Space-adaptive simulation of transition and turbulence in shear flows

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ERCOFTAC Autumn Festival, London, UK, October 10th, 2024

Outline

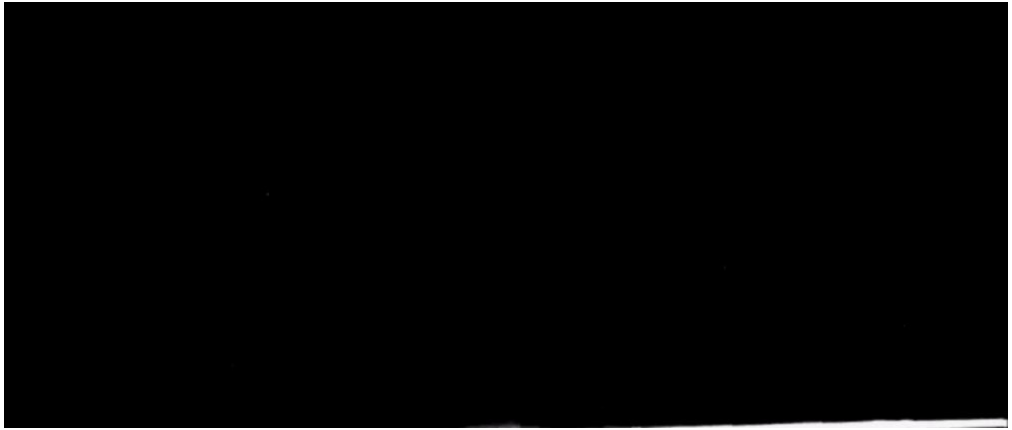
- **Introduction**
- Space-adaptive numerical framework
- Transitional and turbulent coherent structures in shear flows
- Conclusions and outlook

Introduction

Shear flows are ubiquitous and can be categorised as

- Free shear flows
- Wall-bounded shear flows

Vortex shedding around Jeju Island, Korean strait

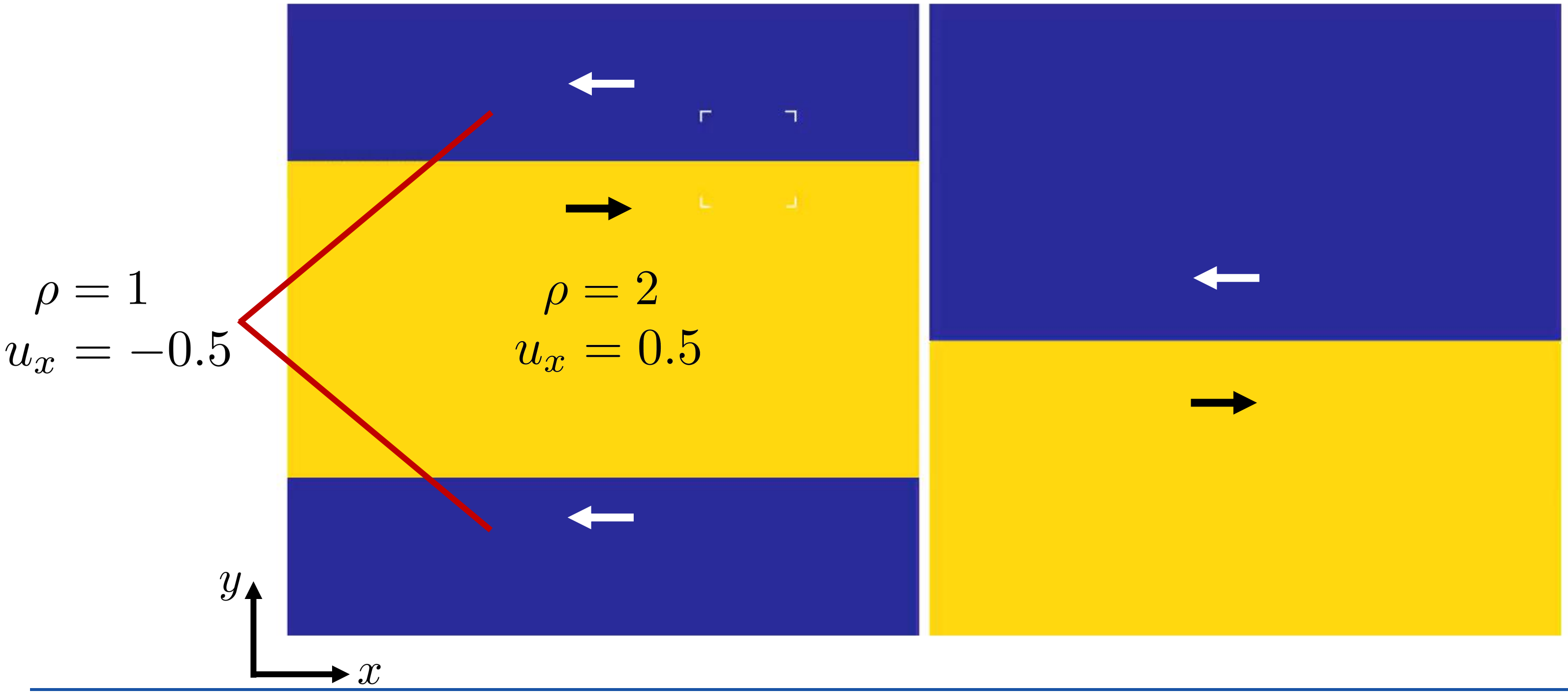


Spatially developing turbulent boundary layer on a flat plate

NASA Worldview, Himawari satellite showing the Von Kármán vortex street on December 12th, 2019
J.H. Lee *et al.*, Spatially developing turbulent boundary layer on a flat plate, V0054 Gallery of Fluid Motion, 2014

