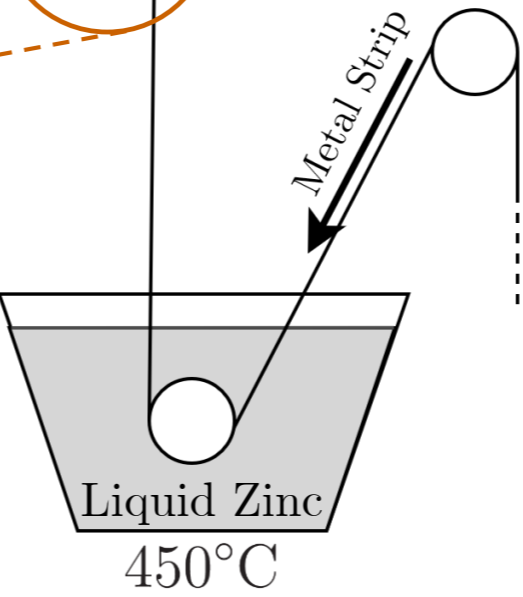


Increased production rates ( $U_p \uparrow$ )

**Thicker** Films ( $h \uparrow$ )

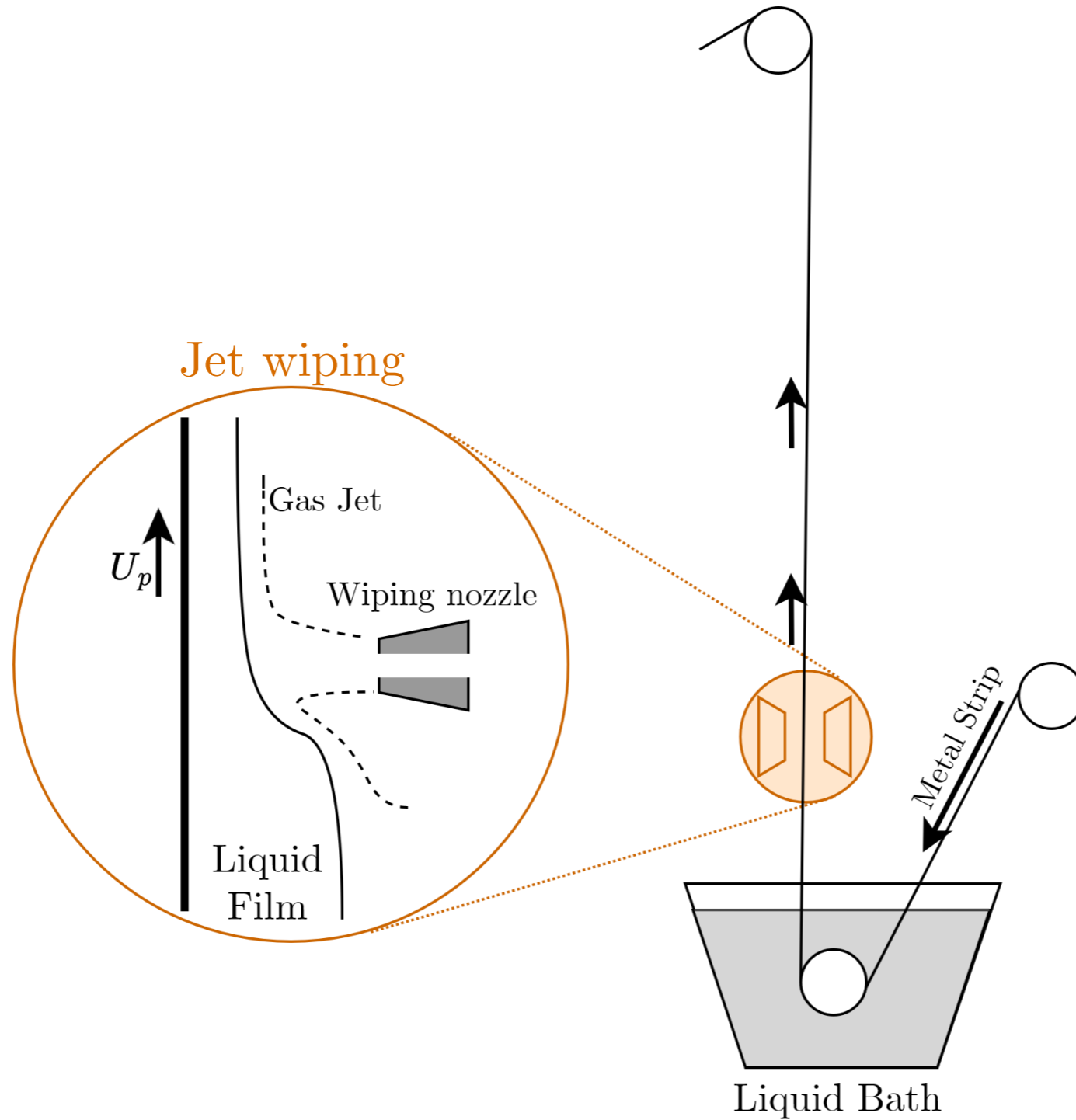
Increased production rates ( $U_p \uparrow$ )

**Thinner** Films ( $h \uparrow$ )

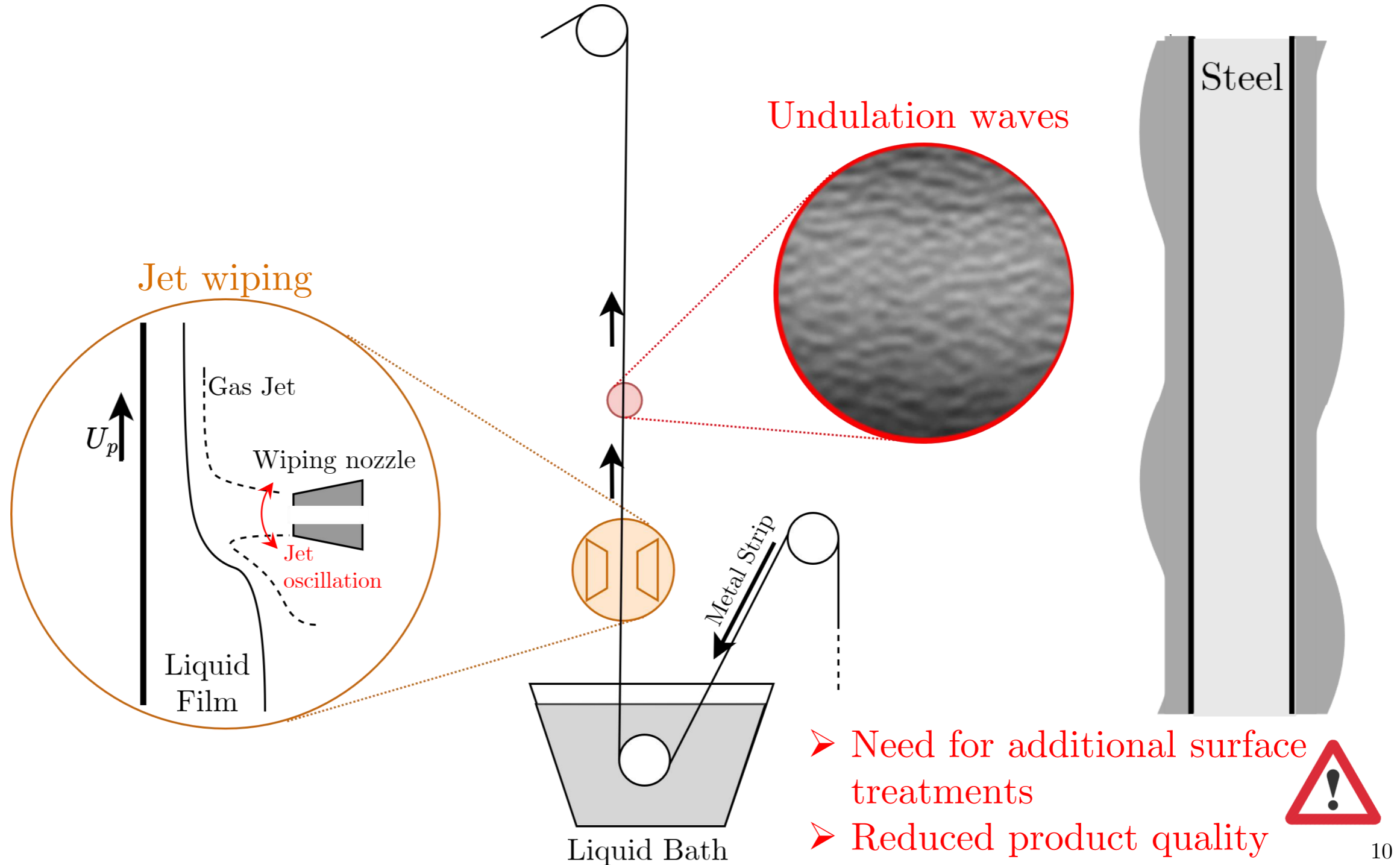




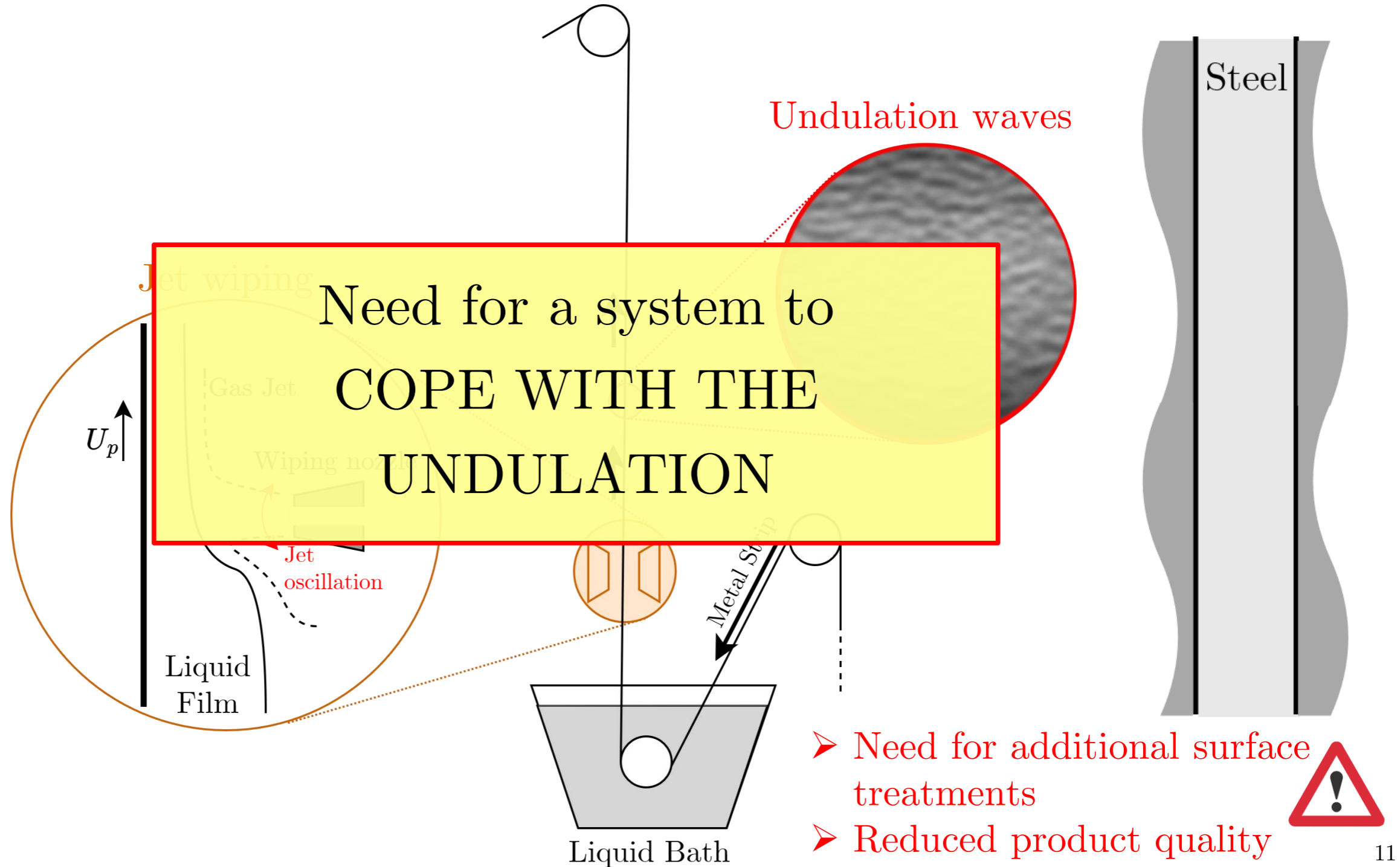
# Hot-Dip Galvanization



# Hot-Dip Galvanization

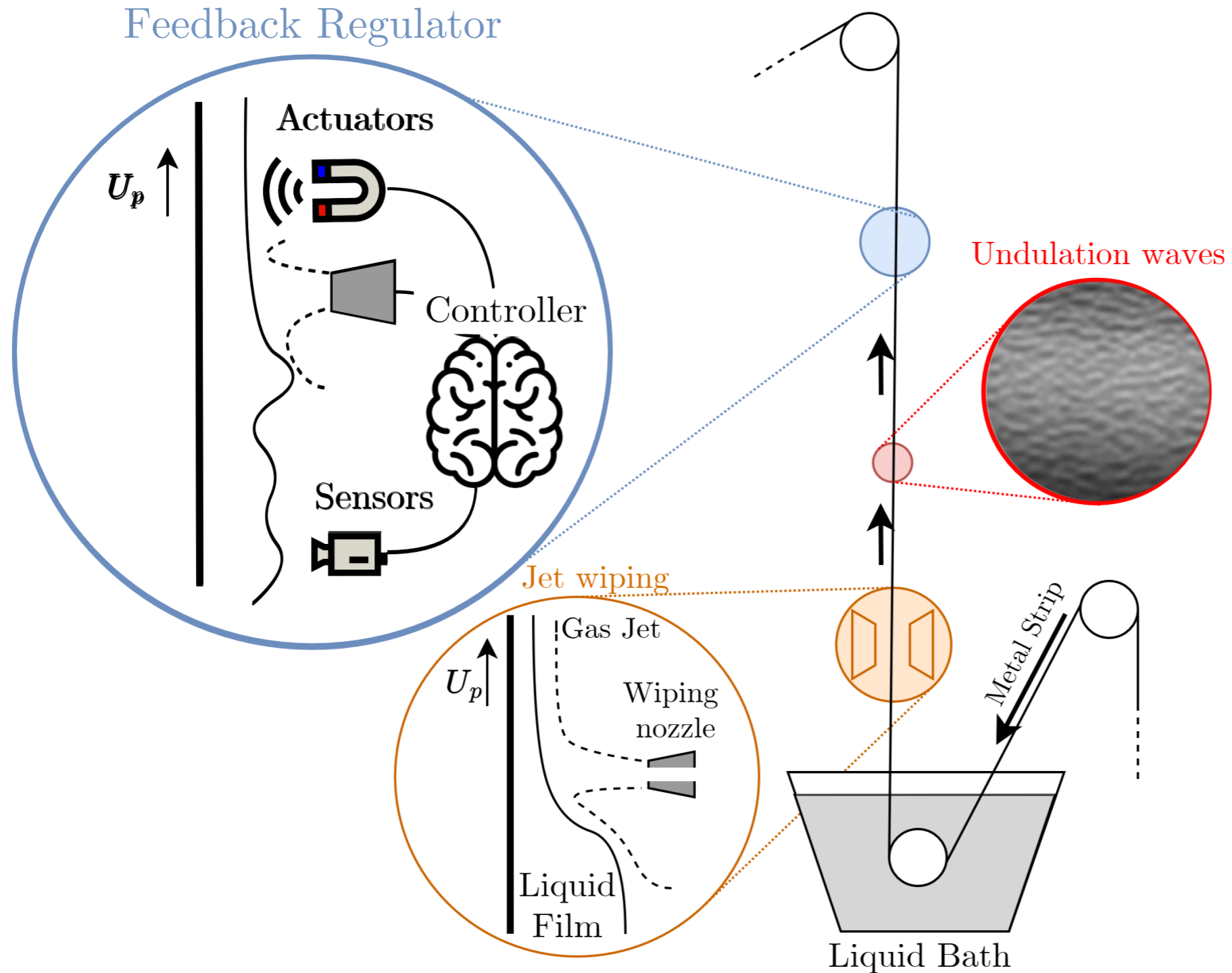


# Hot-Dip Galvanization



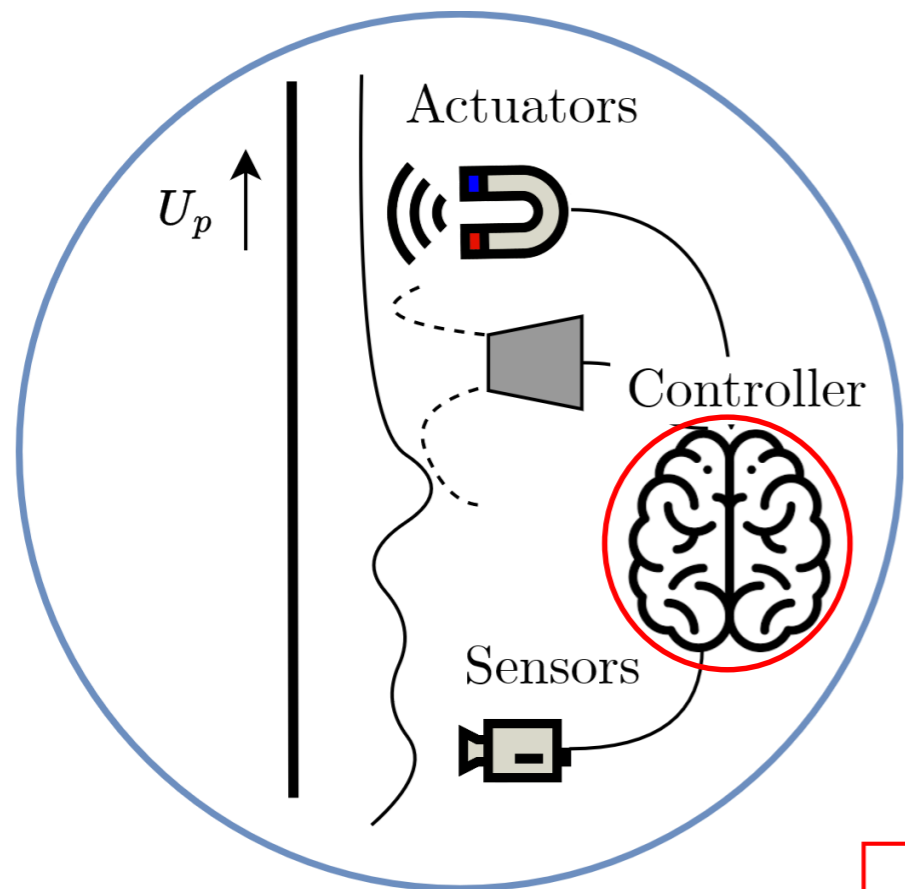
- Need for additional surface treatments
- Reduced product quality

# Hot-Dip Galvanization



# Aim of the project

## Feedback Regulator



Develop an optimal control strategy for a feedback regulator aimed at minimizing undulation waves downstream of the jet-wiping region, using gas jets and electromagnetic actuators.