Introduction

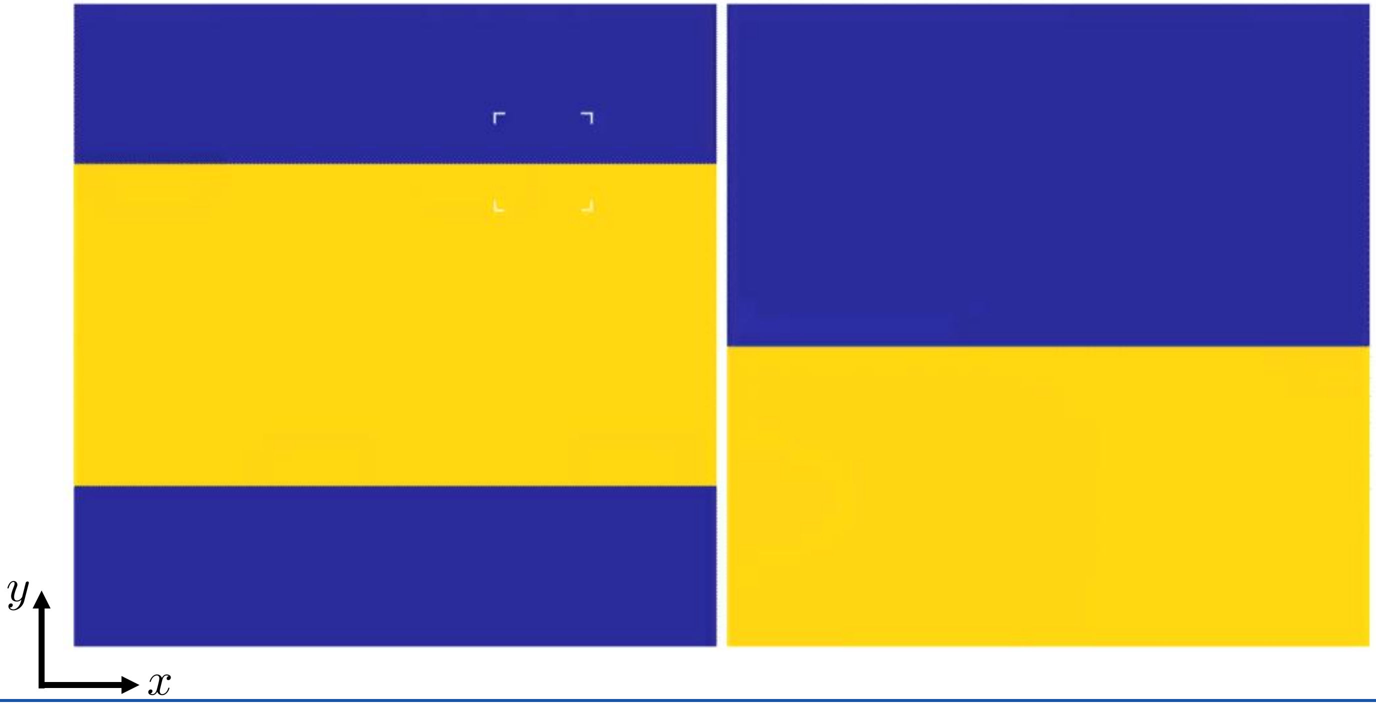
Two-dimensional simulation of the Kelvin-Helmholtz instability, one of the most important flow instabilities



K. Schaal *et al.*, Astrophysical hydrodynamics with a high-order discontinuous Galerkin scheme and adaptive mesh refinement, MNRSA, 2015

Introduction

Two-dimensional simulation of the Kelvin-Helmholtz instability, one of the most important flow instabilities



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- **Space-adaptive numerical framework**
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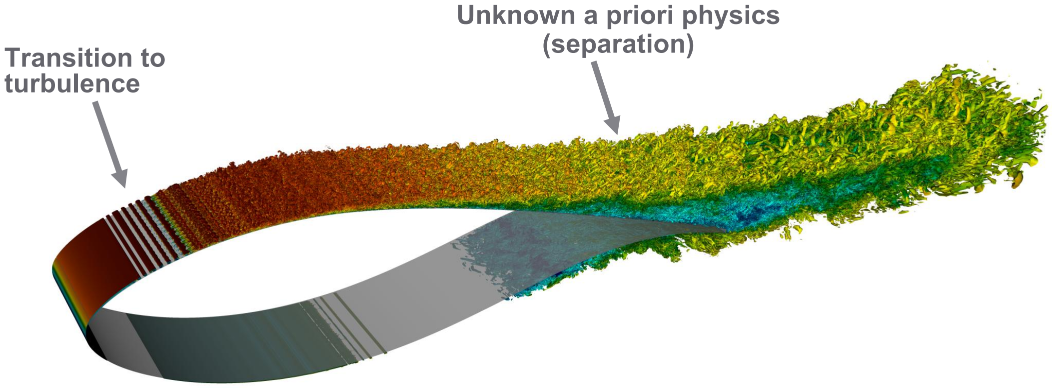
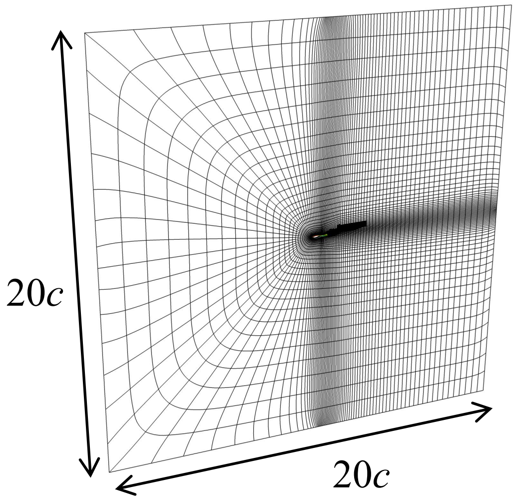
Direct numerical simulation

Spectral element method code

- Dimensionless incompressible NS equations
- **Nek5000**, open-source code for CFD
  - Minimal numerical viscosity & dispersion
  - High-order implicit/explicit time integration
  - High-order spatial discretisation (SEM)
- Adaptive Mesh Refinement (AMR)



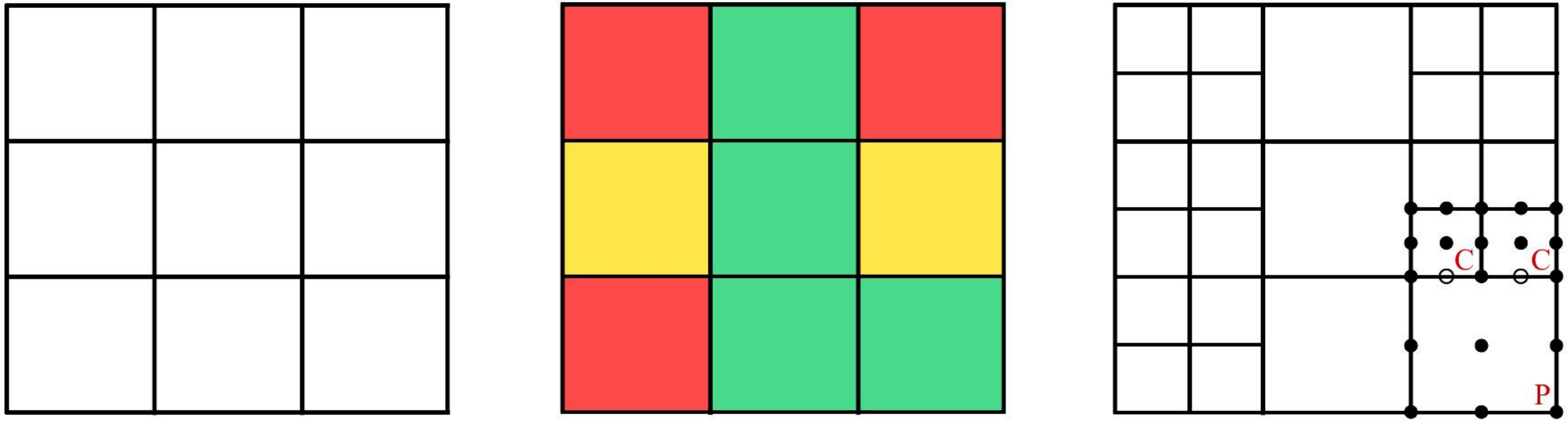


P.F. Fischer, J. Lottes, S.G. Kerkemeier, Nek5000: fast high-order scalable CFD, 2008
T.C. Fava, D. Massaro *et al.*, Transition to turbulence in a rotating wind turbine blade at Re=300,000, JFM (To appear), 2024 **(Thesis paper)**