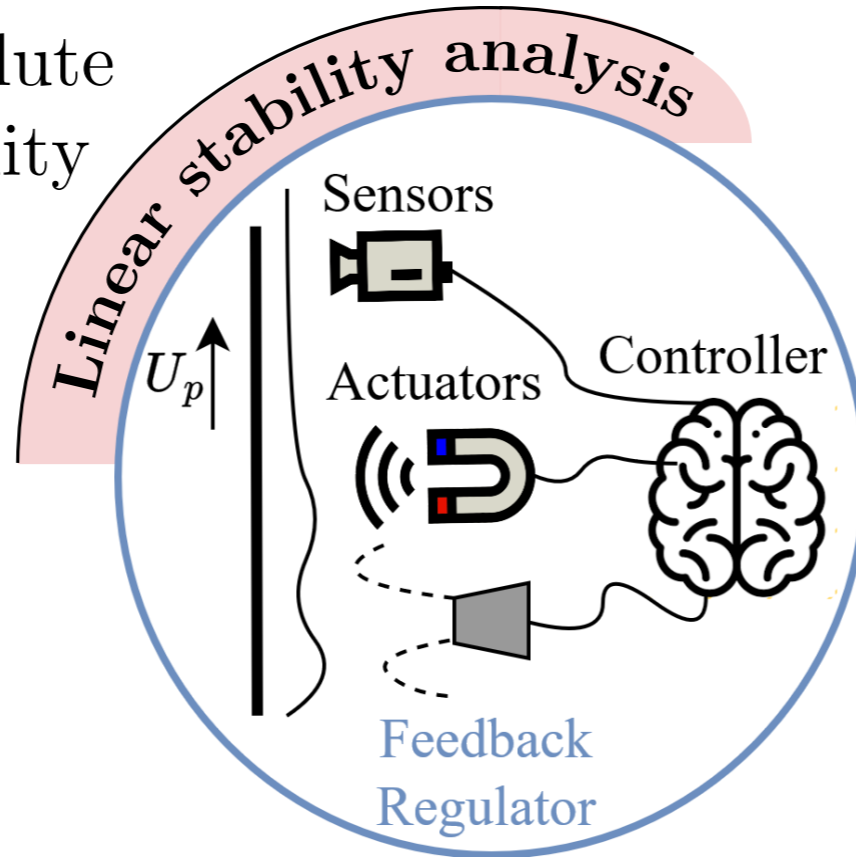
## Objectives

- ❑ Carry out a **linear stability analysis**
- ❑ **Derive a simplified model** of the liquid film
- ❑ Explore different **control methods**

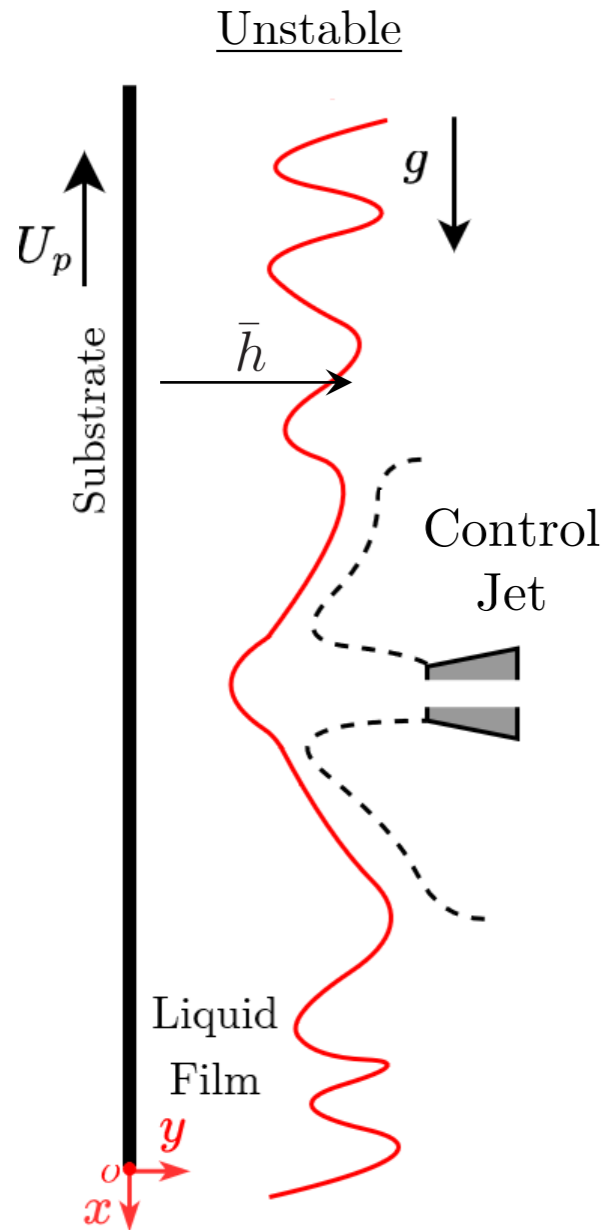
# Contents

## Linear stability analysis

Computation of the threshold between absolute and convective instability

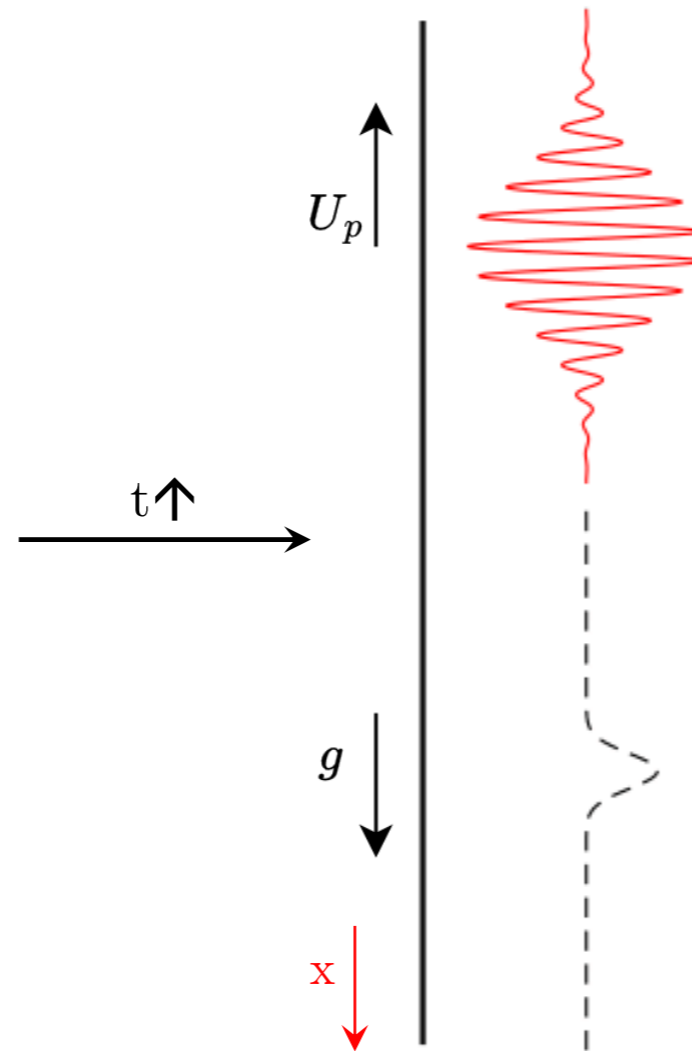


# Dynamic of the liquid film



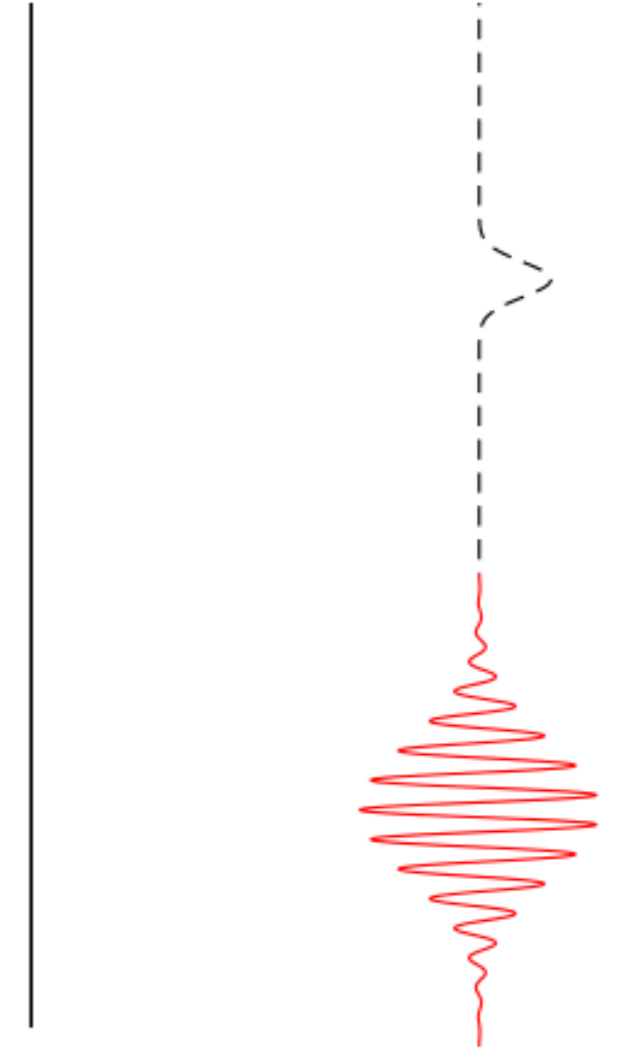
Very thin liquid film

Entrainment regime



Very thick liquid film

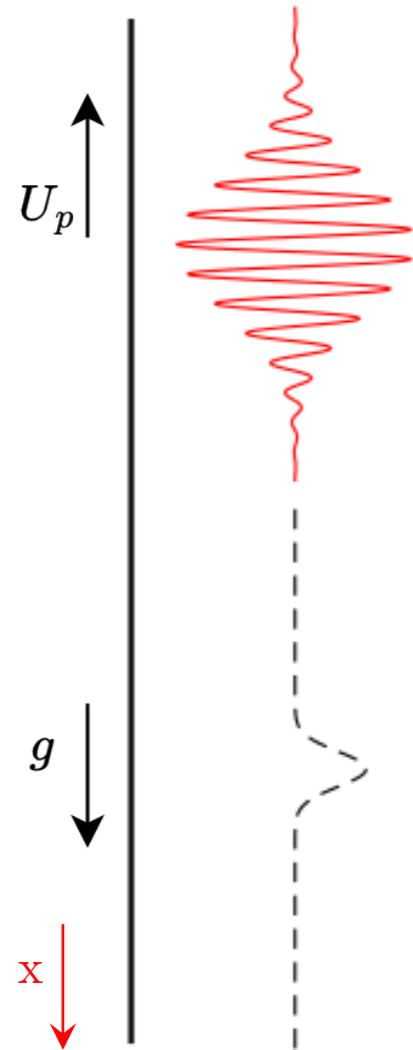
Gravity regime



# Dynamic of the liquid film

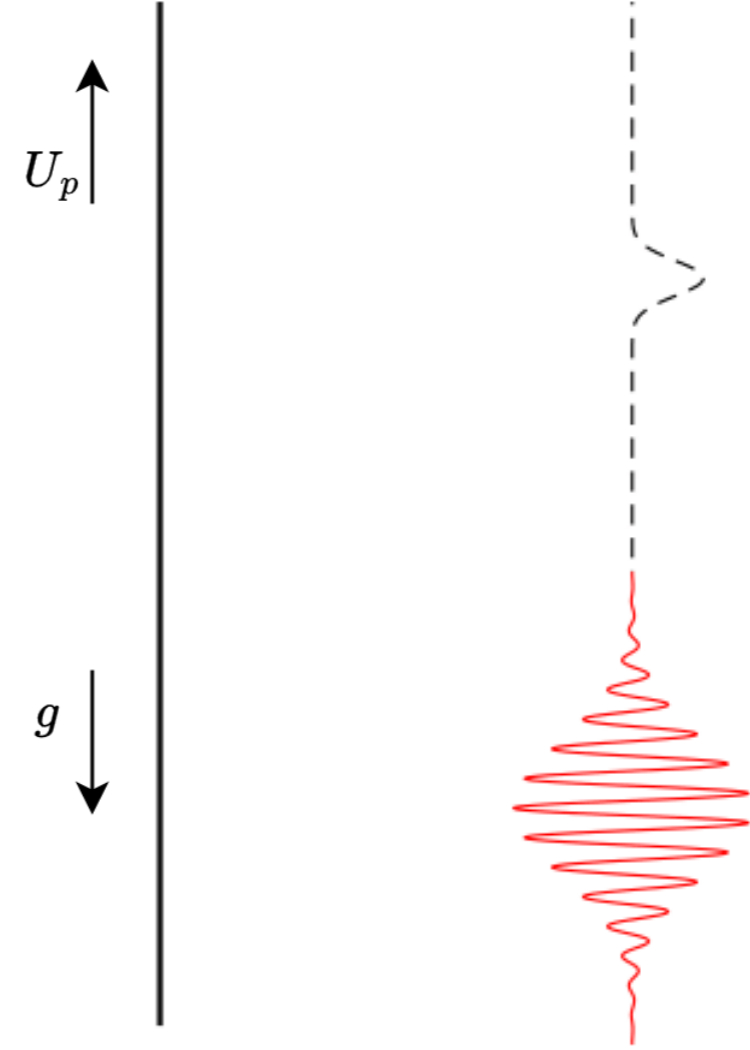
Very thin liquid film

Entrainment regime



Very thick liquid film

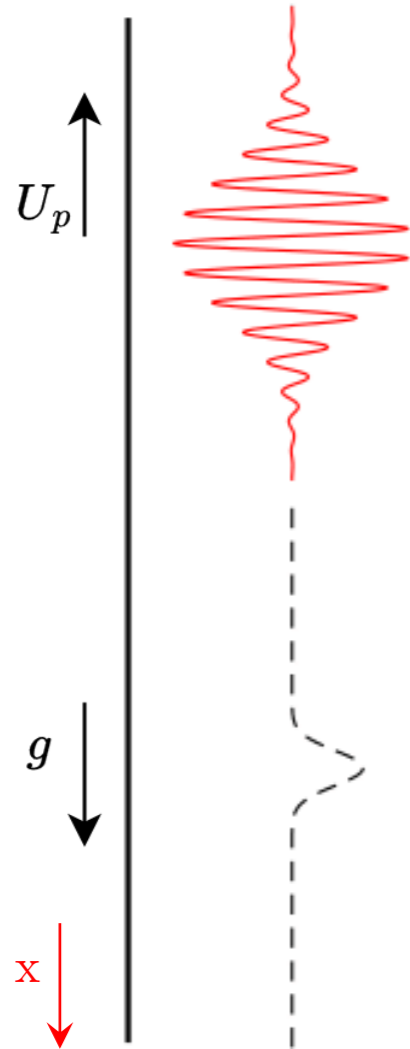
Gravity regime





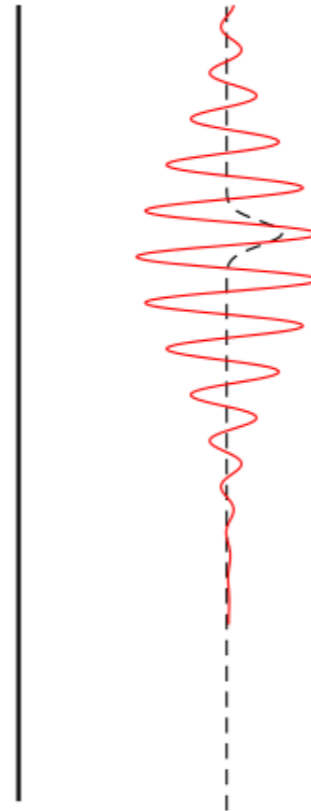
# Dynamic of the liquid film

Very thin liquid film  
Entrainment regime

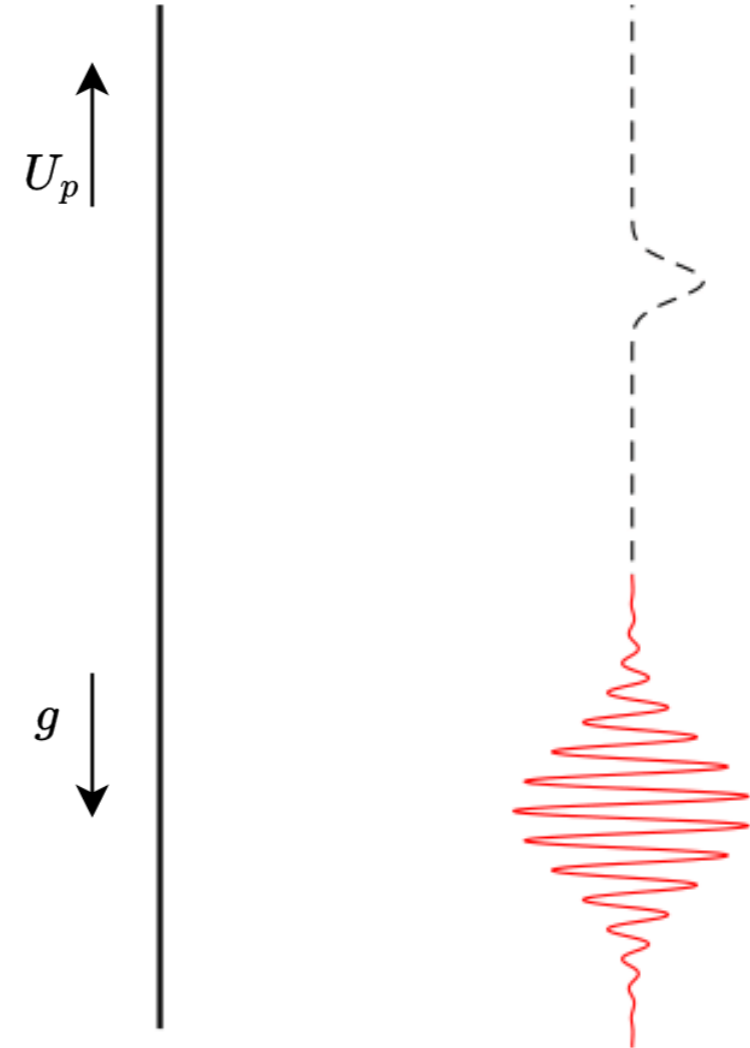


Equilibrium  
between gravity  
and entrainment

Region of absolute  
instability

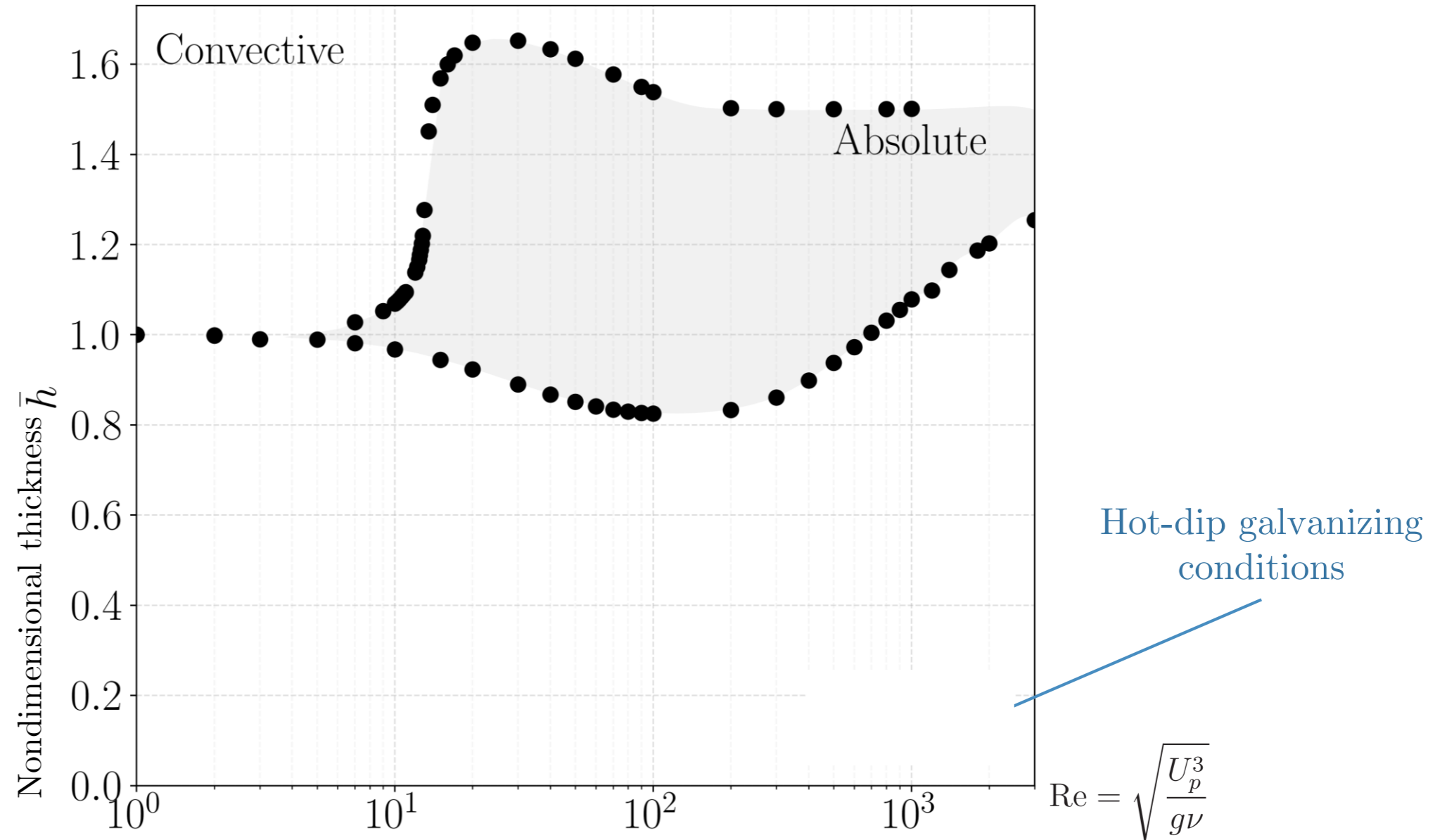


Very thick liquid film  
Gravity regime



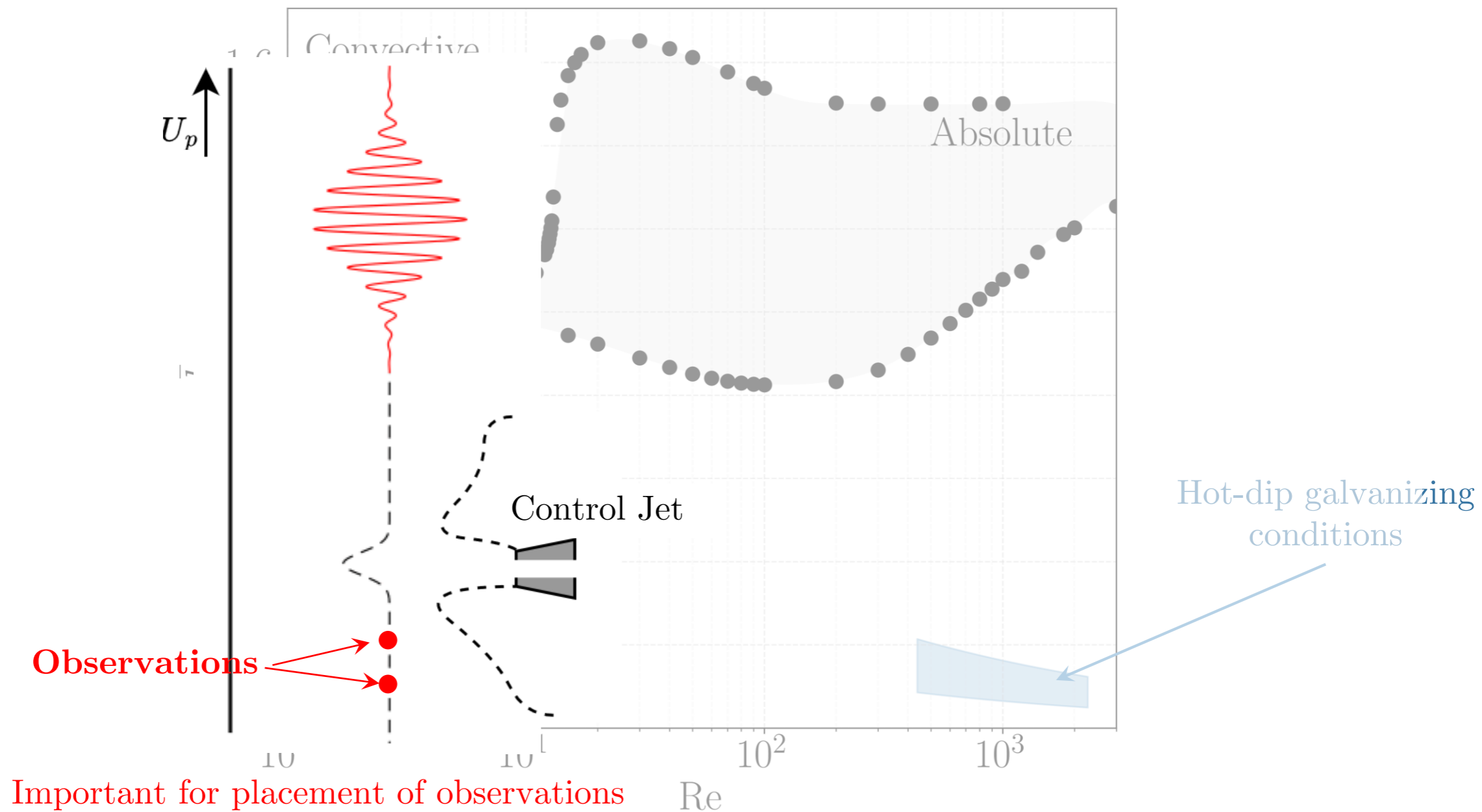
# Dynamic of the liquid film

We found the region of absolute instability studying the linearized 2D Navier-Stokes equations