Adaptive mesh refinement

Introduction

- Error measurement
- Adaptive mesh refinement strategy
- Mesh refinement/coarsening methodology



***h*-refinement**

N. Offermans, Aspects of adaptive mesh refinement in the spectral element method, PhD thesis, KTH, 2019
D. Massaro *et al.*, Interface discontinuities in Spectral-Element simulations with Adaptive Mesh Refinement, ICOSAHOM, 2022 **(Thesis paper 3)**

Adaptive mesh refinement

Error measurement

- **Spectral error indicator**: truncation and quadrature error
- **Adjoint error estimator**: driven by the adjoint-sensitivity to a functional of interest
- **Causality-based error indicator**: driven by the causality-sensitivity (Shannon transfer entropy)

Shannon entropy: the average amount of information as a measure of uncertainty

$$I(y) = -\log[p(y)] \qquad H(Y) = \overline{I(Y)} = \sum_y -p(y)\log[p(y)] \geq 0$$

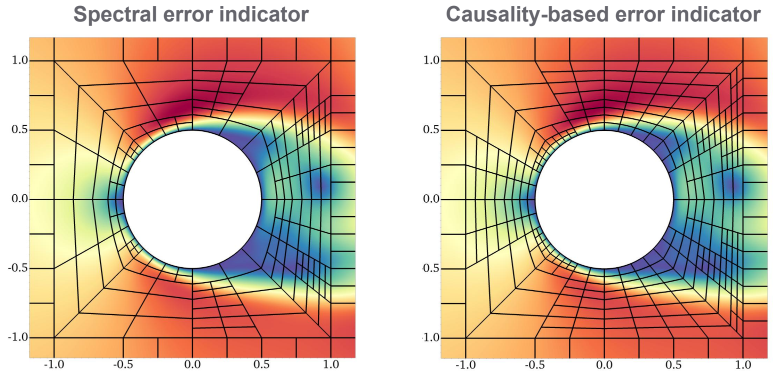
What is the entropy of a coin toss experiment?

$$p(\text{heads}) = 0.5,\ p(\text{tails}) = 0.5 \qquad H(Y) = 1,$$

$$p(\text{heads}) = 1,\ p(\text{tails}) = 0 \qquad H(Y) = 0$$

Does correlation measure causality?

$$y^+ = 30, \quad \rho_{u-u_\tau} = 0.541,\ \rho_{u_\tau - u} = 0.541 \qquad TE_{u\to u_\tau} = 0.0448,\ TE_{u_\tau \to u} = 0.353$$

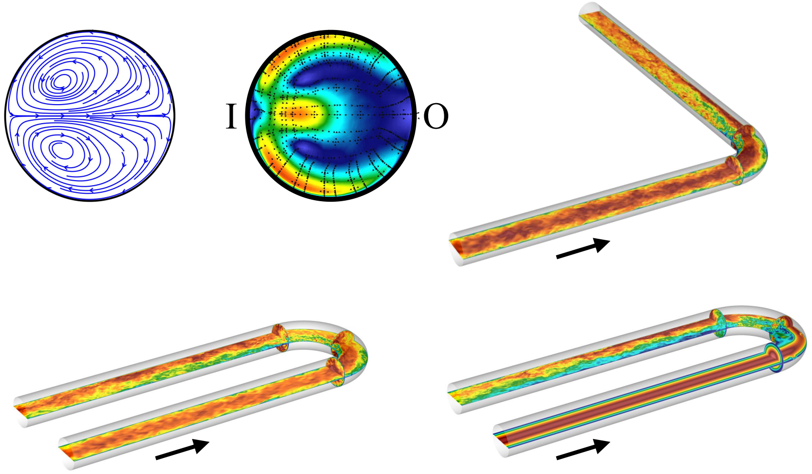


N. Offermans, D. Massaro *et al.*, Error-driven adaptive mesh refinement for unsteady turbulent flows in spectral-element simulations, Comput. Fluids, 2023 **(Thesis paper 1)**
D. Massaro *et al.*, On the potential of transfer entropy in turbulent dynamical systems, Nat. Sci. Rep., 2024 **(Thesis paper 2)**

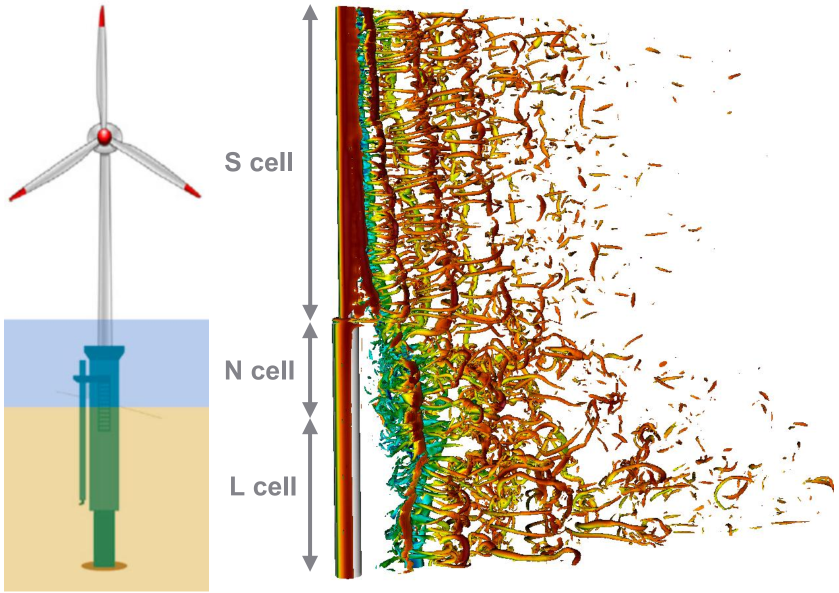
Transitional and turbulent shear flows

Bent pipe and stepped cylinder flows

- Spatially developing bent pipe flows
  - Swirl switching phenomenon in the transitional and turbulent regime: bending angle and inflow effect



- Flow around a stepped cylinder
  - Investigation of the three wake cells appearing in the transitional and turbulent regime



V. Lupi, D. Massaro *et al.*, Swirl switching in spatially developing bent pipes, PRF (Submitted), 2024 **(Thesis paper 8)**
D. Massaro & P. Schlatter, Global stability of the flow past a stepped cylinder, JFM, 2024 **(Thesis paper 9)**

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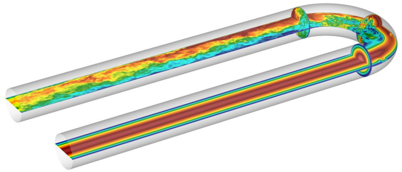
Transition in wall-bounded shear flows

Turbulent bent pipe flows



- Unknown physical mechanism responsible for the transition
- Global stability analysis
  - Base flow calculation from non-linear incompressible Navier-Stokes equations

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = -\boldsymbol{\nabla} p + \dfrac{1}{Re_b} \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, \end{cases}$$

  - Solution of direct linear problem

$$\begin{cases} \dfrac{\partial \boldsymbol{u}'}{\partial t} + (\boldsymbol{u}' \cdot \nabla)\, \boldsymbol{U} + (\boldsymbol{U} \cdot \nabla)\, \boldsymbol{u}' = -\boldsymbol{\nabla} p' + \dfrac{1}{Re_b} \nabla^2 \boldsymbol{u}', \\ \boldsymbol{\nabla} \cdot \boldsymbol{u}' = 0, \end{cases}$$

$$\boldsymbol{u}'(\mathbf{x}, t) = \hat{\boldsymbol{u}}(\mathbf{x}) e^{\lambda t}, \quad \lambda \in \mathbb{C}$$

$$p'(\mathbf{x}, t) = \hat{p}(\mathbf{x}) e^{\lambda t}, \quad \lambda \in \mathbb{C}$$

$$\lambda \mathcal{M} \hat{\mathbf{r}} = \mathcal{J} \hat{\mathbf{r}}$$

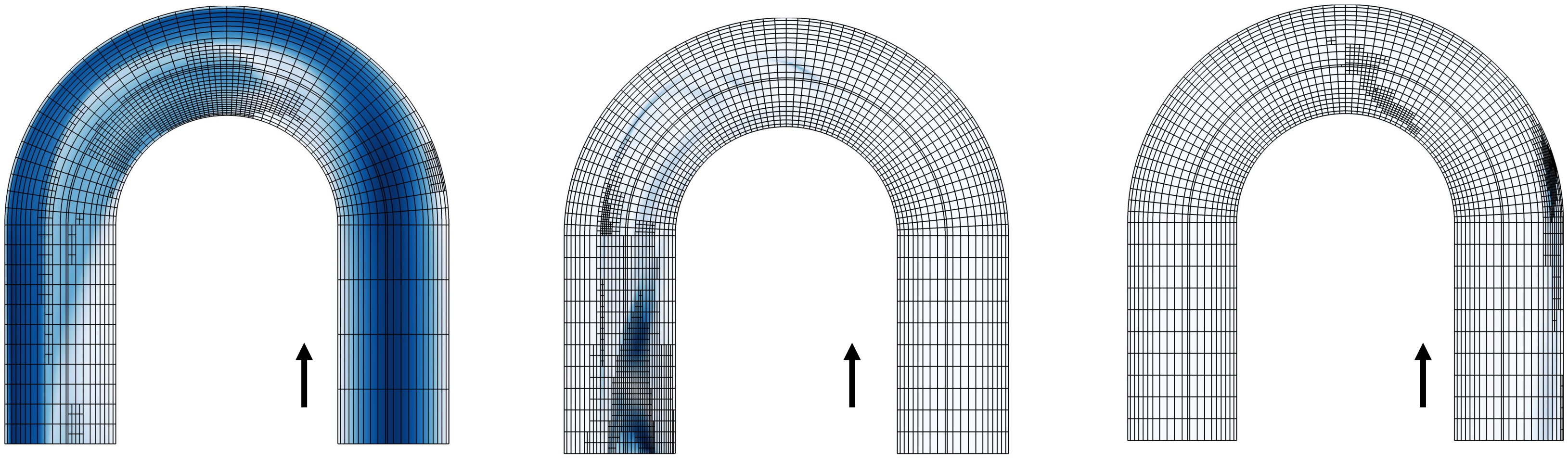
  - Solution of adjoint linear problem (for structural sensitivity)

$$\begin{cases} -\dfrac{\partial \boldsymbol{u}^\dagger}{\partial t} - (\boldsymbol{U} \cdot \nabla)\, \boldsymbol{u}^\dagger + (\nabla \boldsymbol{U})^T \boldsymbol{u}^\dagger = \boldsymbol{\nabla} p^\dagger + \dfrac{1}{Re_b} \nabla^2 \boldsymbol{u}^\dagger, \\ -\boldsymbol{\nabla} \cdot \boldsymbol{u}^\dagger = 0, \end{cases}$$

$$\sigma = \frac{\ln(|k|)}{\Delta t}, \quad \omega = \frac{\arg(k)}{\Delta t},$$

D.C. Sorensen, Implicit application of polynomial filters in a k-step Arnoldi method, SIAM J. Matrix Anal. Appl., 1992
E. Åkervik, *et al.*, Steady solutions of the Navier–Stokes equations by selective frequency damping, Phys. Fluids, 2006

Transition in wall-bounded shear flows
Turbulent bent pipe flows



Non-linear solution | Linear direct solution | Linear adjoint solution

$\delta = R_p/R_c = 1/3$

$Re_b = 2550$

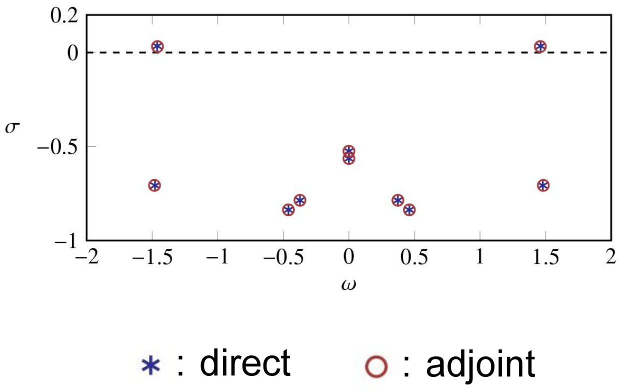
D. Massaro *et al.*, Adaptive mesh refinement for global stability analysis of transitional flows, ICOSAHOM, 2023 **(Thesis paper 6)**
D. Massaro *et al.*, Global stability of 180°-bend pipe flow with mesh adaptivity, PRF, 2023 **(Thesis paper 7)**