# Dynamic of the liquid film

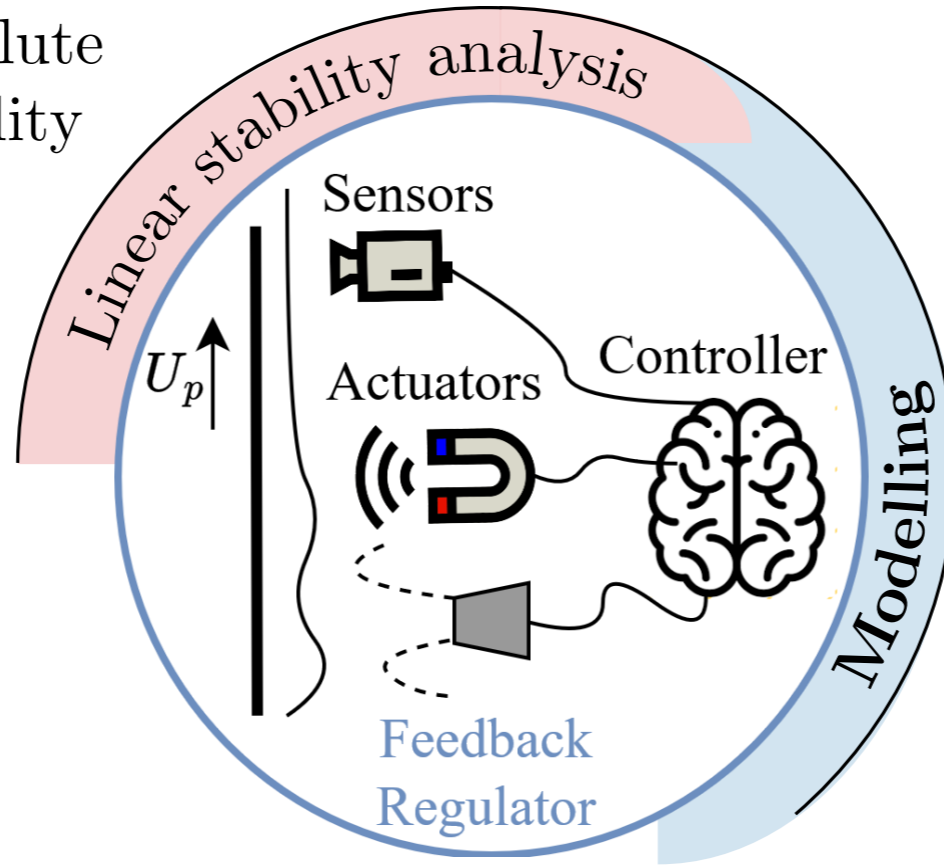
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# Contents

## Linear stability analysis

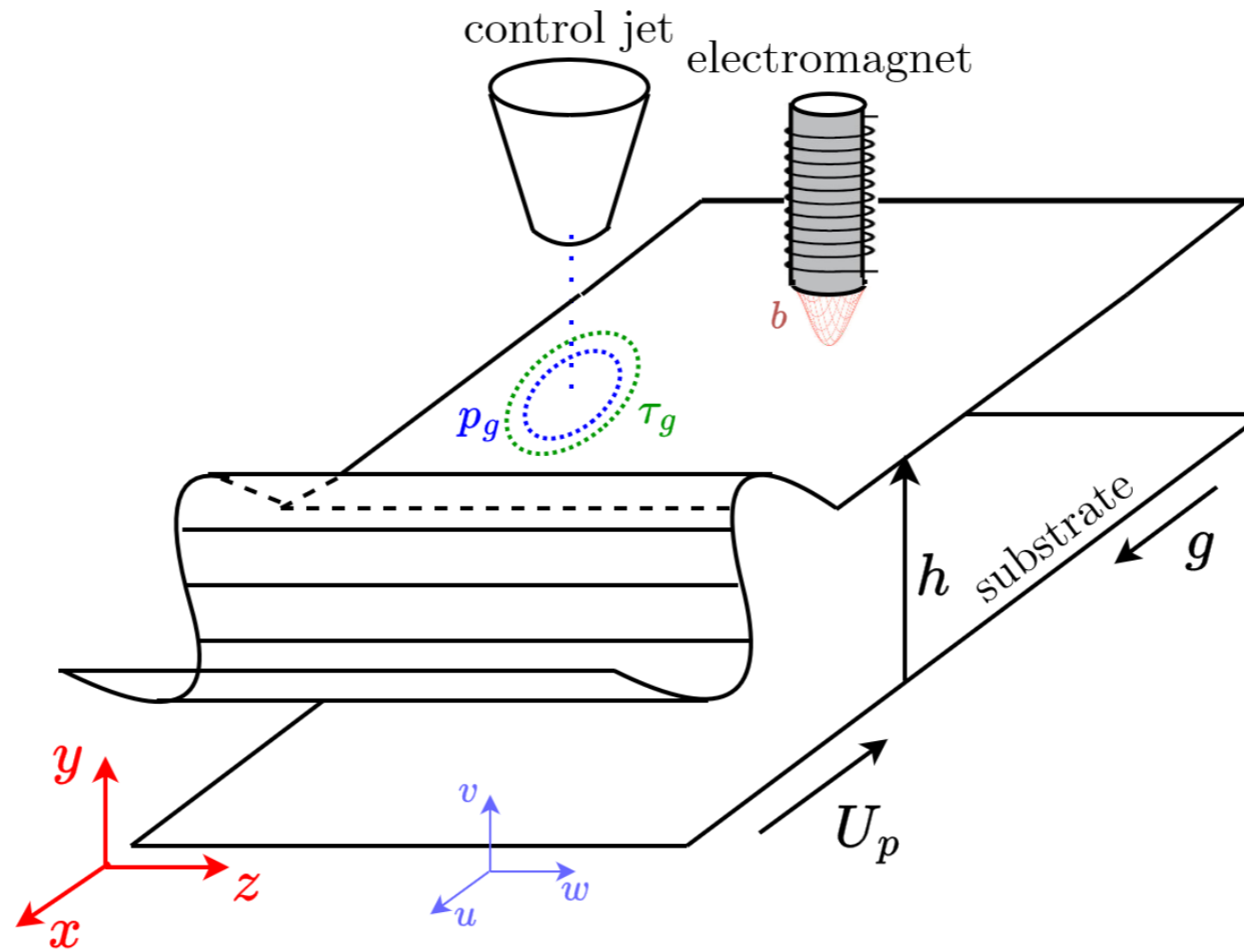
Computation of the threshold between absolute and convective instability



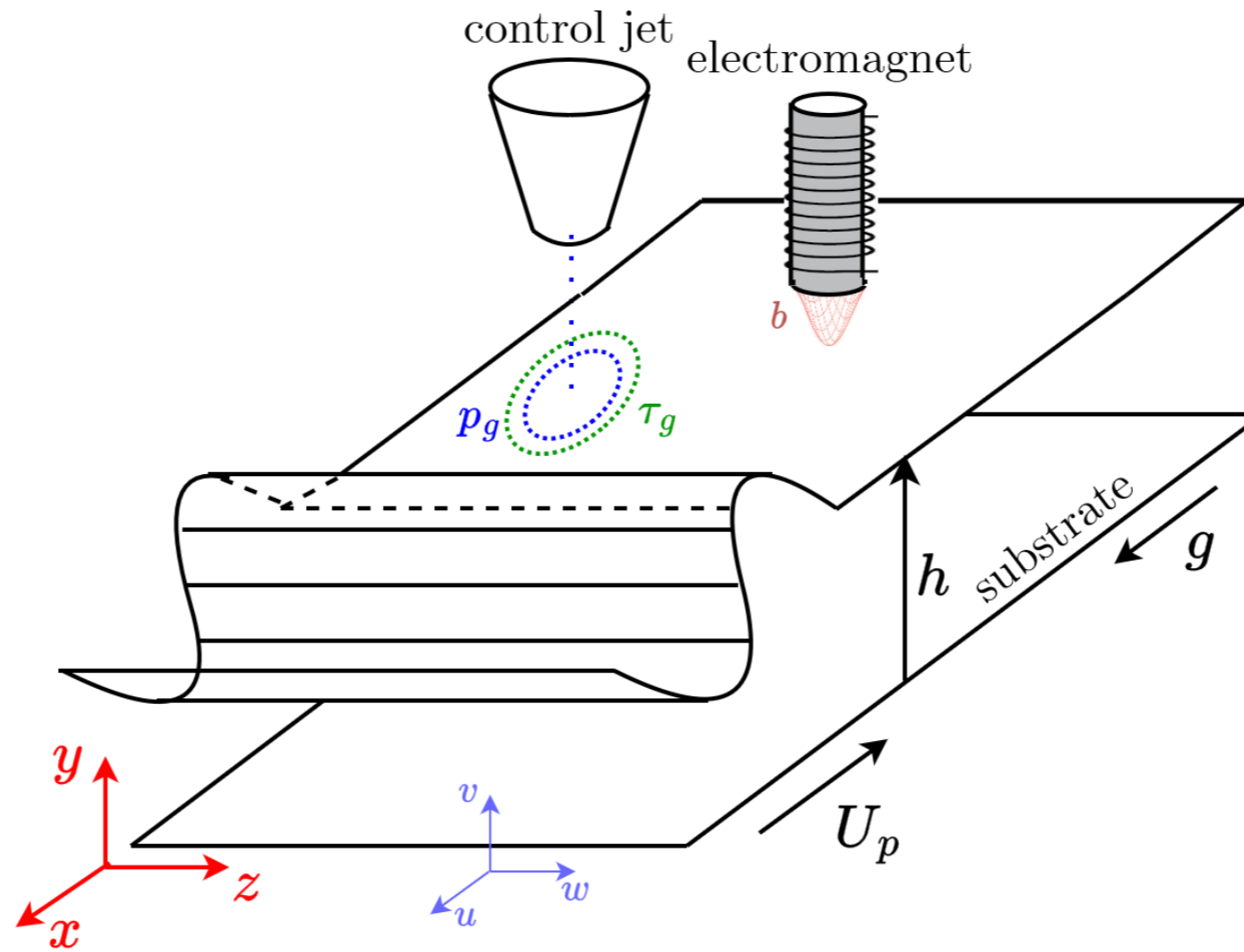
## Derive simplified model

Derivation liquid film  
reduced order model  
with actuators modelling

# Liquid film modelling



# Liquid film modelling



# Liquid film modelling

Long-wave assumption

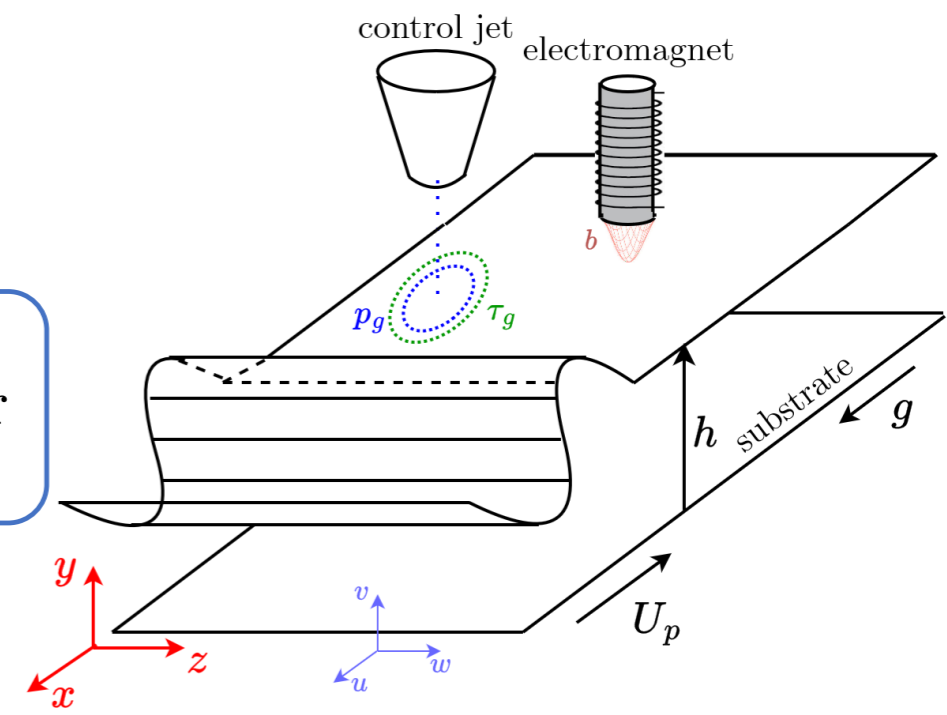
$$h_{\text{ref}}/x_{\text{ref}} = \varepsilon \ll 1$$

Integrate over liquid film thickness

Navier-Stokes equations

Boundary-Layer equations

Integral Boundary-Layer (IBL) model



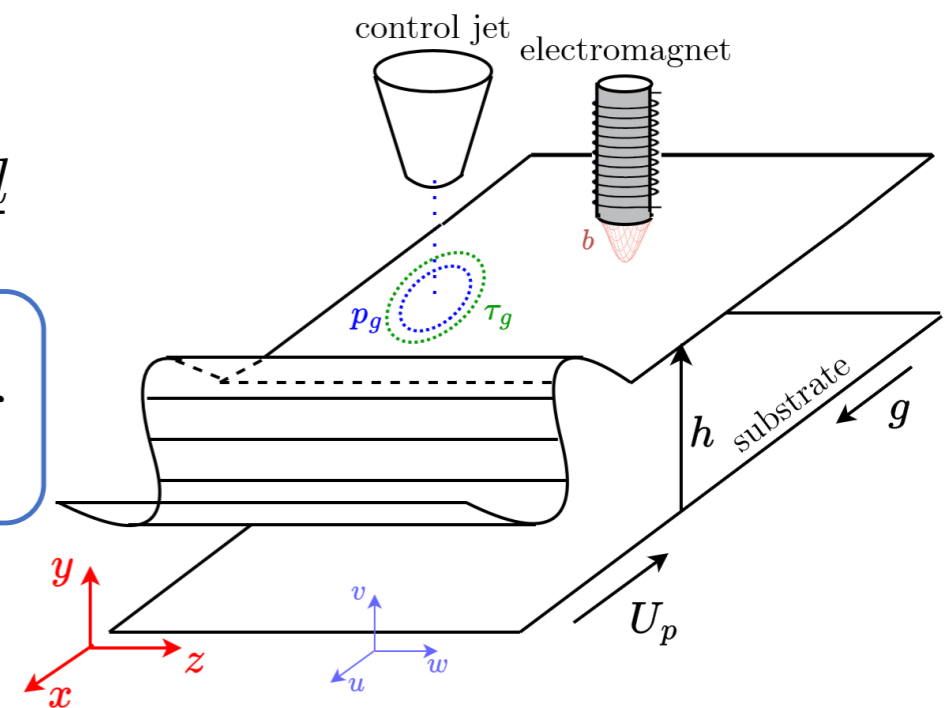
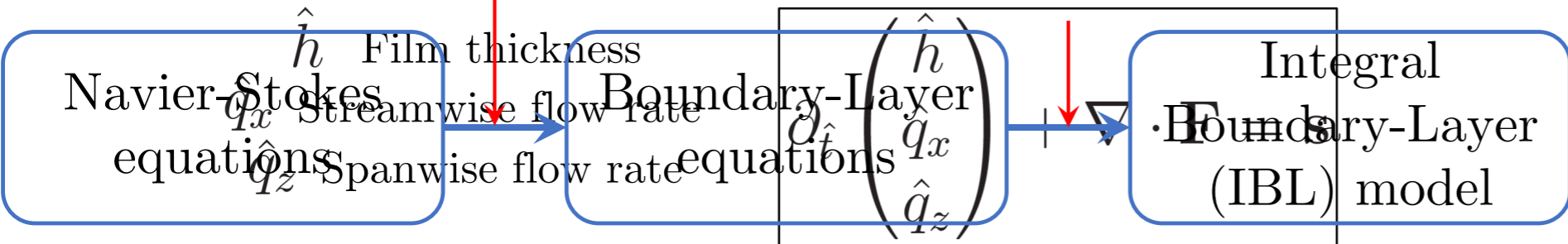
# Liquid film modelling

Long-wave assumption

$$h_{\text{ref}}/x_{\text{ref}} = \varepsilon \ll 1$$

Integral Boundary Layer model

Integrate over liquid film thickness



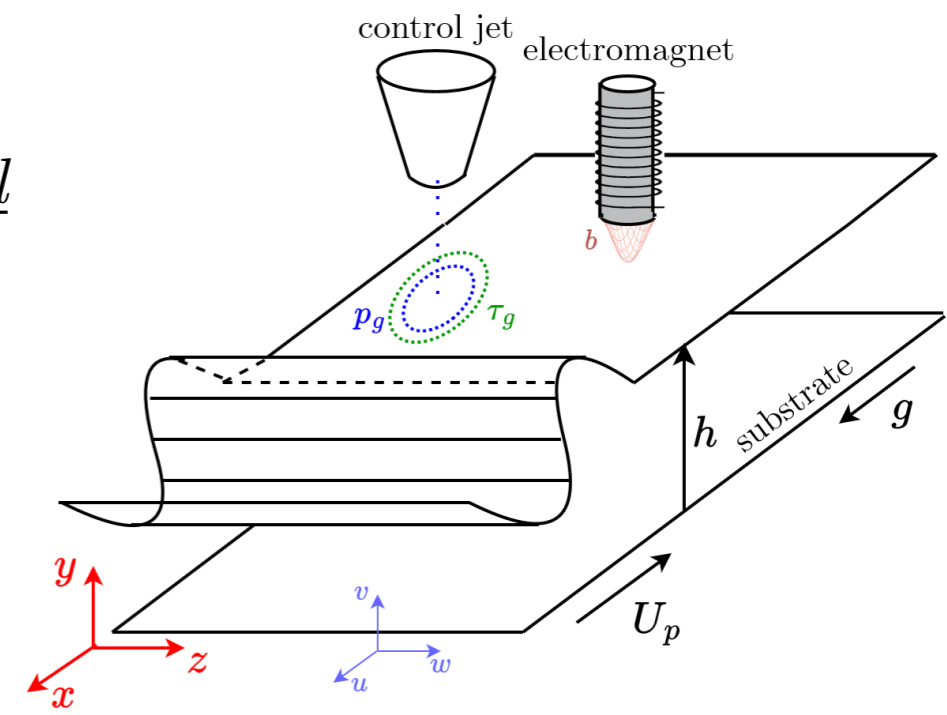


# Liquid film modelling

## 3D nondimensional Integral Boundary Layer model

$\hat{h}$  Film thickness  
 $\hat{q}_x$  Streamwise flow rate  
 $\hat{q}_z$  Spanwise flow rate