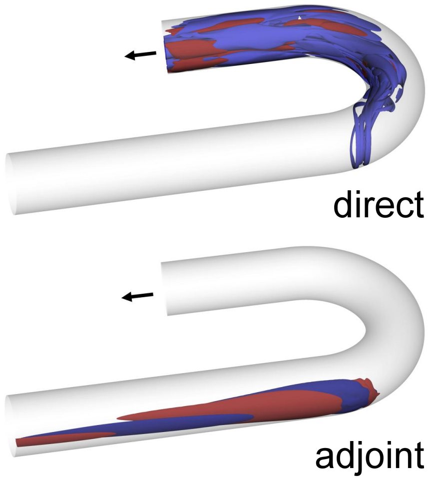
Transition in wall-bounded shear flows

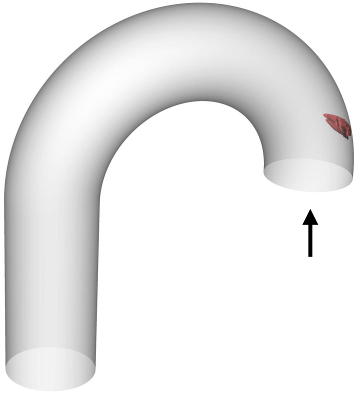
Turbulent bent pipe flows

- A pair of unstable complex conjugate eigenvalues found (Hopf bifurcation)
- Critical Reynolds number $Re_{b,cr} \approx 2528$
- Same frequency as in non-linear simulations, with period $T \approx 4.3$ convective time units at $Re_b = 2550$
- Structural sensitivity to spatially localised feedback to identify the core of the instability:
  - the wavemaker corresponds to the recirculation bubble

$$\eta(\mathbf{x}) = \frac{\|\hat{\boldsymbol{u}}^\dagger(\mathbf{x})\| \|\hat{\boldsymbol{u}}'(\mathbf{x})\|}{\int_\Omega \hat{\boldsymbol{u}}^\dagger(\mathbf{x})\hat{\boldsymbol{u}}'(\mathbf{x})\mathrm{d}\Omega}$$







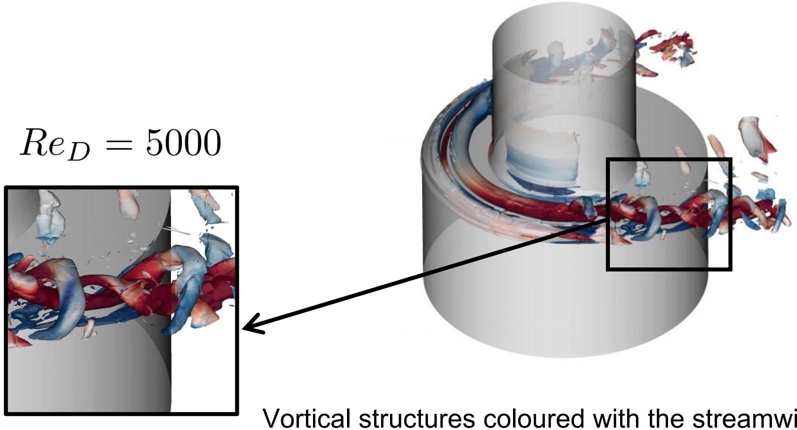
F. Giannetti & P. Luchini, Structural sensitivity of the first instability of the cylinder wake, JFM, 2007
D. Massaro *et al.*, Global stability of 180°-bend pipe flow with mesh adaptivity, PRF, 2023 **(Thesis paper 7)**

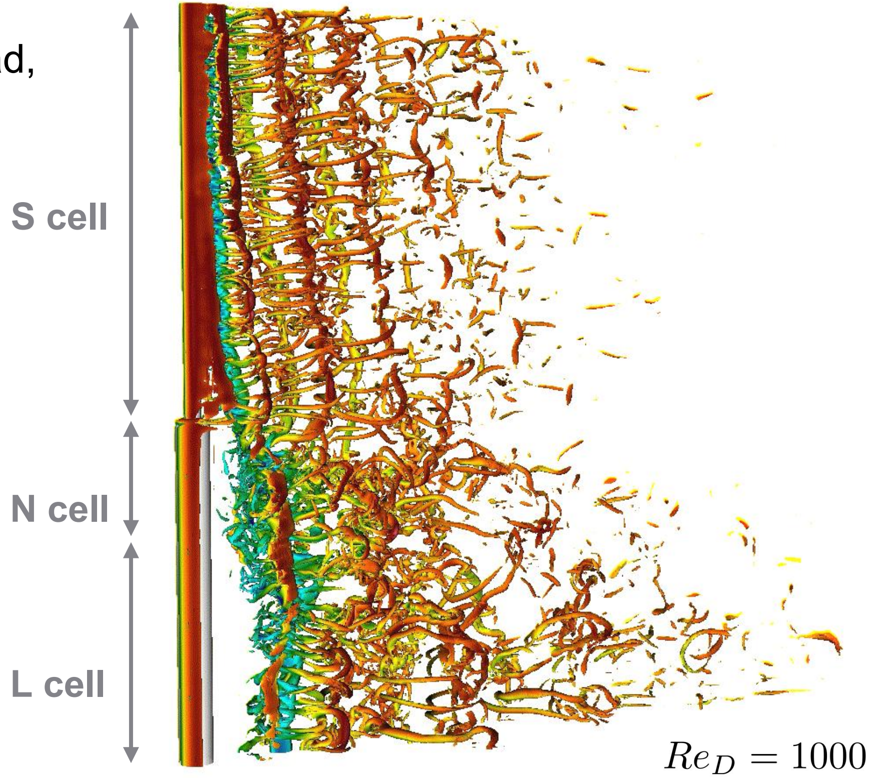
Turbulence in free shear flows

Flow around a stepped cylinder

- Lack of studies for more complicated, but widespread, cylindrical geometries

- Junction and wake vortex dynamics investigation
- S-N-L cells interaction in various regimes
- Stability analysis





Vortical structures coloured with the streamwise vorticity (on the junction) and the velocity (in the wake)

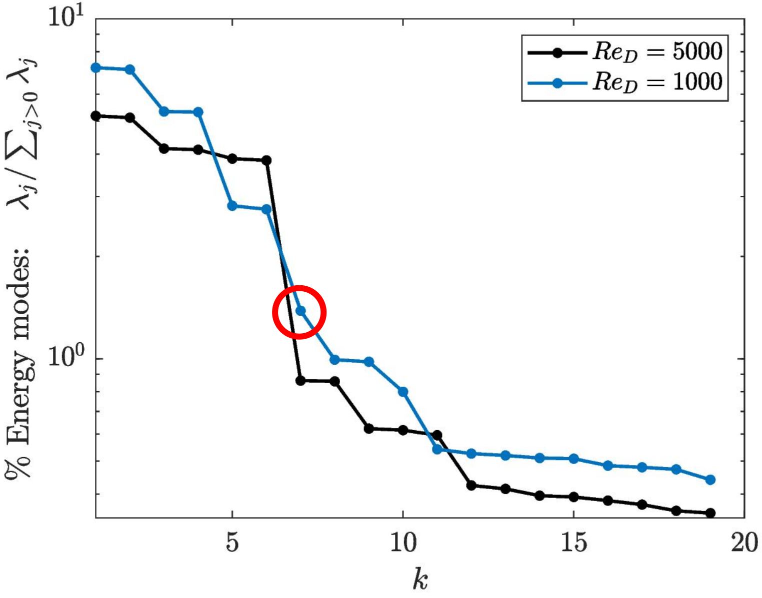
N. Ko & A. Chan, Wakes behind circular cylinders with stepwise change of diameter, Exp. Therm. Fluid Sci., 1992
C. Tian *et al.*, How vortex dynamics affects the structural load in step cylinder flow, JFM, 2023

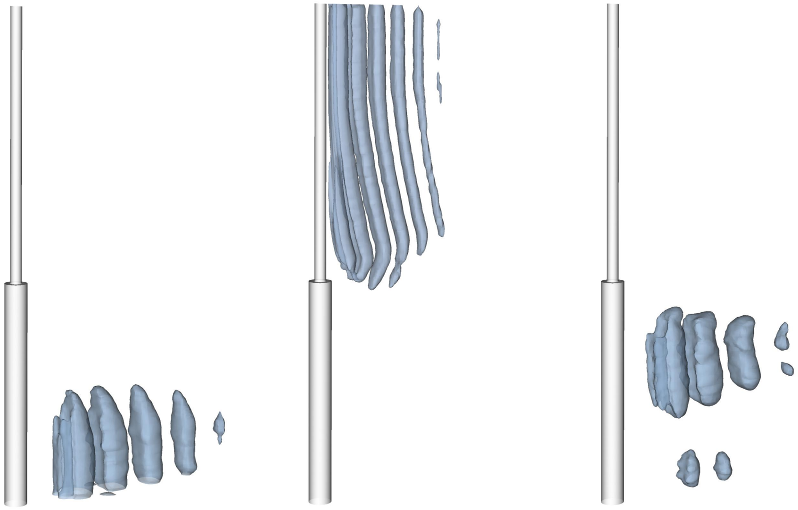
Turbulence in free shear flows

Proper orthogonal decomposition

$$U_m = [u_0, u_1, \ldots, u_{m-1}] \in \mathbb{R}^{n \times m}$$

$$U_m = XT = X\Sigma W^T$$

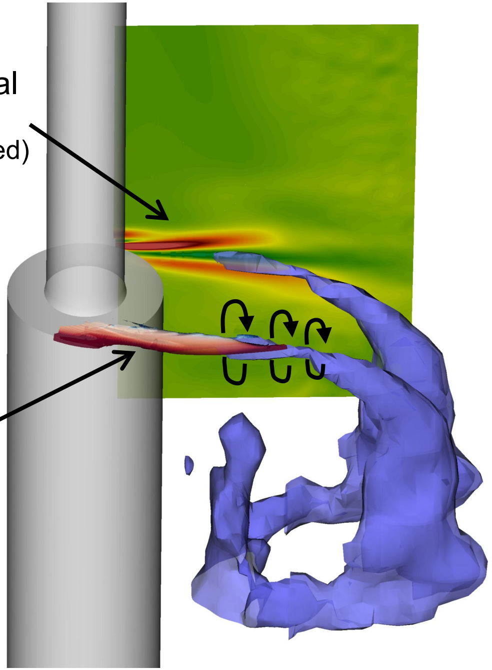




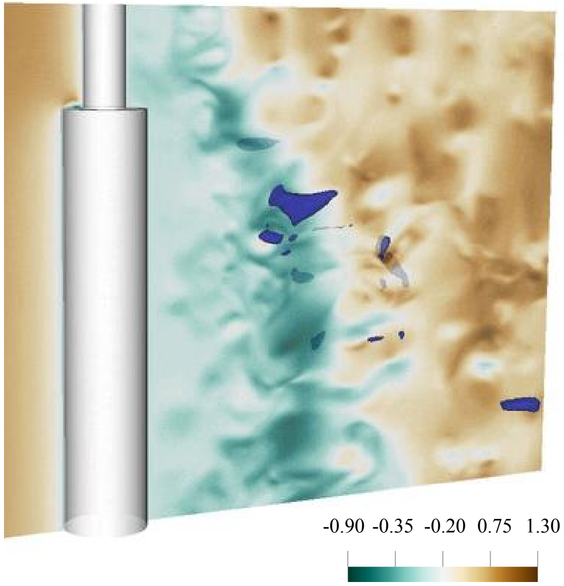
D. Massaro *et al.*, The flow around a stepped cylinder with turbulent wake and stable shear layer, JFM, 2023 **(Thesis paper 10)**
D. Massaro *et al.*, Coherent structures in the turbulent stepped cylinder flow at Re_D=5000, Int. J. Heat Fluid Flow, 2023 **(Thesis paper 11)**