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$



Source term

Flux matrix

$$\mathbf{F} = \begin{pmatrix} \hat{q}_x & \int_0^{\hat{h}} \hat{u}^2 d\hat{y} & \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} \\ \hat{q}_z & \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} & \int_0^{\hat{h}} \hat{w}^2 d\hat{y} \end{pmatrix}$$

$$\mathbf{s} = \begin{pmatrix} 0 \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{x}} \hat{p} + \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \partial_{\hat{x}\hat{z}\hat{z}} \hat{h} + 1 \right) + \hat{\tau}_{g,x} + \hat{\tau}_{w,x} - \text{Ha}^2 \hat{b}^2 \hat{q}_x \right] \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{z}} \hat{p} + \partial_{\hat{z}\hat{z}\hat{z}} \hat{h} + \partial_{\hat{z}\hat{x}\hat{x}} \hat{h} \right) + \hat{\tau}_{g,z} + \hat{\tau}_{w,z} - \text{Ha}^2 \hat{b}^2 \hat{q}_z \right] \end{pmatrix}$$

Reduced Reynolds number

Hartmann number  
(ratio electromagnetic forces to viscous forces)

# Liquid film modelling

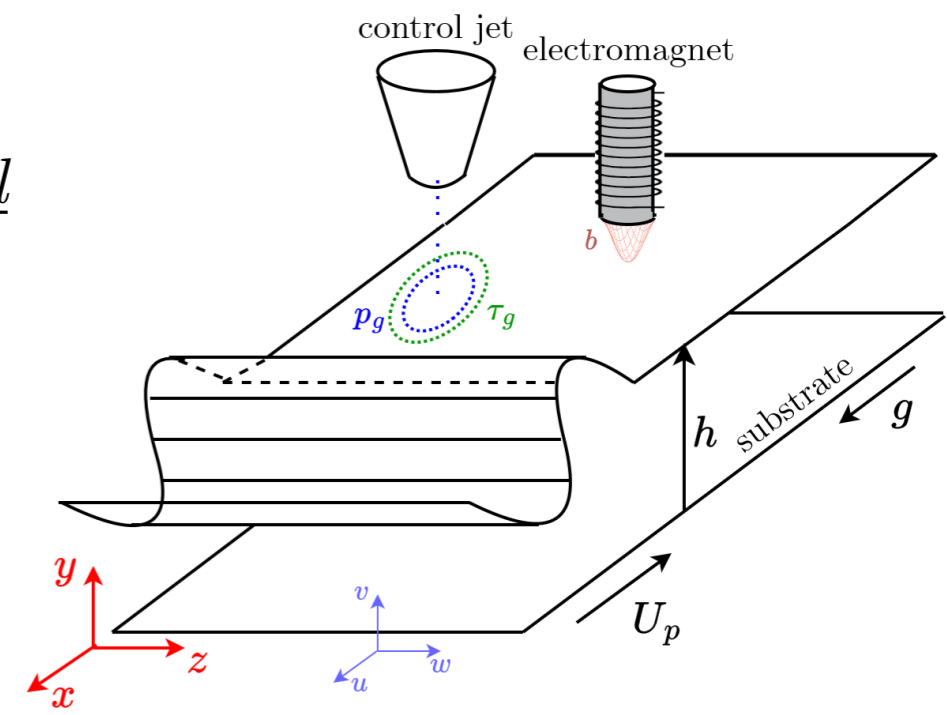
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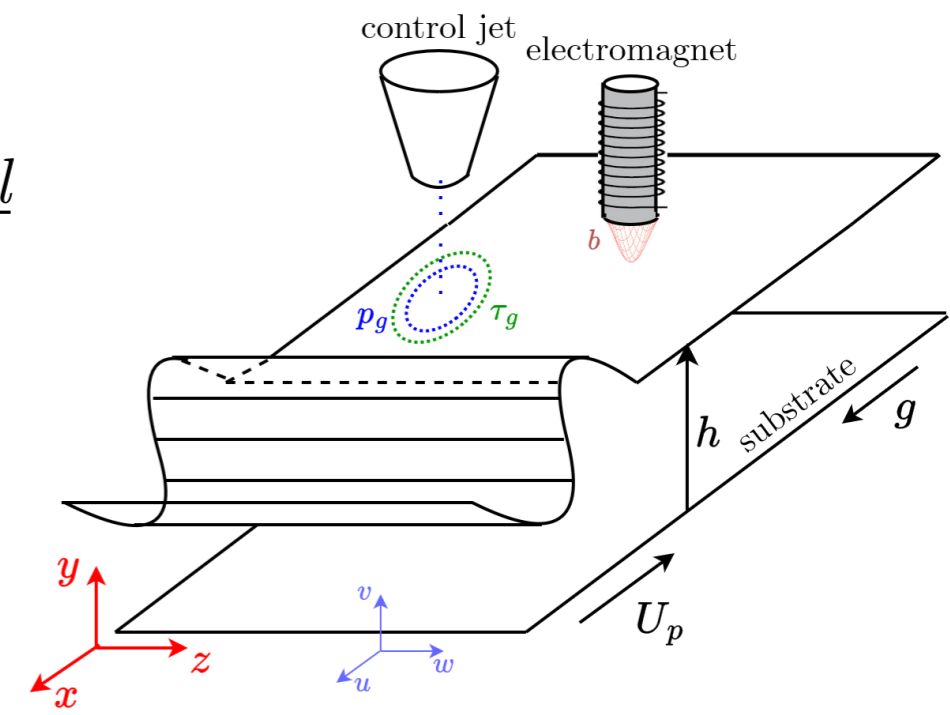
$$\mathbf{s} = \begin{pmatrix} 0 \text{ Gravity} \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{x}} \hat{p} + \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \partial_{\hat{x}\hat{z}\hat{z}} \hat{h} + \boxed{1} \right) + \hat{\tau}_{g,x} + \hat{\tau}_{w,x} - \text{Ha}^2 \hat{b}^2 \hat{q}_x \right] \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{z}} \hat{p} + \partial_{\hat{z}\hat{z}\hat{z}} \hat{h} + \partial_{\hat{z}\hat{x}\hat{x}} \hat{h} \right) + \hat{\tau}_{g,z} + \hat{\tau}_{w,z} - \text{Ha}^2 \hat{b}^2 \hat{q}_z \right] \end{pmatrix}$$

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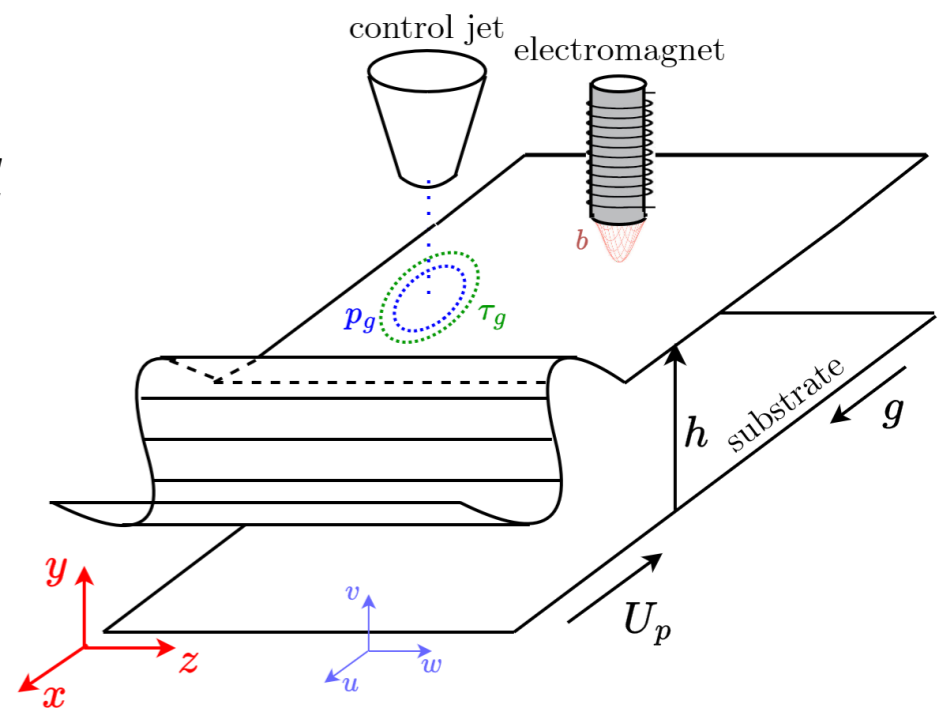
Shear at the substrate

# Liquid film modelling

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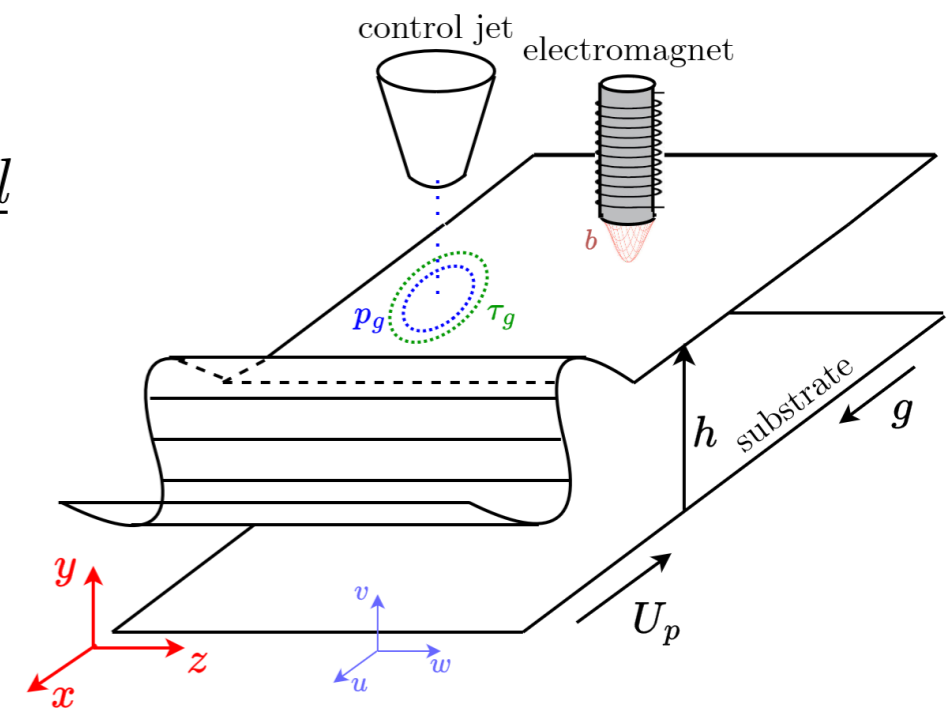
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# Liquid film modelling

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$$\mathbf{s} = \begin{pmatrix} \text{Gas-jets} & 0 & \text{Gravity} & \text{Lorentz Force} \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{x}} \hat{p} + \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \partial_{\hat{x}\hat{z}\hat{z}} \hat{h} \right) + 1 \right] + \hat{\tau}_{g,x} + \hat{\tau}_{w,x} - \text{Ha}^2 \hat{b}^2 \hat{q}_x \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{z}} \hat{p} + \partial_{\hat{z}\hat{z}\hat{z}} \hat{h} + \partial_{\hat{z}\hat{x}\hat{x}} \hat{h} \right) \right] + \hat{\tau}_{g,z} + \hat{\tau}_{w,z} - \text{Ha}^2 \hat{b}^2 \hat{q}_z \end{pmatrix}$$

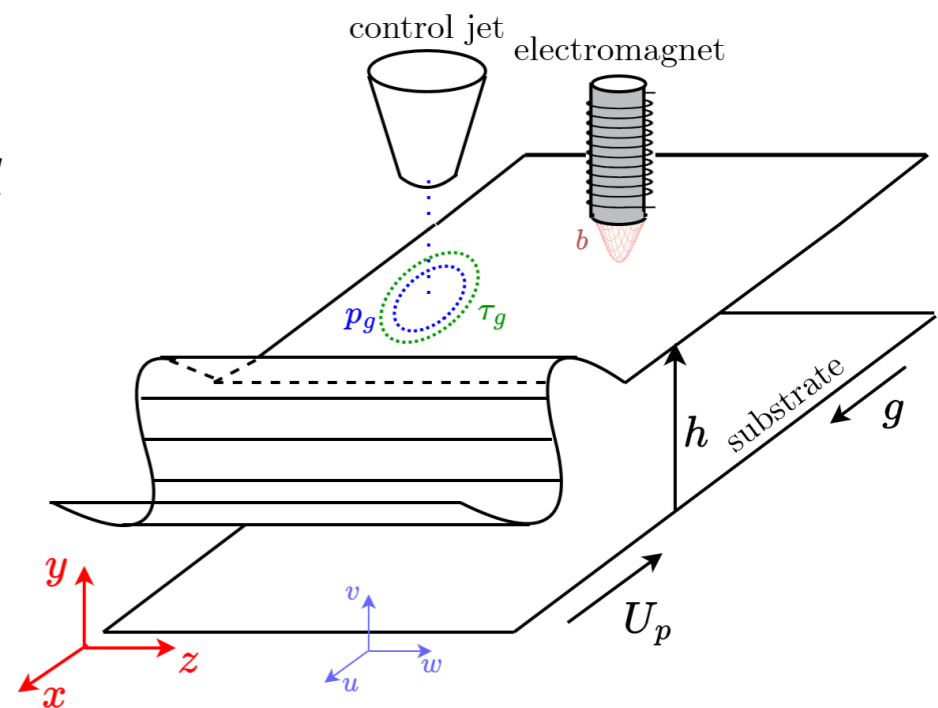
Surface Tension
Shear at the substrate

# Liquid film modelling

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Source term

Flux matrix

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Surface Tension      Shear at the substrate

Closures relations

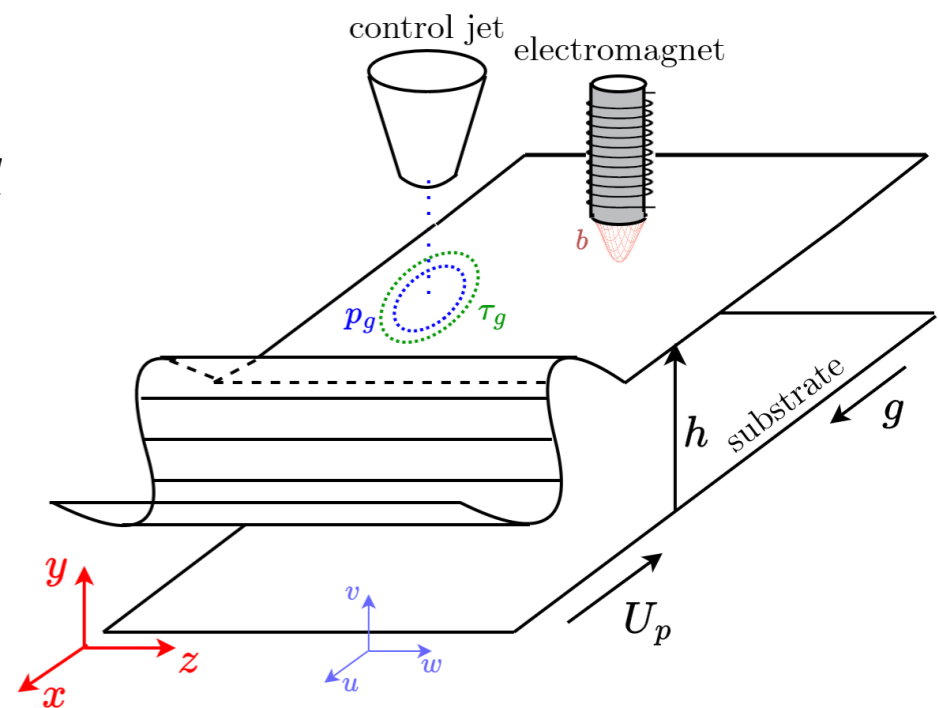
$$\hat{u} = \hat{u}(\hat{y}; \hat{h}, \hat{q}_x, \hat{q}_z) \quad \hat{w} = \hat{w}(\hat{y}; \hat{h}, \hat{q}_x, \hat{q}_z)$$

# Liquid film modelling

## 3D nondimensional Integral Boundary Layer model

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Source term

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Closures relations

**Self-similarity hypothesis**

$$\hat{u} = \hat{u}(\hat{y}; \hat{h}, \hat{q}_x, \hat{q}_z) \quad \hat{w} = \hat{w}(\hat{y}; \hat{h}, \hat{q}_x, \hat{q}_z)$$

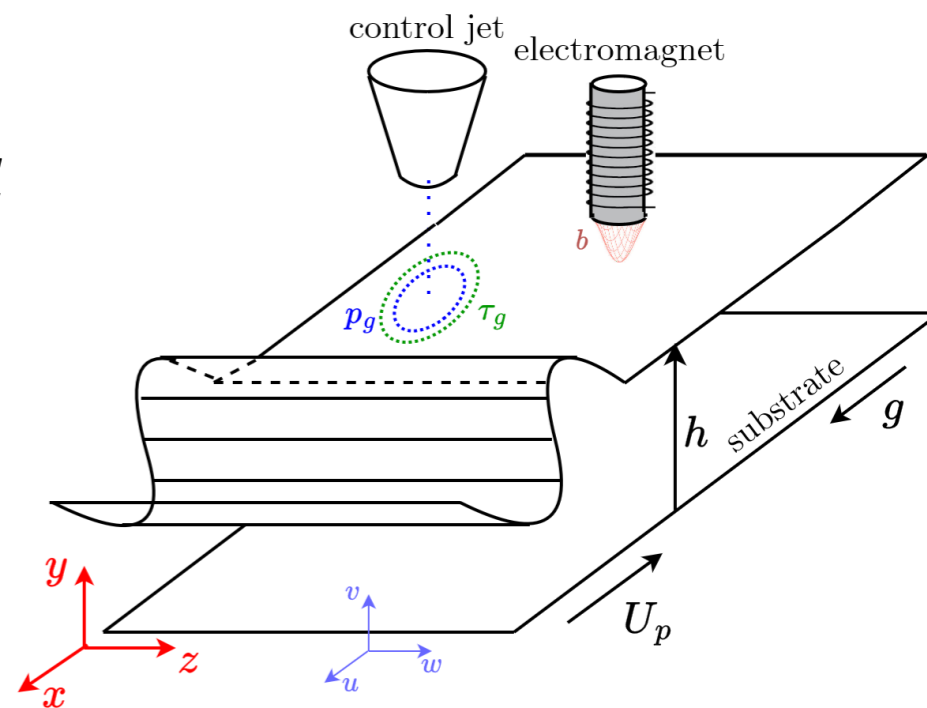
*The velocity profile is everywhere parabolic and self-similar.*

# Liquid film modelling

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$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$



Source term

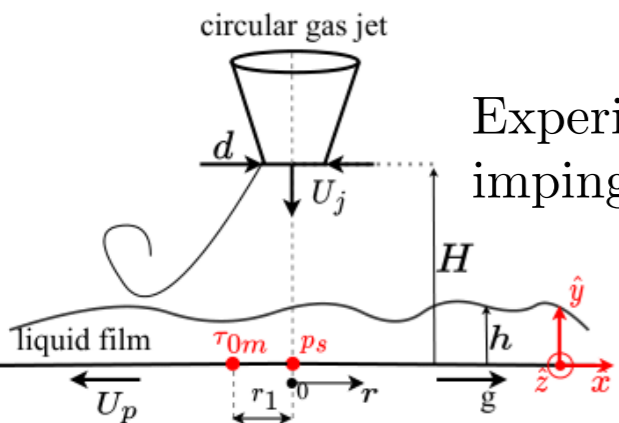
Flux matrix

$$\mathbf{F} = \begin{pmatrix} \hat{q}_x \\ \hat{q}_z \end{pmatrix} \begin{pmatrix} \int_0^{\hat{h}} \hat{u}^2 d\hat{y} & \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} \\ \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} & \int_0^{\hat{h}} \hat{w}^2 d\hat{y} \end{pmatrix}$$

$$\mathbf{s} = \begin{pmatrix} \text{Gas-jets} & 0 & \text{Gravity} & \text{Lorentz Force} \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{x}} \hat{p} + \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \partial_{\hat{x}\hat{z}\hat{z}} \hat{h} \right) + \hat{1} \right] + \hat{\tau}_{g,x} + \hat{\tau}_{w,x} - \text{Ha}^2 \hat{b}^2 \hat{q}_x \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{z}} \hat{p} + \partial_{\hat{z}\hat{z}\hat{z}} \hat{h} + \partial_{\hat{z}\hat{x}\hat{x}} \hat{h} \right) \right] + \hat{\tau}_{g,z} + \hat{\tau}_{w,z} - \text{Ha}^2 \hat{b}^2 \hat{q}_z \end{pmatrix}$$