Turbulence in free shear flows

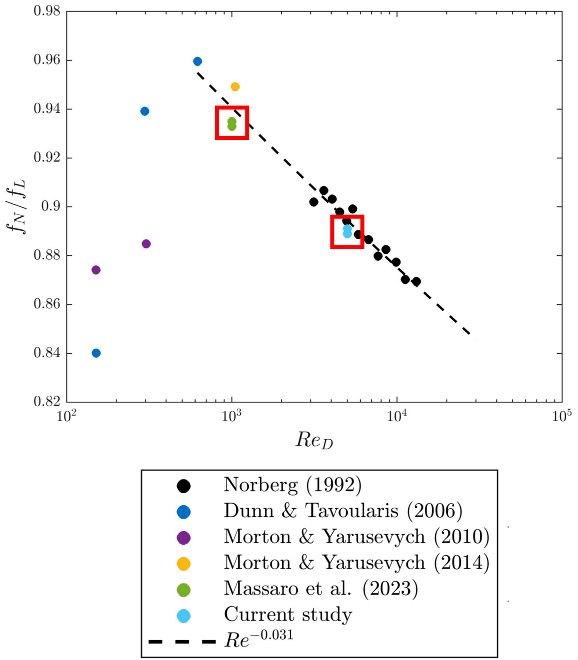
Flow around a stepped cylinder



Negative isosurface of downwash mode



Instantaneous streamwise velocity with intense downward flow motions in blue



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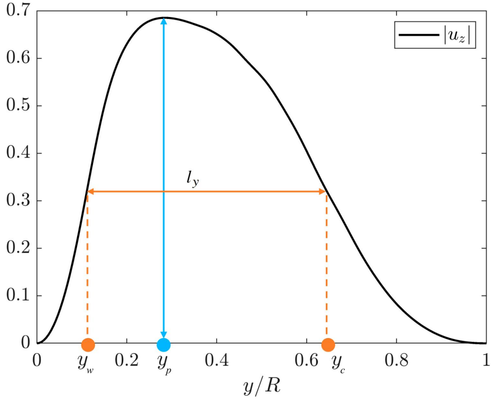
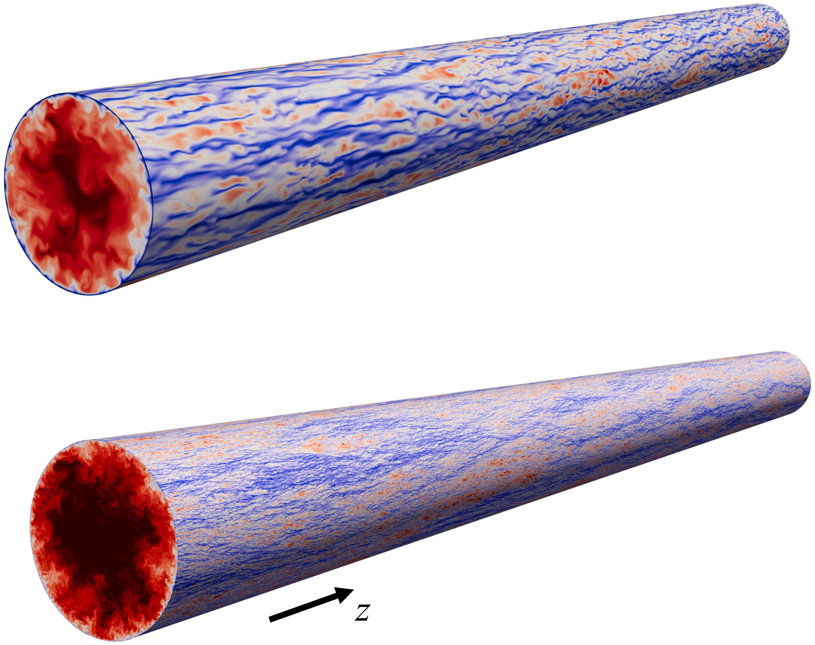
Coherence in wall-bounded shear flows

Turbulent straight pipe flow

- Pseudo-spectral code **Openpipeflow**
- Five friction Reynolds numbers: 180 to 5200
- $L_z = 10\pi R$
- Karhunen-Loève decomposition

$$p = (\kappa_\theta, \kappa_z), \quad \hat{\boldsymbol{u}}_p(r,t) = \sum_{q=1}^{\infty} \hat{a}_{(q,p)}(t) \hat{\Phi}_{(q,p)}(r)$$



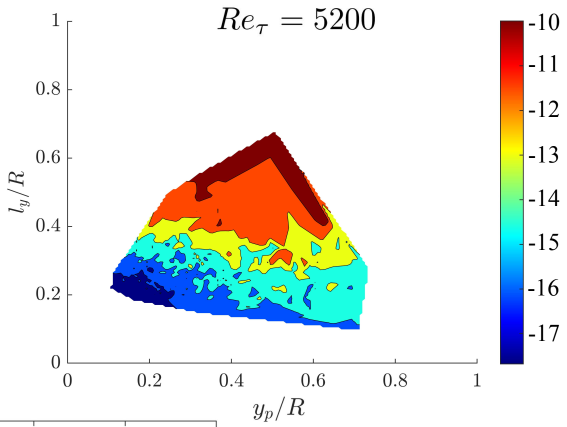


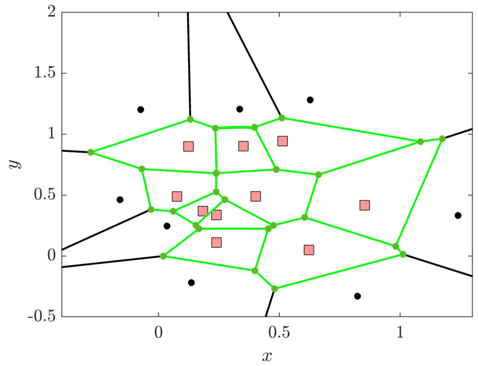
A.P. Willis, The Openpipeflow Navier-Stokes solver. SoftwareX, 2017
D. Massaro *et al.*, Energy-based characterisation of large-scale coherent structures in turbulent pipe flows, JFM (To appear), 2024 **(Thesis paper 5)**

Coherence in wall-bounded shear flows

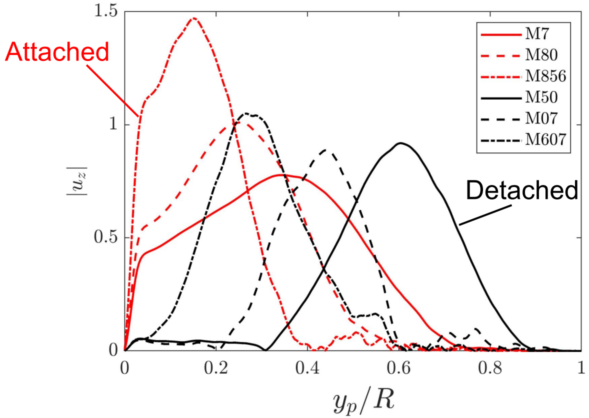
Turbulent straight pipe flow

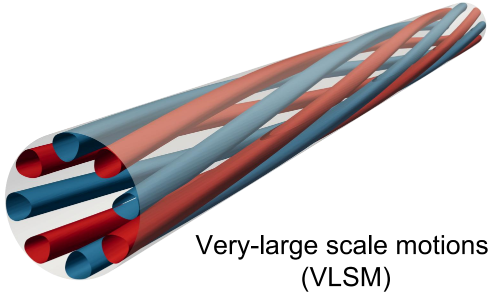
Novel energy-based classification between attached and detached eddies



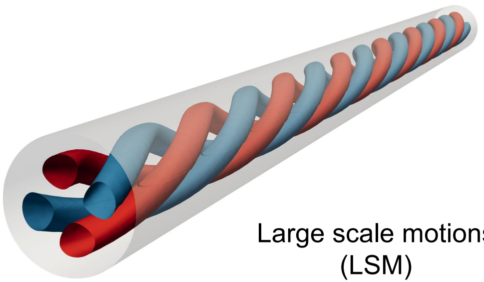


Voronoi diagram





Very-large scale motions (VLSM)



Large scale motions (LSM)

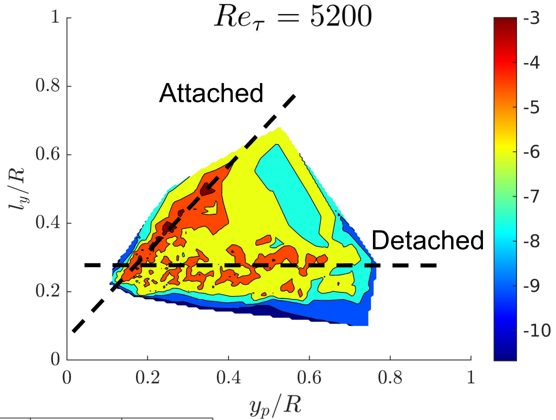
D. Massaro *et al.*, Karhunen-Loève decomposition of high Reynolds number turbulent pipe flows: a Voronoi analysis, JOP, 2024 **(Thesis paper 4)**
D. Massaro *et al.*, Energy-based characterisation of large-scale coherent structures in turbulent pipe flows, JFM (To appear), 2024 **(Thesis paper 5)**