Surface Tension
Shear at the substrate

## Modelling actuators



Experimental correlations of impinging gas jets

$$\tau_g = f_\tau(\lambda) \tau_{0m} \quad p = f_p(\lambda) p_s$$

$$\lambda = r/H$$

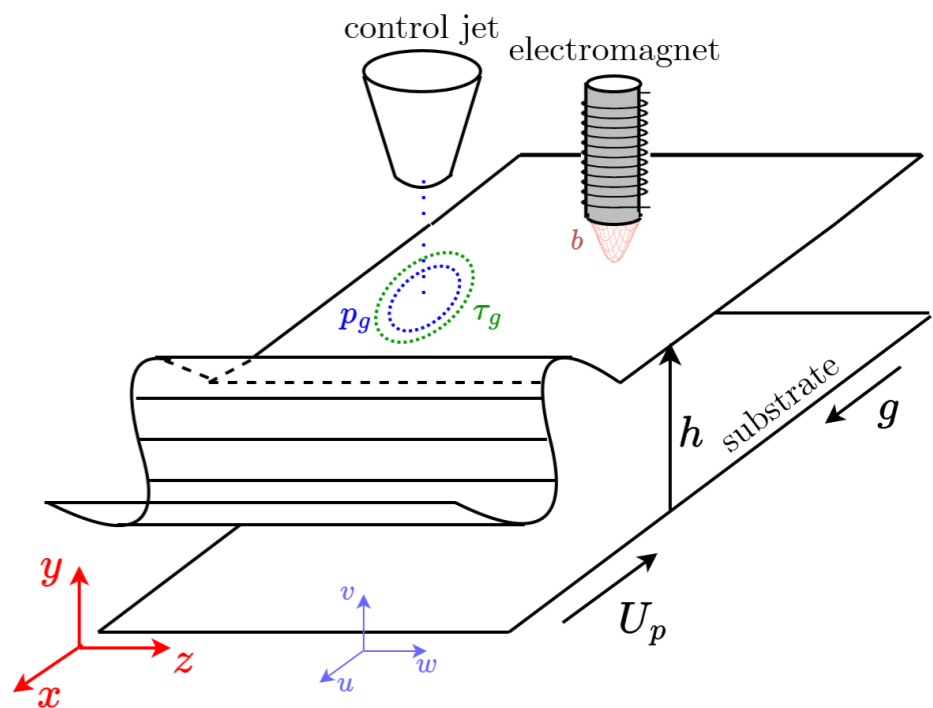


# Liquid film modelling

## 3D nondimensional Integral Boundary Layer model

$\hat{h}$  Film thickness  
 $\hat{q}_x$  Streamwise flow rate  
 $\hat{q}_z$  Spanwise flow rate

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$



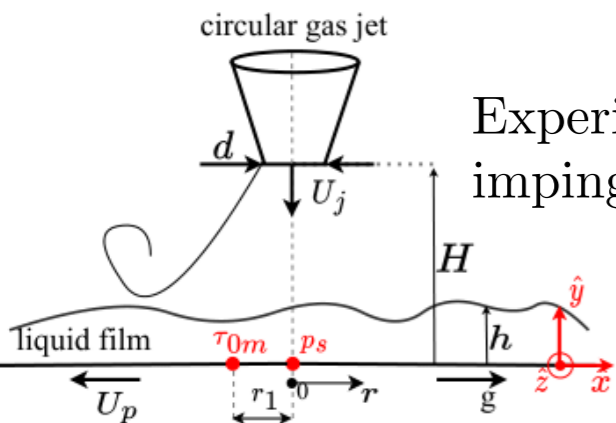
Source term

Flux matrix

$$\mathbf{F} = \begin{pmatrix} \hat{q}_x & \int_0^{\hat{h}} \hat{u}^2 d\hat{y} & \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} \\ \hat{q}_z & \int_0^{\hat{h}} \hat{u}\hat{w} d\hat{y} & \int_0^{\hat{h}} \hat{w}^2 d\hat{y} \end{pmatrix}$$

$$\mathbf{s} = \begin{pmatrix} \text{Gas-jets} & 0 & \text{Gravity} & \text{Lorentz Force} \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{x}} \hat{p} + \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \partial_{\hat{x}\hat{z}\hat{z}} \hat{h} + 1 \right) + \hat{\tau}_{g,x} + \hat{\tau}_{w,x} - \text{Ha}^2 \hat{b}^2 \hat{q}_x \right] \\ \delta^{-1} \left[ \hat{h} \left( -\partial_{\hat{z}} \hat{p} + \partial_{\hat{z}\hat{z}\hat{z}} \hat{h} + \partial_{\hat{z}\hat{x}\hat{x}} \hat{h} \right) + \hat{\tau}_{g,z} + \hat{\tau}_{w,z} - \text{Ha}^2 \hat{b}^2 \hat{q}_z \right] \end{pmatrix}$$

Surface Tension      Shear at the substrate



Experimental correlations of impinging gas jets

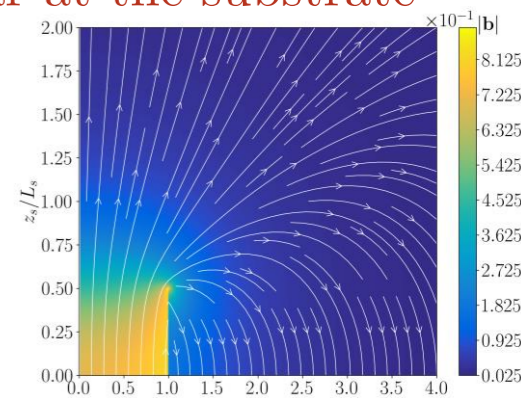
$$\tau_g = f_\tau(\lambda) \tau_{0m} \quad p = f_p(\lambda) p_s$$

$$\lambda = r/H$$

## Modelling actuators

Approximation magnetic field of a solenoid

$$b(x, z, t) = b_t(t) e^{-\frac{(\hat{x}-\hat{x}_0)^2}{2\gamma_x^2} + \frac{-(\hat{z}-\hat{z}_0)^2}{2\gamma_z^2}}$$

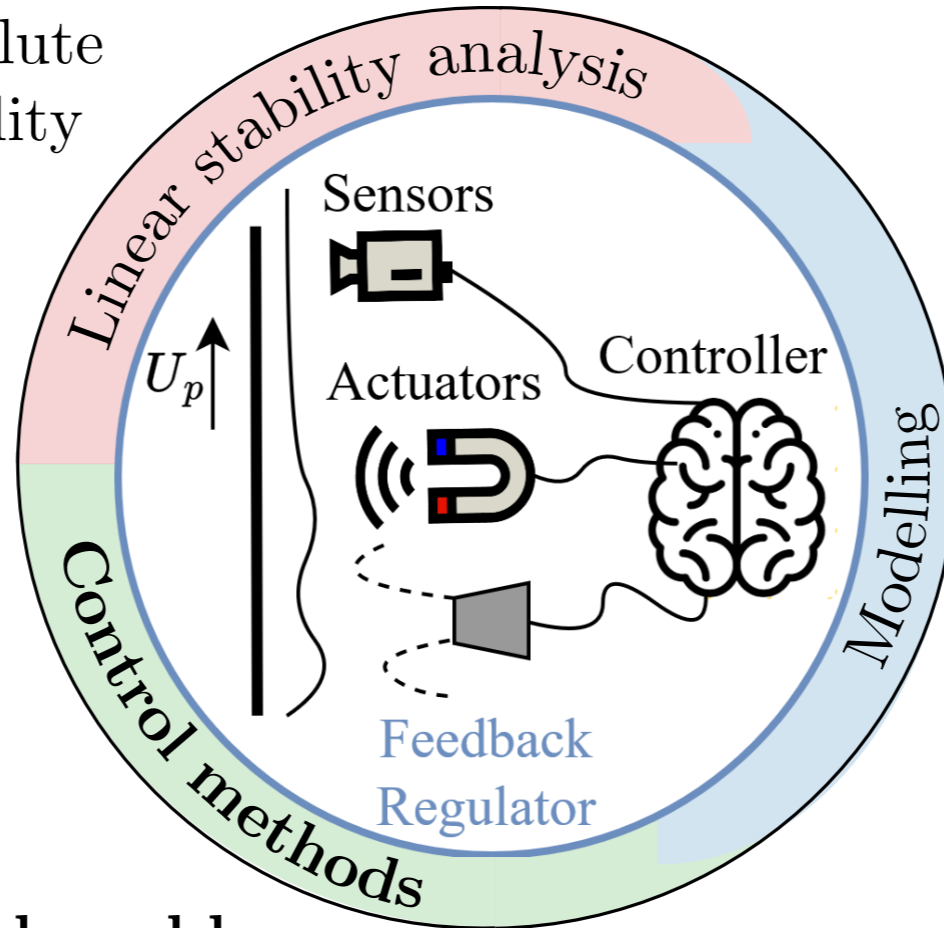


Pino, Fabio, Benoit Scheid, and Miguel Alfonso Mendez. "Multi-objective optimization of the magnetic wiping process in dip-coating." *arXiv preprint*

# Contents

## Linear stability analysis

Computation of the threshold between absolute and convective instability



## Control methods

□ Definition optimal control problem

## Derive simplified model

Derivation liquid film reduced order model with actuators modelling

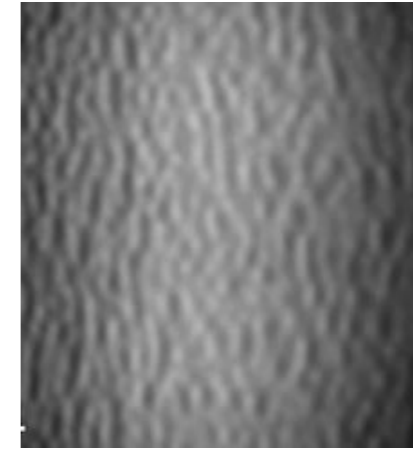
# Definition optimal control problem

# Definition optimal control problem

Integral Boundary Layer model

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

Undulation waves



# Definition optimal control problem

Integral Boundary Layer model

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

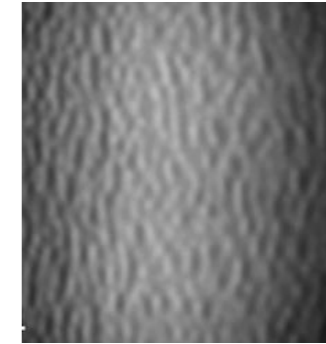
Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t}$$

$\Omega_r$  Reward area

Undulation waves



**a** Control Parameter

$U_j$  nozzle exit velocity

$p_s$  jet's impingement pressure

$\hat{b}$  Intensity magnetic field

# Definition optimal control problem

Integral Boundary Layer model

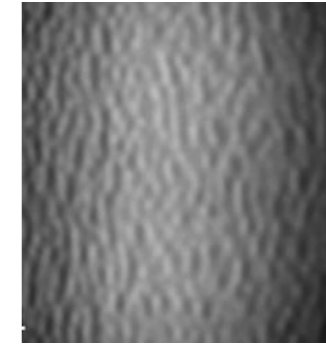
$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t} \quad \Omega_r \text{ Reward area}$$

Undulation waves



**a** Control Parameter

$U_j$  nozzle exit velocity

$p_s$  jet's impingement pressure

$\hat{b}$  Intensity magnetic field

---

Model based control approach (White box)

Governing equations

- ❑ Full knowledge of the Integral Boundary Layer equations
- ❑ Linearization around desired flat state  $\bar{h}$
- ❑ Identify feedback gains that maintain the system within the stable region.

# Definition optimal control problem

Integral Boundary Layer model

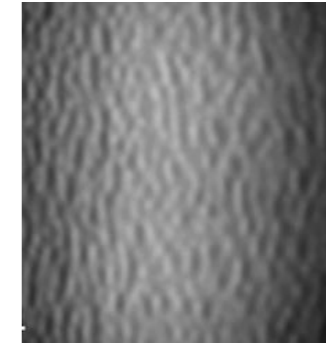
$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

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Undulation waves



**a** Control Parameter

$U_j$  nozzle exit velocity

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---

Machine learning approach (Black box)

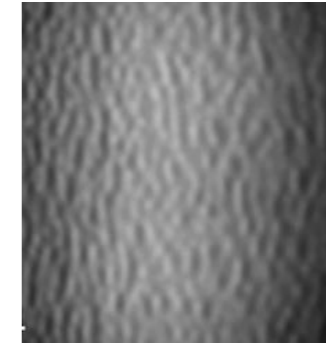


# Definition optimal control problem

Integral Boundary Layer model

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

Undulation waves



Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t} \quad \Omega_r \text{ Reward area}$$

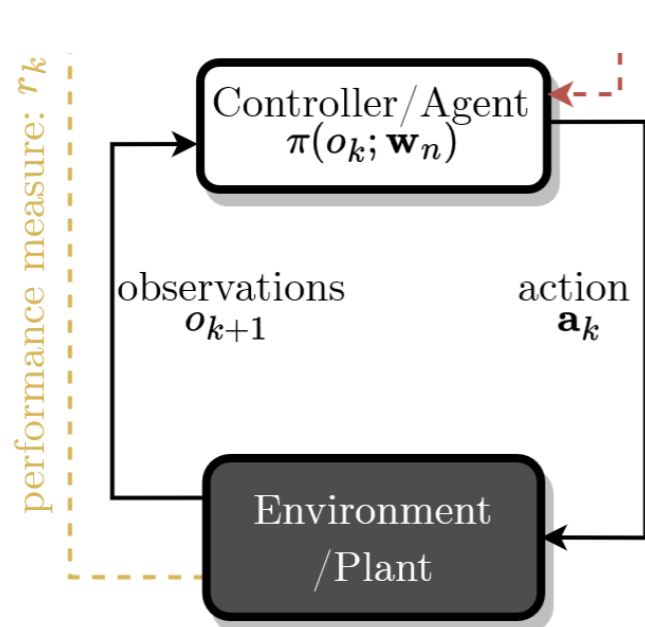
**a** Control Parameter

$U_j$  nozzle exit velocity

$p_s$  jet's impingement pressure

$\hat{b}$  Intensity magnetic field

Machine learning approach (Black box)



Free surface observations

$$o_k = m(\hat{h}(\hat{x}, \hat{t}_k))$$

Instantaneous reward

$$r_k = \int_{\Omega_r} \mathcal{L}(\hat{h}(\hat{x}, \hat{t}_k) - \bar{h}) d\hat{x}$$

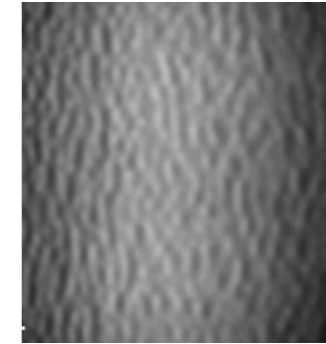


# Definition optimal control problem

Integral Boundary Layer model

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

Undulation waves



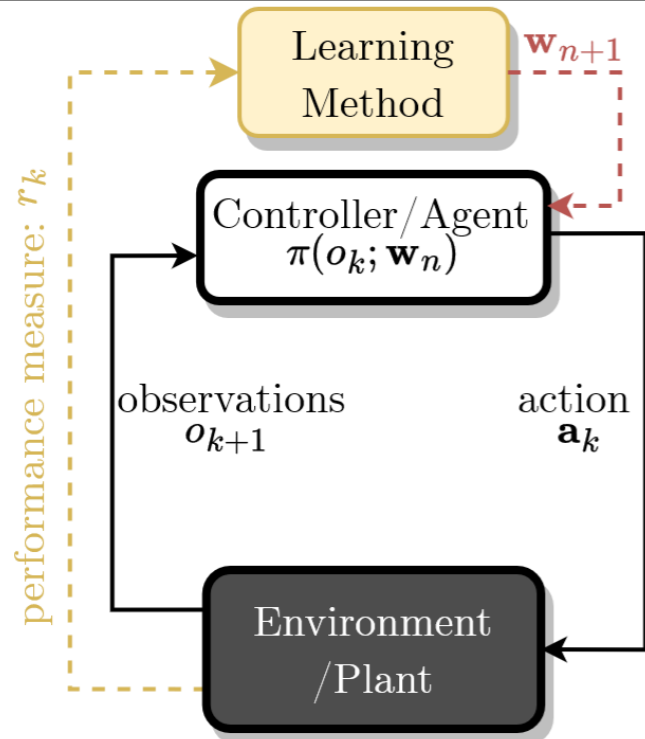
Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t} \quad \Omega_r \text{ Reward area}$$

**a** Control Parameter

- $U_j$  nozzle exit velocity
- $p_s$  jet's impingement pressure
- $\hat{b}$  Intensity magnetic field



Machine learning approach (Black box)

Free surface observations

$$o_k = m(\hat{h}(\hat{x}, \hat{t}_k))$$

Feedback control action

$$\mathbf{a} = \pi(o_k | \mathbf{w}_n)$$

Machine learning control methods vary in the way they adjust the controller action parameters.

Instantaneous reward

$$r_k = \int_{\Omega_r} \mathcal{L}(\hat{h}(\hat{x}, \hat{t}_k) - \bar{h}) d\hat{x}$$

Control action parameters

$$\mathbf{w}_n$$

# Definition optimal control problem

Liquid film Governing Equations

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

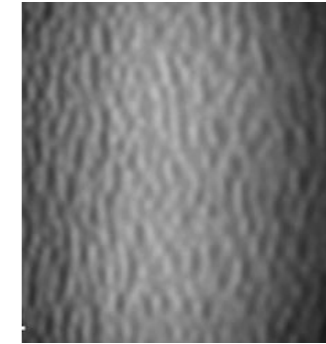
Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t}$$

$\Omega_r$  Reward area

Undulation waves



**a** Control Parameter

$U_j$  nozzle exit velocity

$p_s$  jet's impingement pressure

$\hat{b}$  Intensity magnetic field

Model based control approach (White box)

## Advantages

- Fast and Efficient
- Stability Guarantees
- Established literature

## Disadvantages

- Linear Systems Limitation
- Model Dependency
- Difficulty with uncertainties and noise

Governing equations

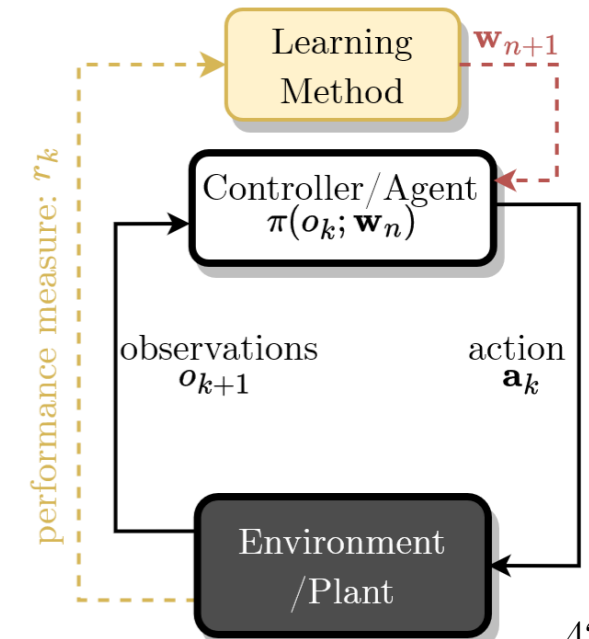
Machine learning approach (Black box)

## Advantages

- Model-Free Learning
- Nonlinear Systems
- Adaptability

## Disadvantages

- No Stability Guarantees
- Sample Inefficiency
- Computational Complexity

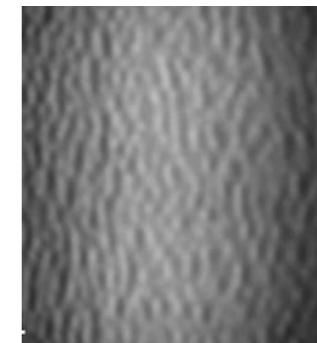


# Definition optimal control problem

Liquid film Governing Equations

$$\partial_{\hat{t}} \begin{pmatrix} \hat{h} \\ \hat{q}_x \\ \hat{q}_z \end{pmatrix} + \nabla \cdot \mathbf{F} = \mathbf{s}$$

Undulation waves



Performance measure

Liquid film as flat as possible in the reward area around target thickness  $\bar{h}$

$$\max_{\mathbf{a}} \int_0^T \int_{\Omega_r} \underbrace{\mathcal{L}(\hat{h}(\hat{x}, \hat{t}) - \bar{h})}_{\text{Running Cost}} d\hat{x} d\hat{t}$$

$\Omega_r$  Reward area

**a** Control Parameter

$U_j$  nozzle exit velocity

$p_s$  jet's impingement pressure

$\hat{b}$  Intensity magnetic field

Model based control approach (White box)

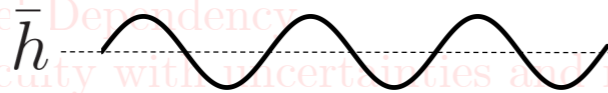
Advantages

- Fast and Efficient
- Stability Guarantees
- Established literature

Control **small** amplitude waves

- Linear Systems Limitation
- Model Dependency
- Difficulty with uncertainties and noise

Governing equations



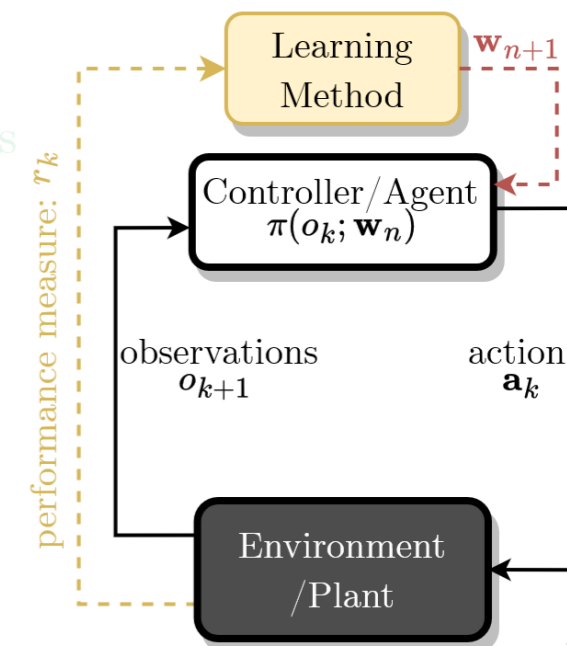
Machine learning approach (Black box)

Advantages

- Model-Free Learning
- Nonlinear and Complex Systems
- Adaptability

Control **large** amplitude waves

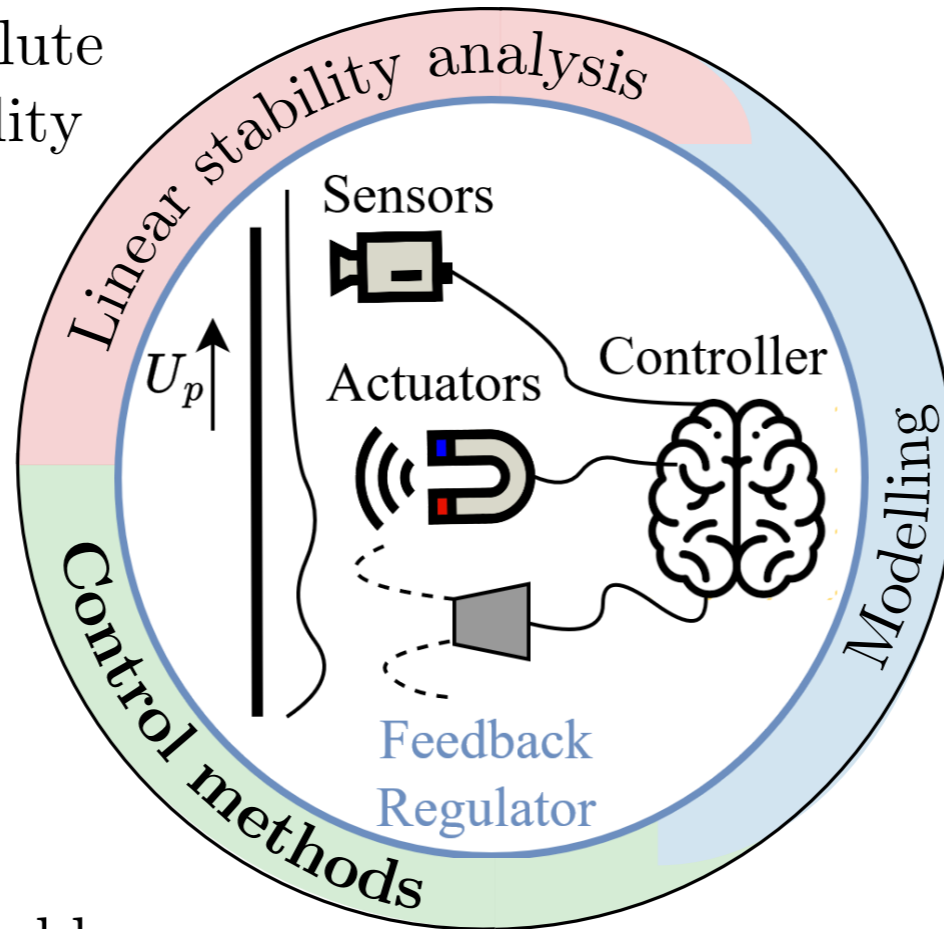
- No Stability Guarantees
- Sample Inefficiency
- Computational Complexity



# Contents

## Linear stability analysis

Computation of the threshold between absolute and convective instability



## Control methods

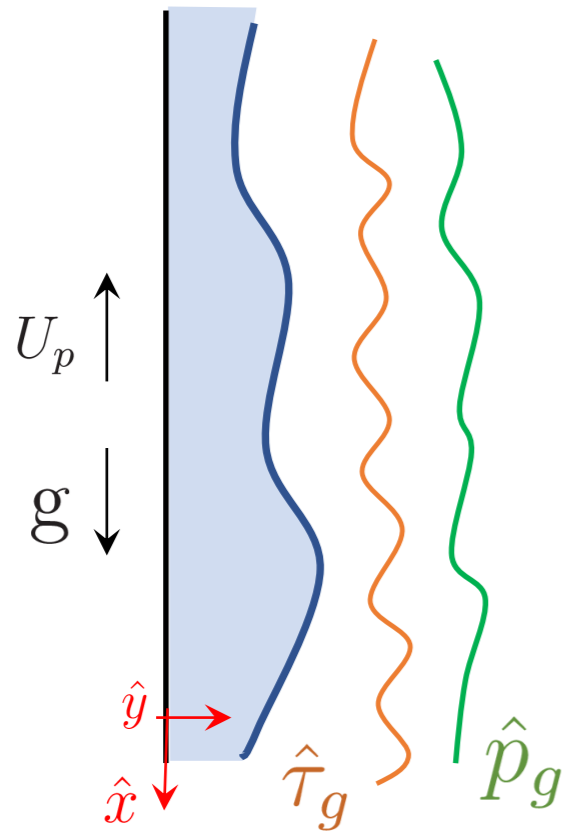
- ❑ Definition optimal control problem
- ❑ Small-amplitude control via linear stability methods
- ❑ Large-amplitude control via machine learning methods