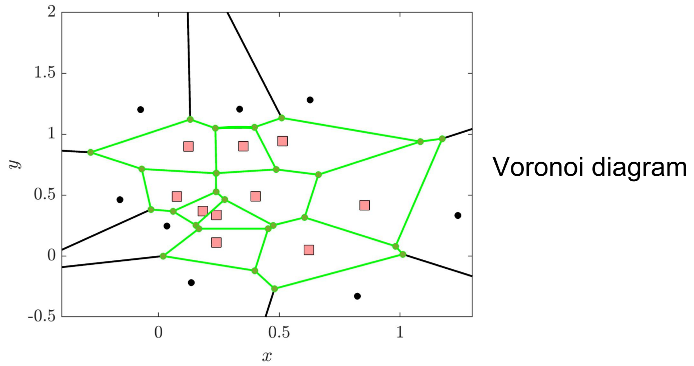
Coherence in wall-bounded shear flows

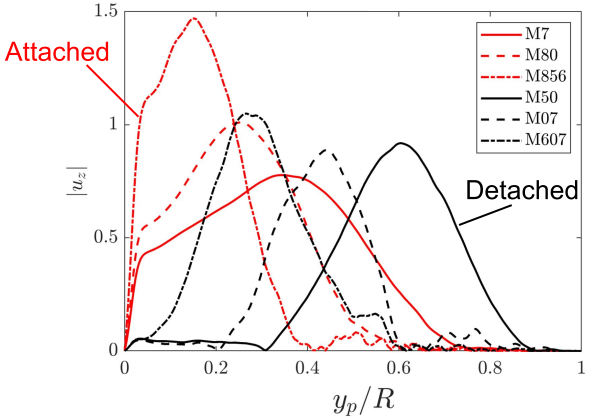
Turbulent straight pipe flow

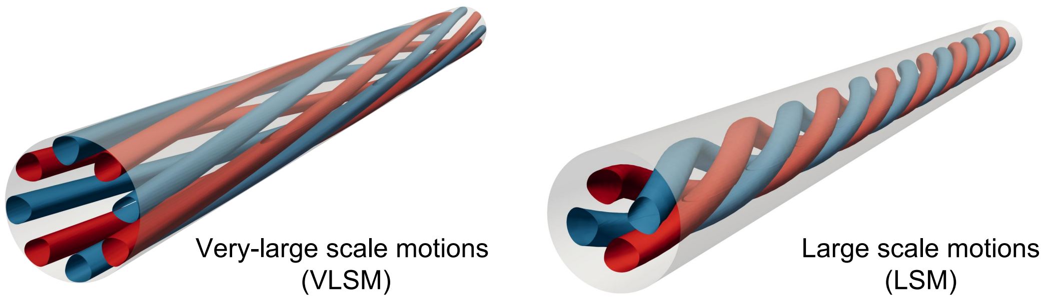
Novel energy-based classification between attached and detached eddies

$log(\lambda_{(q=1,p)}/k \cdot \rho)$









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Conclusions & Outlook

Method development

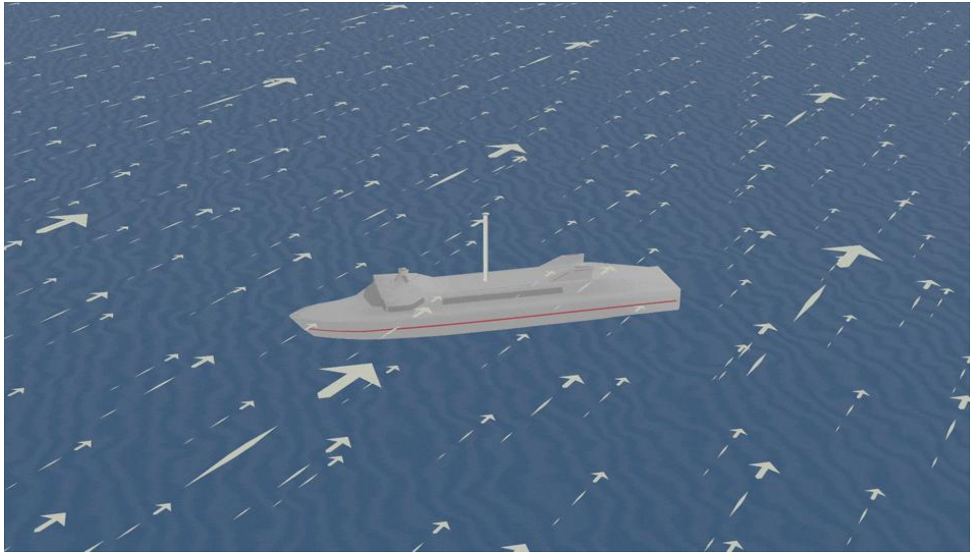
- Space-adaptive numerical framework implementation:
  - Spectral error indicator for a more homogeneous refinement
  - Adjoint error estimator for a goal-oriented refinement
- Information-theoretic causality metric for error indicator and causal structures
- Error-driven mesh design for global stability analysis

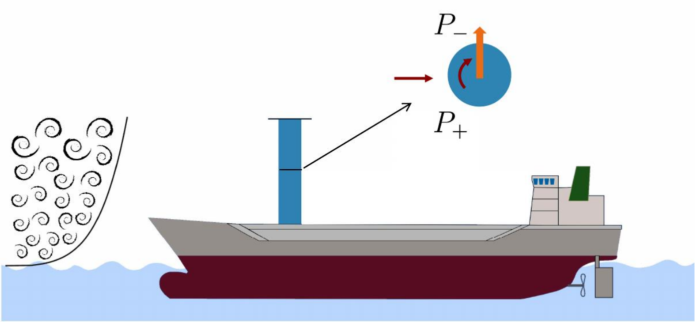
Transitional and turbulent flow physics

- Large-scale coherent structures in turbulent pipe flows: a Voronoi analysis
- Spatially developing bent pipe flows: swirl switching investigation and transition mechanism
- Flow around a stepped cylinder:
  - Threefold cell formation: global instability and turbulent wake
  - Connection between the downwash mechanism and the modulation cell
- Flettner rotor: large-scale motions and local vortex shedding suppression

Conclusions & Outlook

- Flettner rotor: large-scale motions and local vortex shedding suppression
- The inverse Magnus effect, particularly in the flow around a Flettner rotor





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