## Derive simplified model

Derivation liquid film reduced order model with actuators modelling

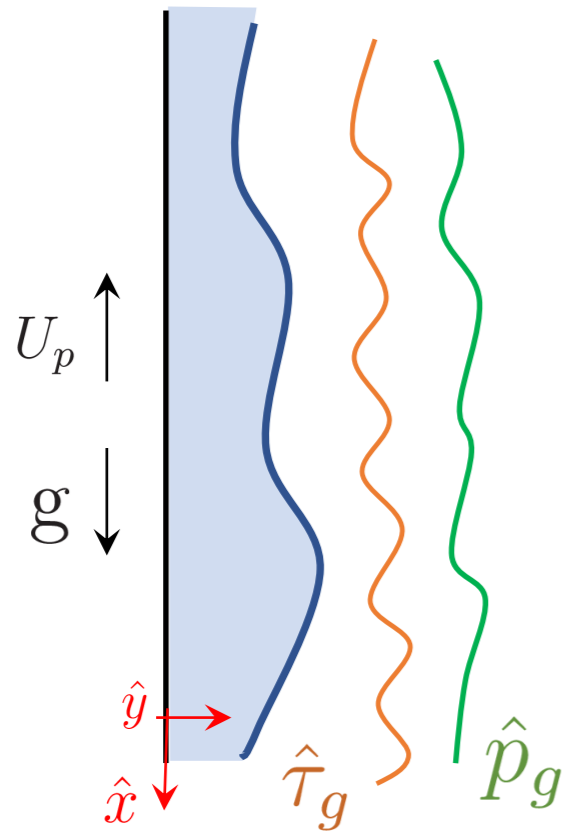
# Linear stability based control

Independent **pressure** and **shear stress** distributions at the free surface



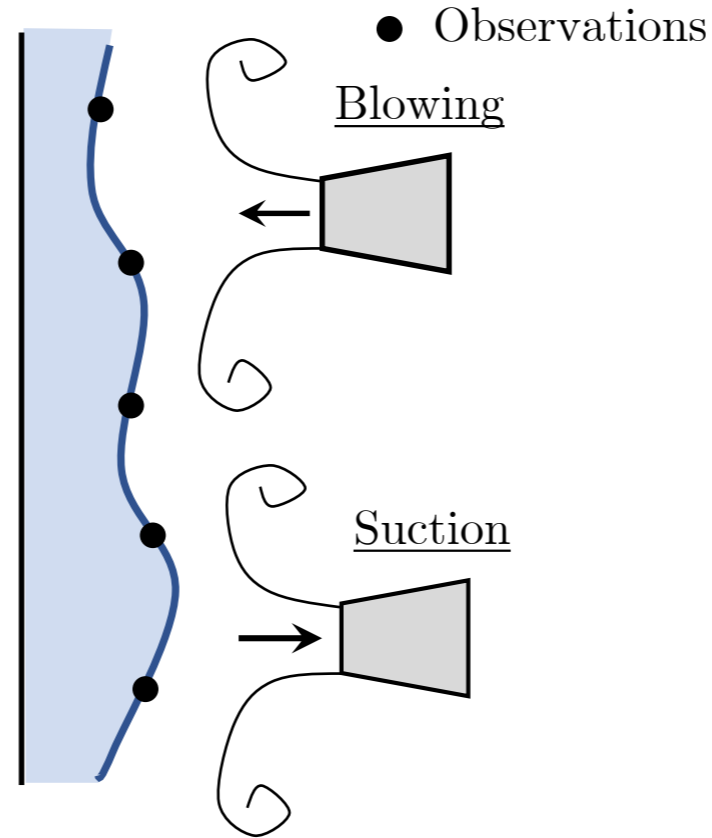
# Linear stability based control

Independent **pressure** and **shear stress** distributions at the free surface



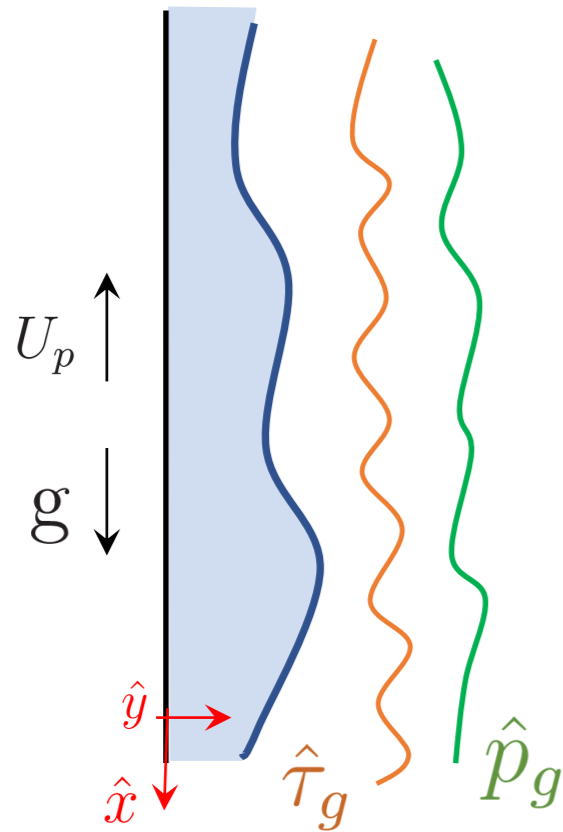
Control jets with blowing and suction

$$\hat{\tau}_g = f(\hat{p}_g)$$



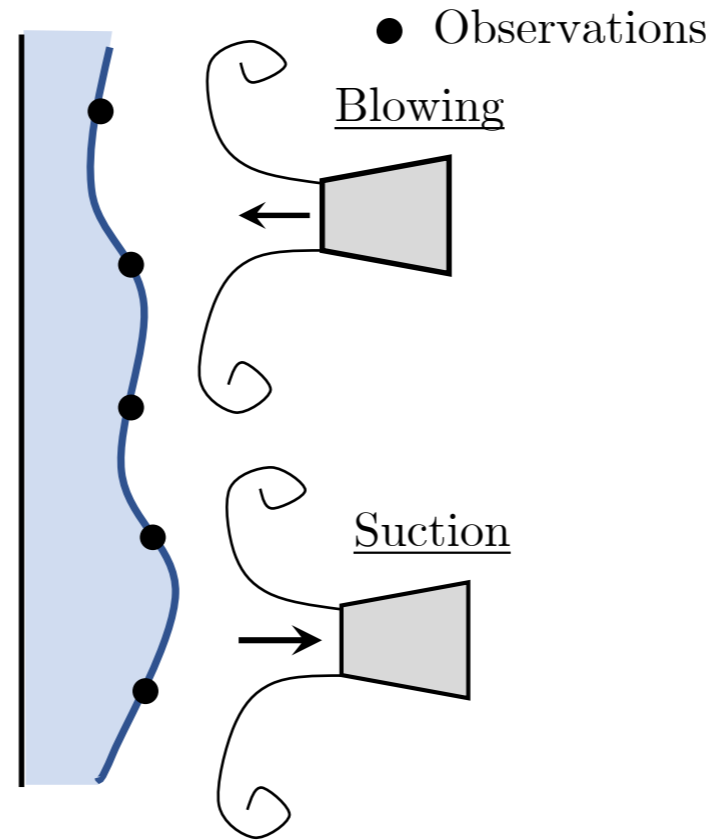
# Linear stability based control

Independent **pressure** and **shear stress** distributions at the free surface

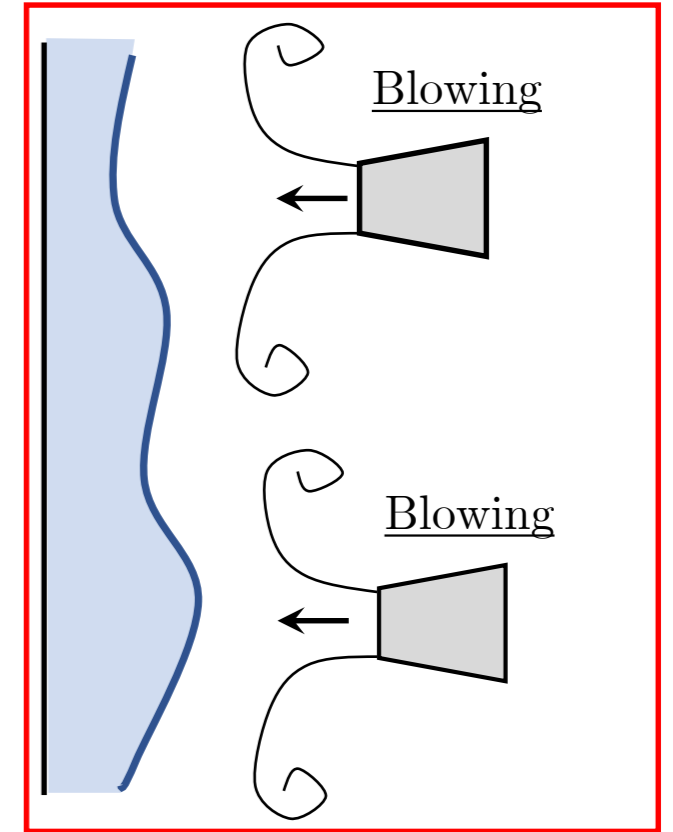


Control jets with blowing and suction

$$\hat{\tau}_g = f(\hat{p}_g)$$



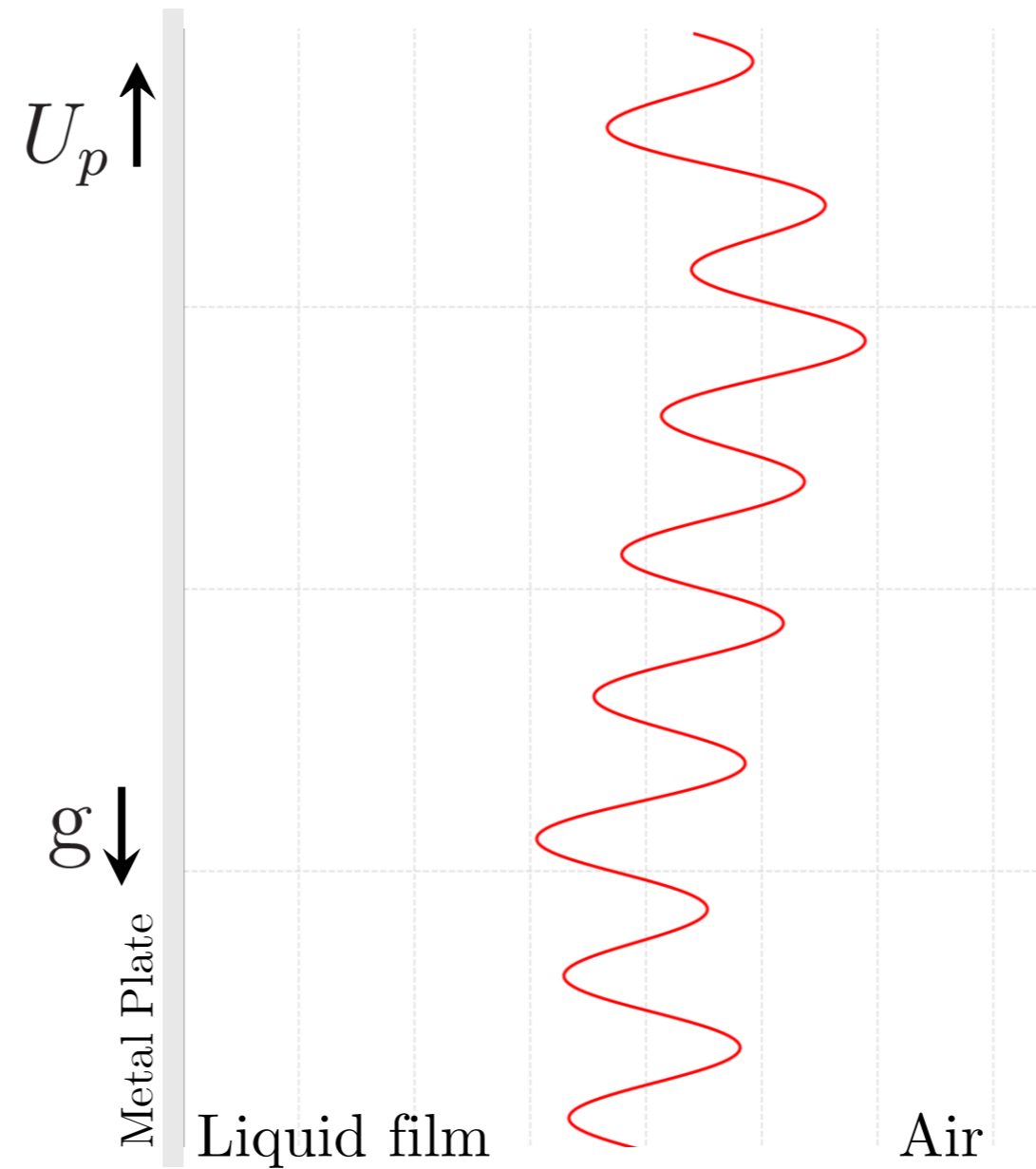
Blowing control jets



Complexity

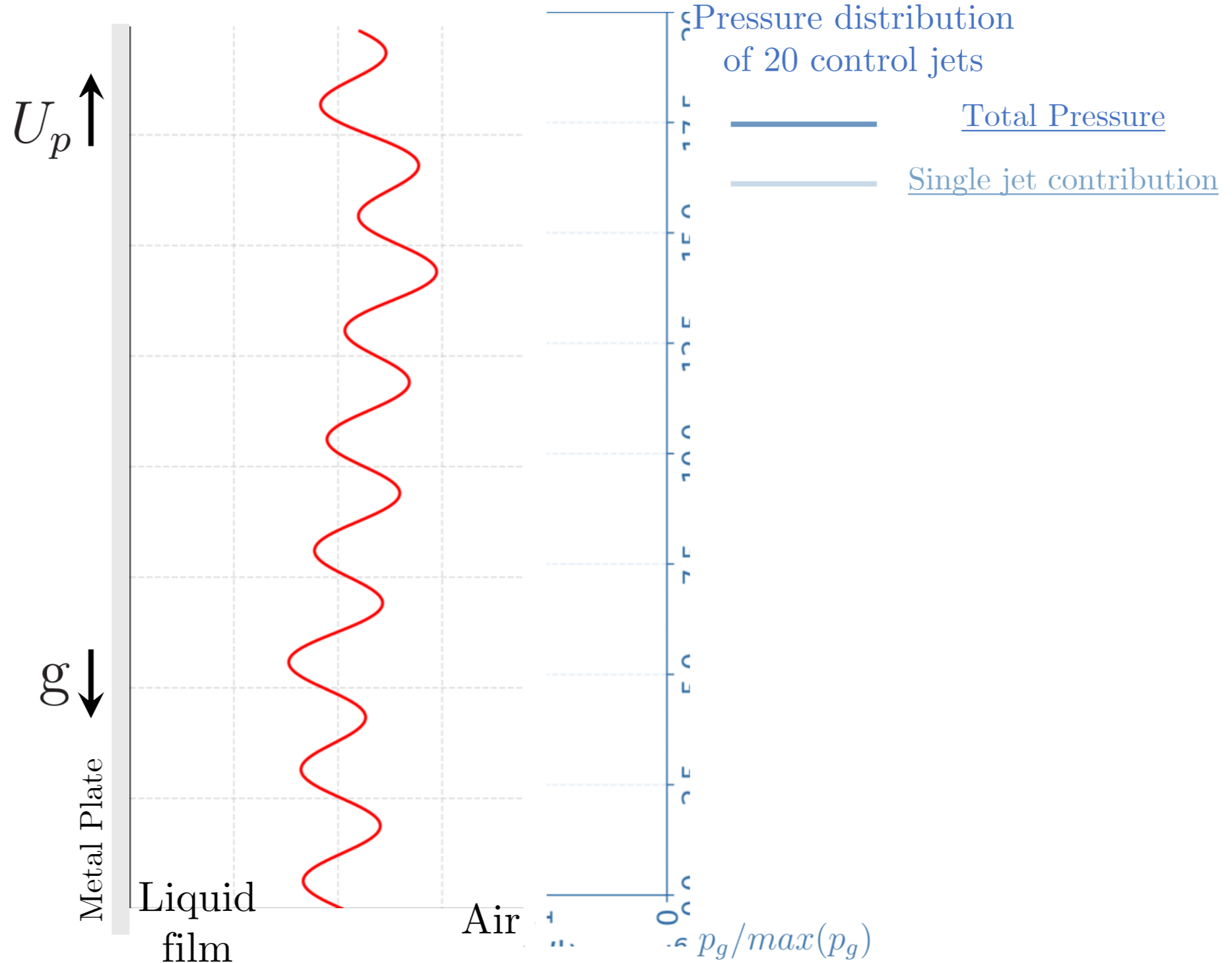
# Linear stability based control

The controller was tested in the stabilization of a highly unstable liquid film





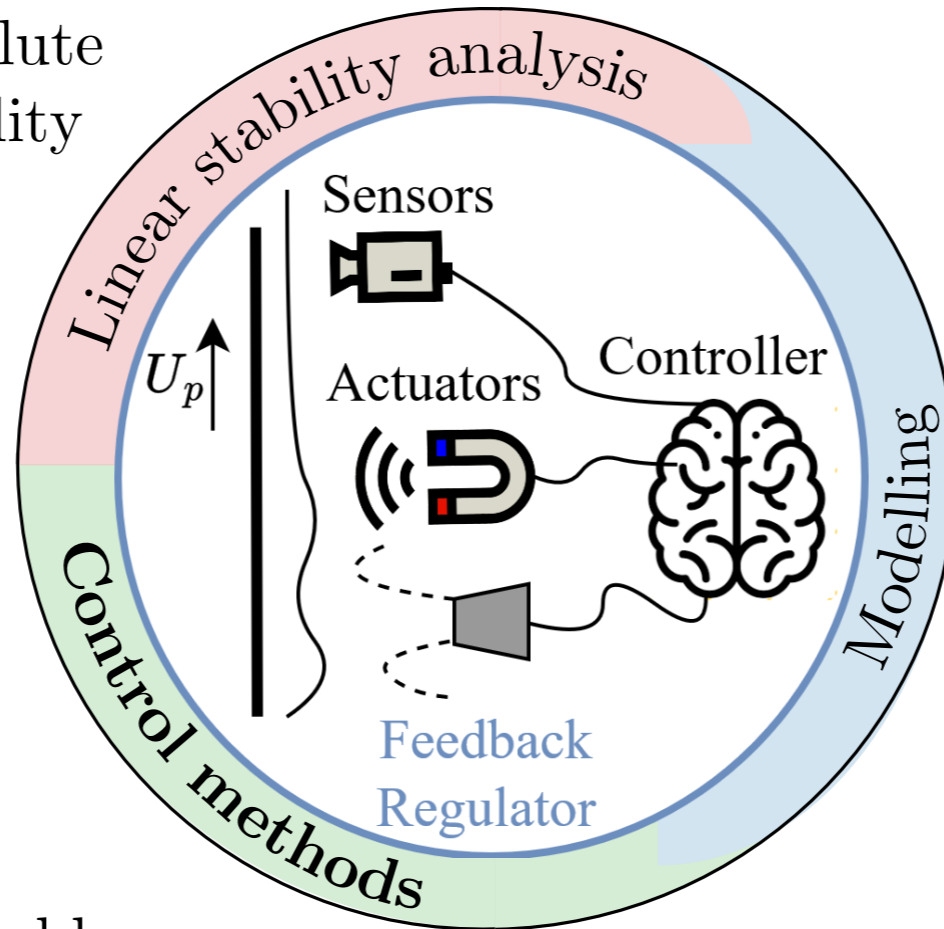
# Model predictive control



# Contents

## Linear stability analysis

Computation of the threshold between absolute and convective instability



## Control methods

- ❑ Definition optimal control problem
- ❑ Small-amplitude control via linear stability methods
- ❑ Large-amplitude control via machine learning methods

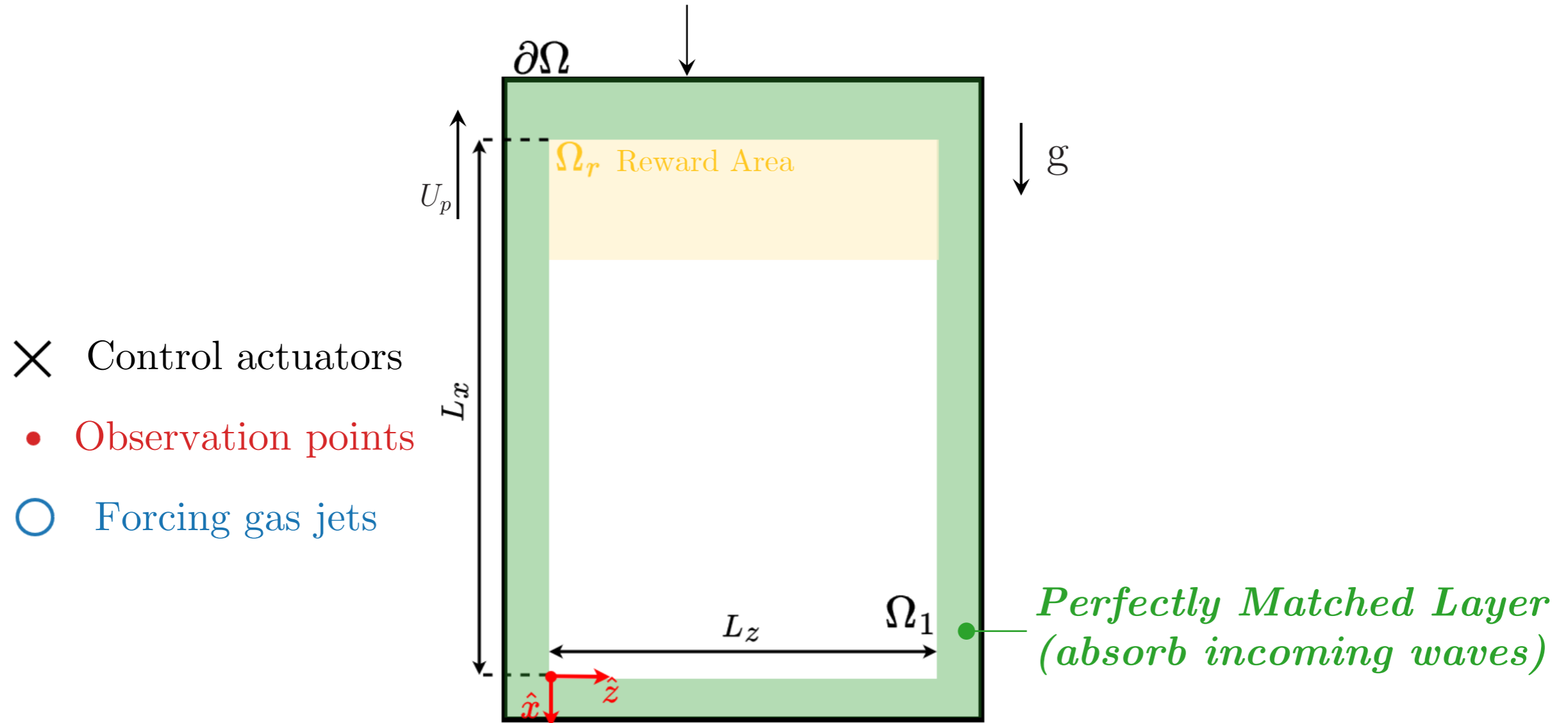
## Derive simplified model

Derivation liquid film reduced order model with actuators modelling

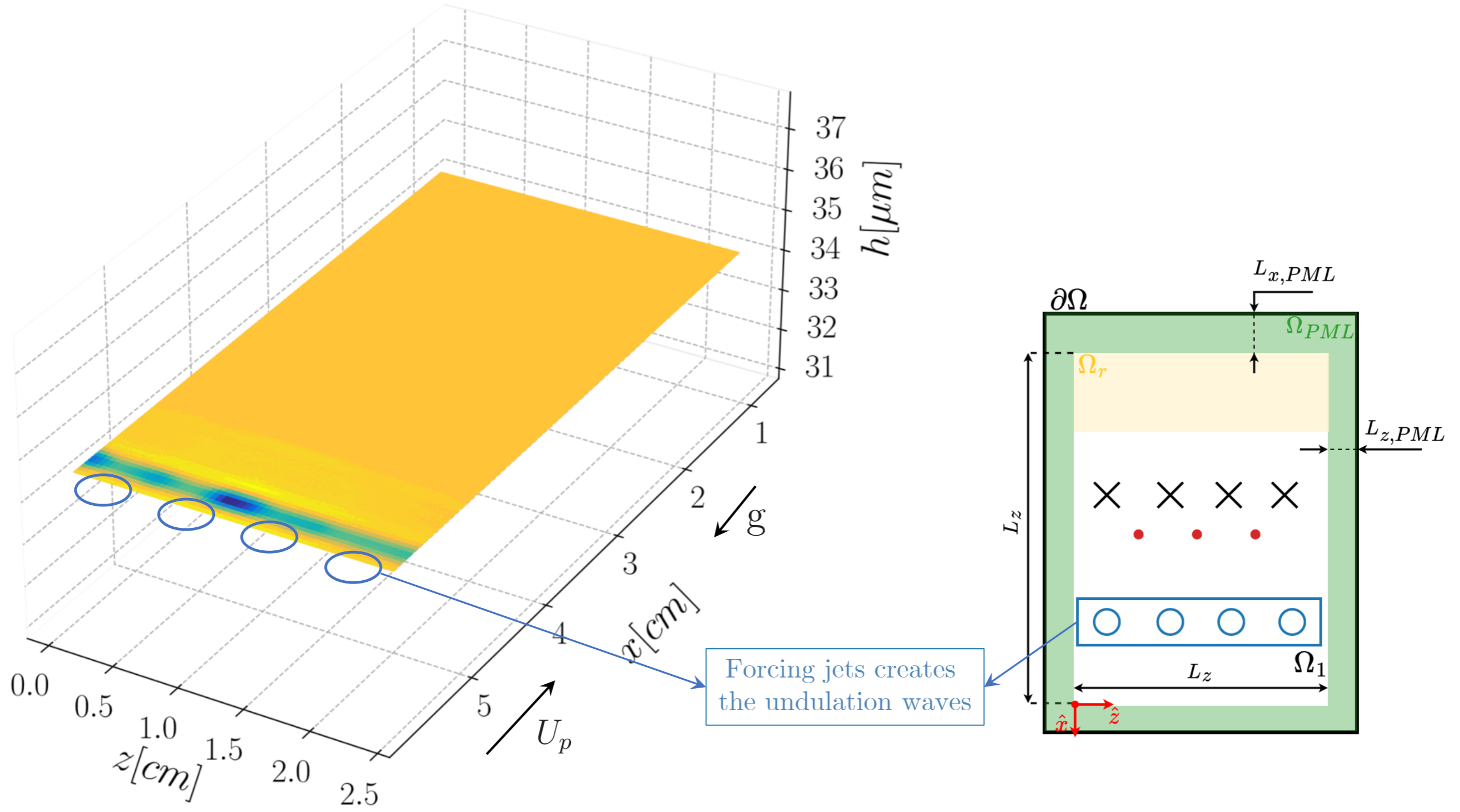
# Undulation control with machine learning methods

*Fourier pseudo-spectral implementation*

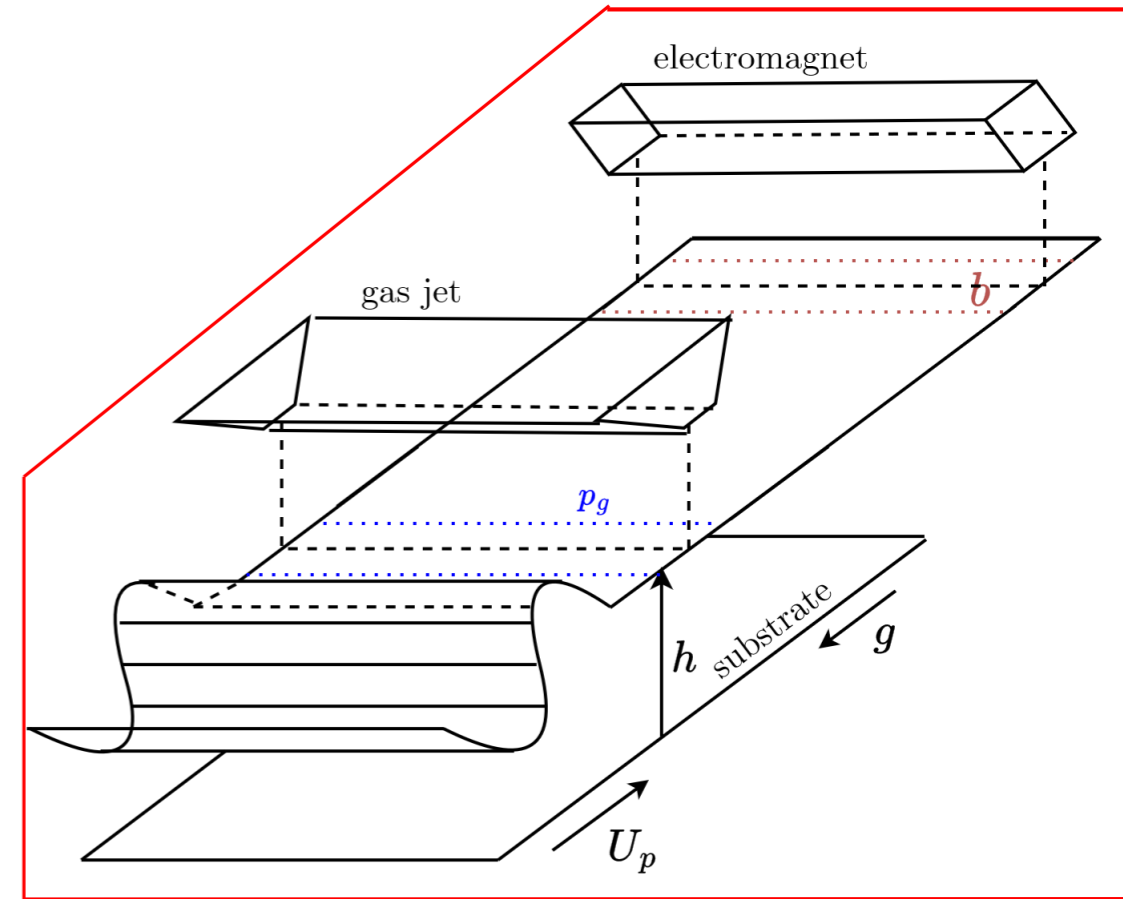
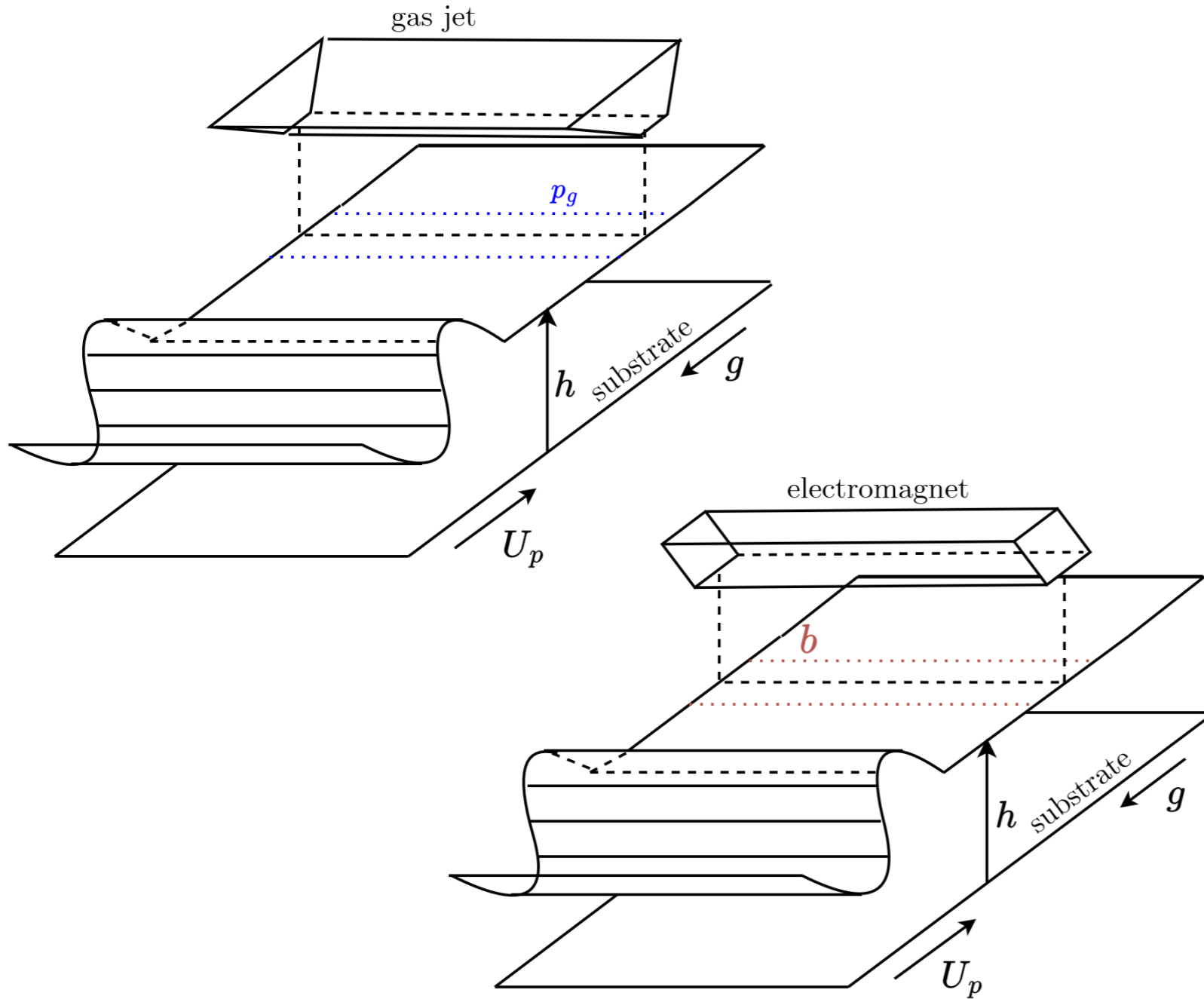
*Periodic boundary conditions*



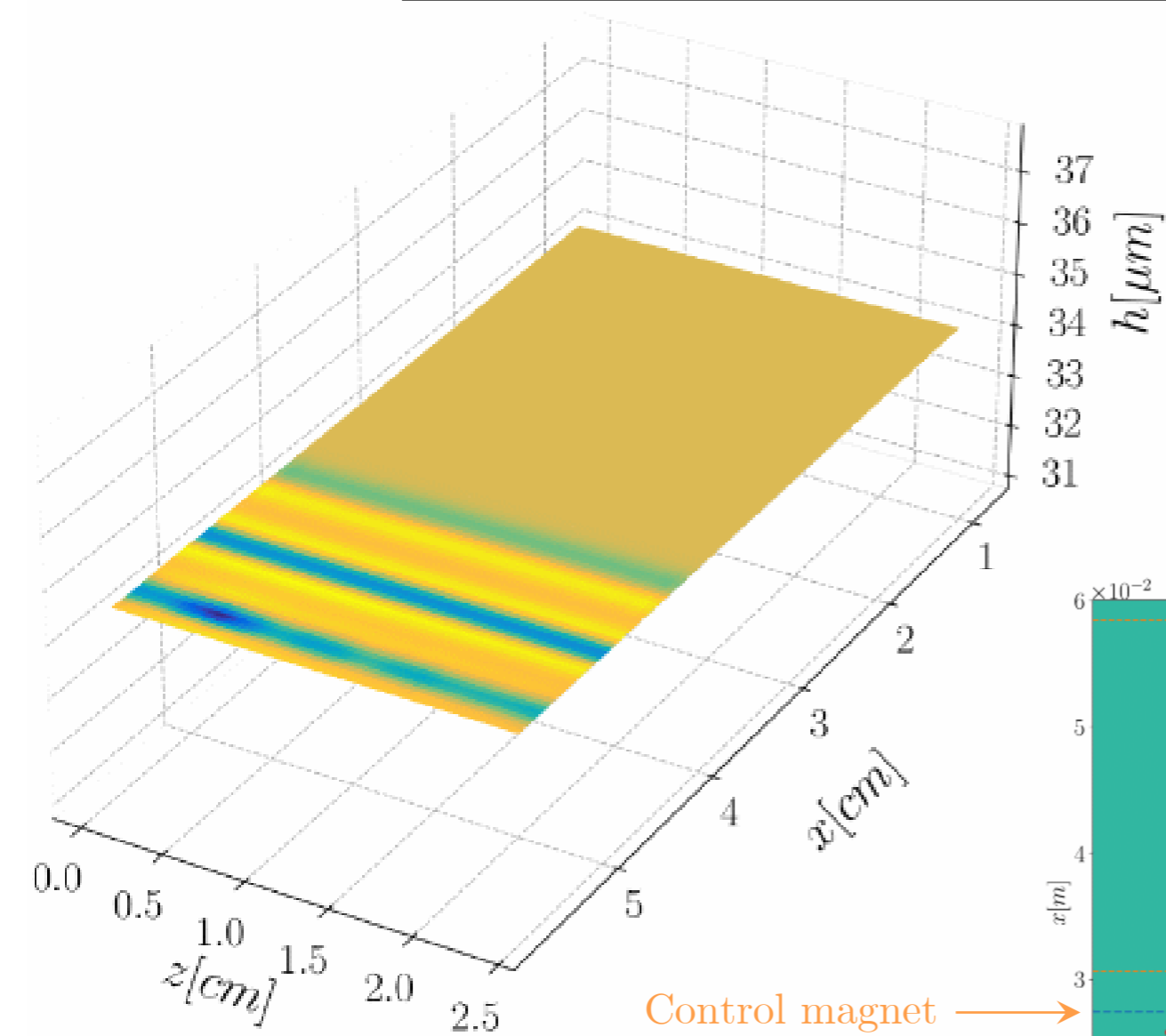
# Undulation control with machine learning methods



# Undulation control with machine learning methods

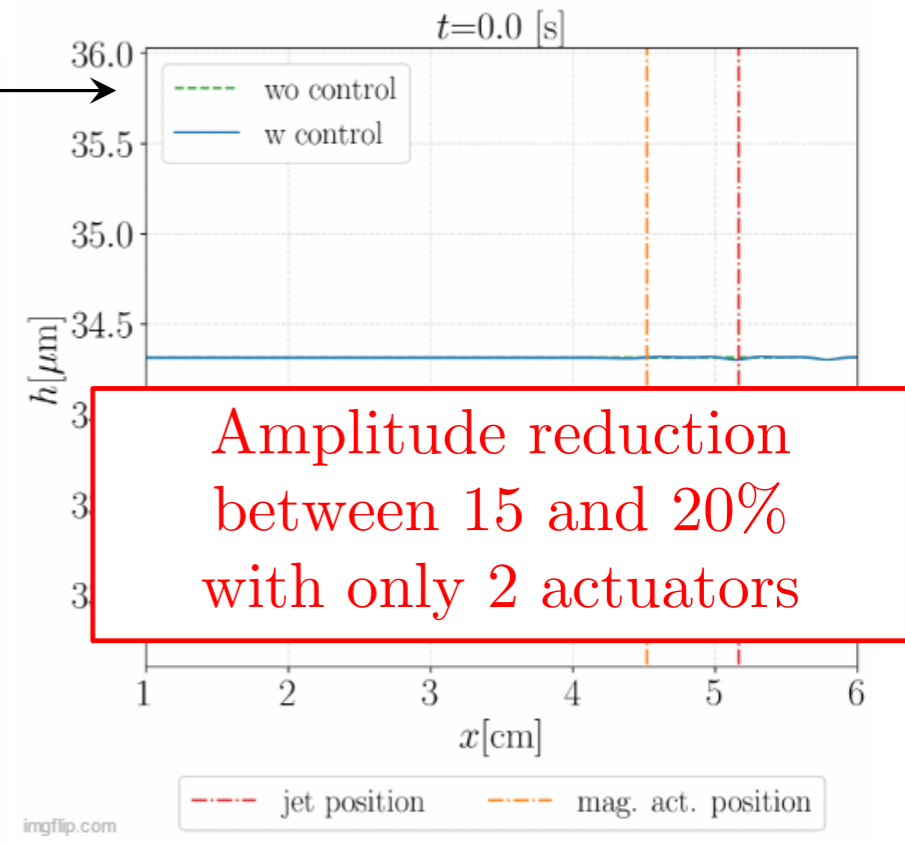
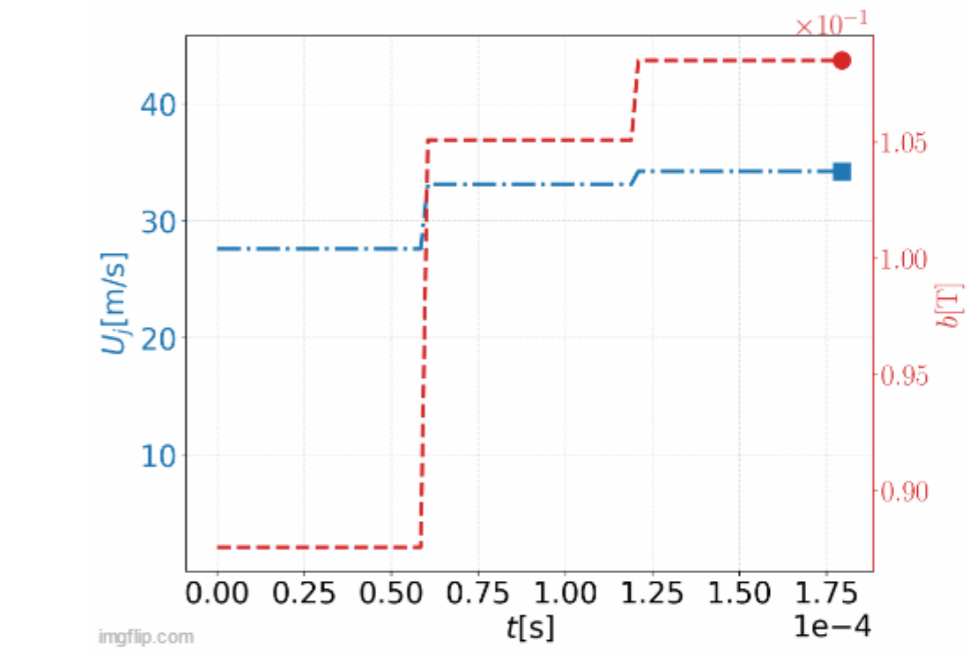
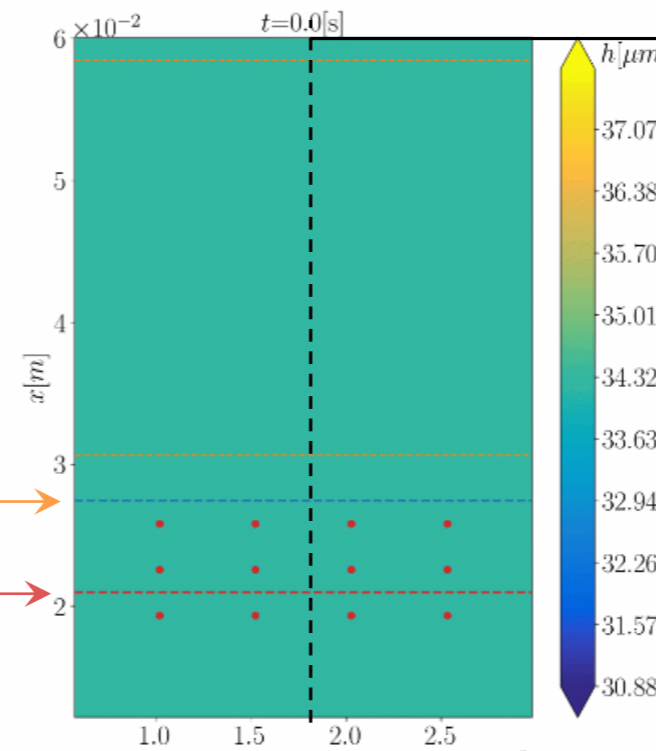


# Undulation control with zinc



Control magnet →

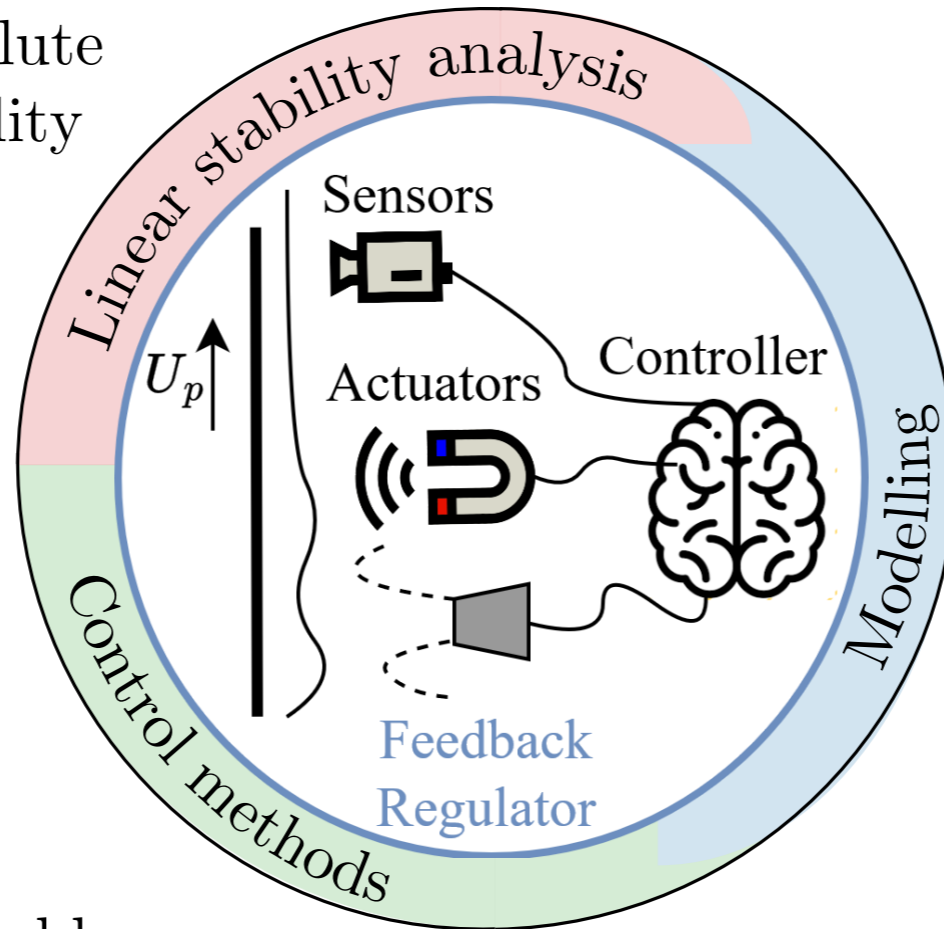
Control jet →



# Contents

## Linear stability analysis

Computation of the threshold between absolute and convective instability

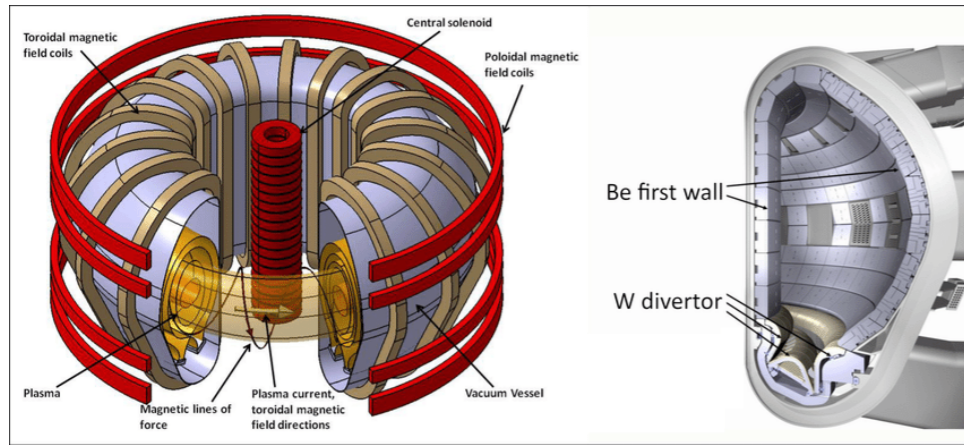


## Derive simplified model

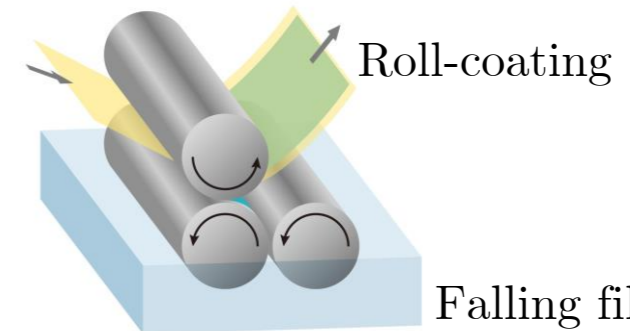
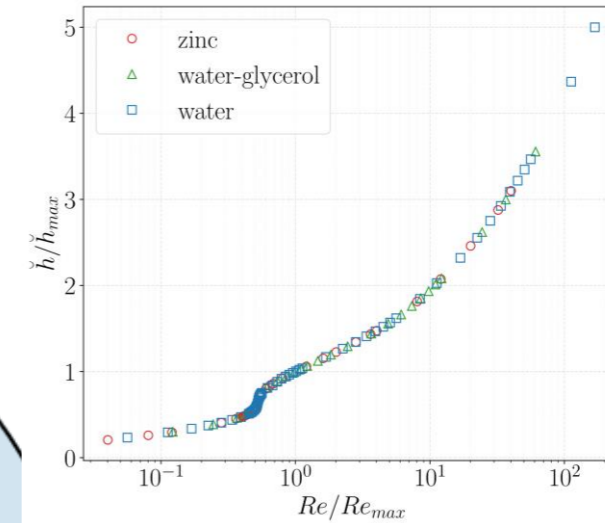
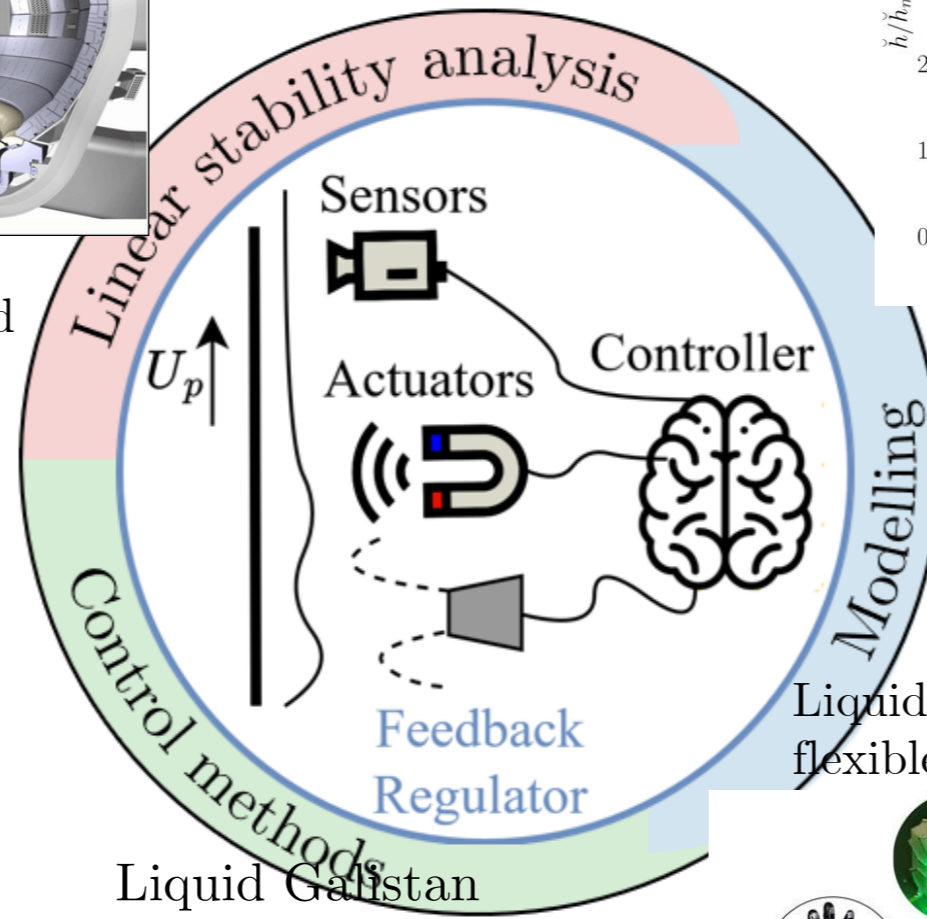
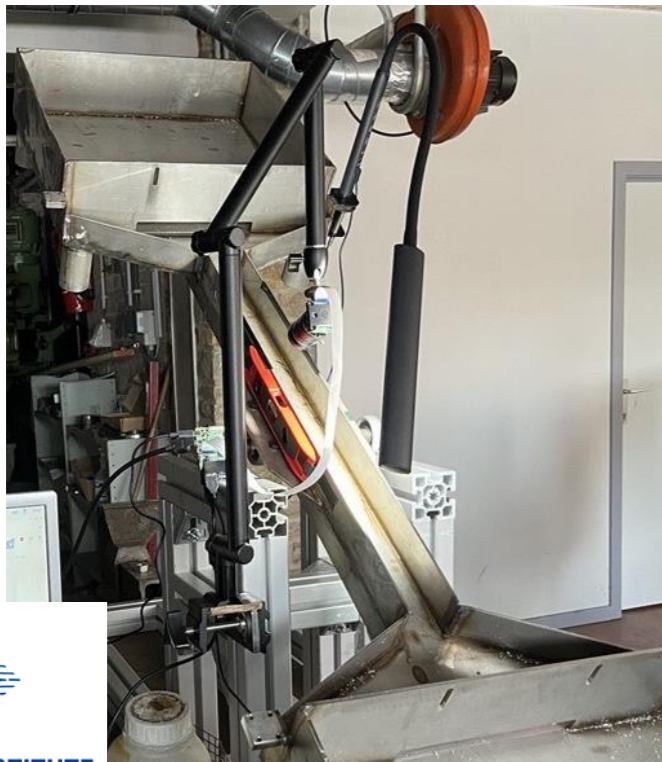
Derivation liquid film reduced order model with actuators modelling

## Control methods

- ❑ Definition optimal control problem
- ❑ Small-amplitude control via linear stability methods
- ❑ Large-amplitude control via machine learning methods

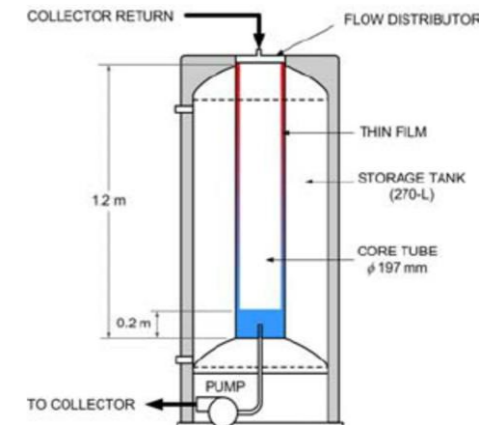


## Active MagnEtic control of LIquid mEtal (AMELIE) facility

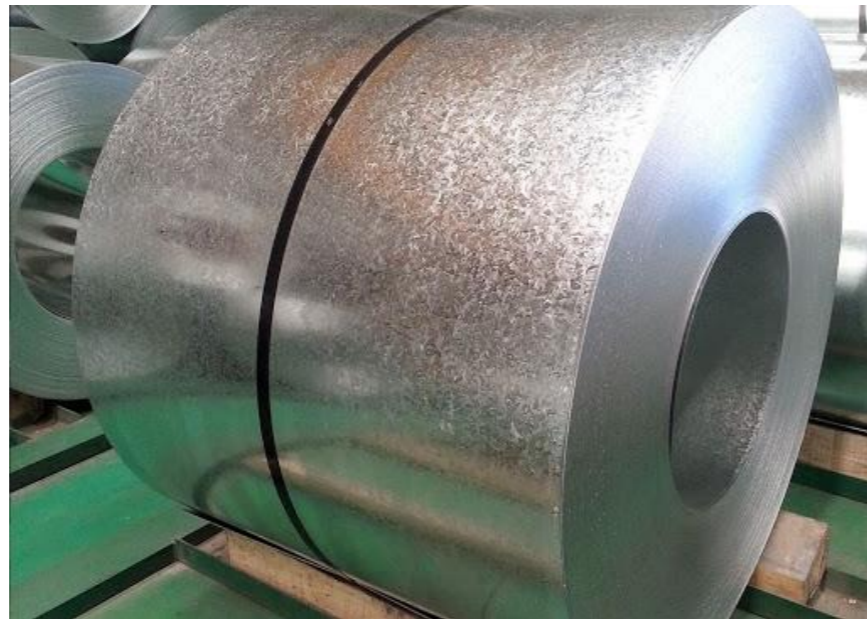


Liquid metal for flexible electronics

Falling film heater exchanger







Thank you for your kind attention.



*The larger the island of knowledge, the longer the shoreline of wonder*