Development of Machine Learning Based Wall Shear Stress Models for LES in the Presence of Adverse Pressure Gradients and Separation

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Doc. ref.: 2019030-THESELDP-SAFRAN

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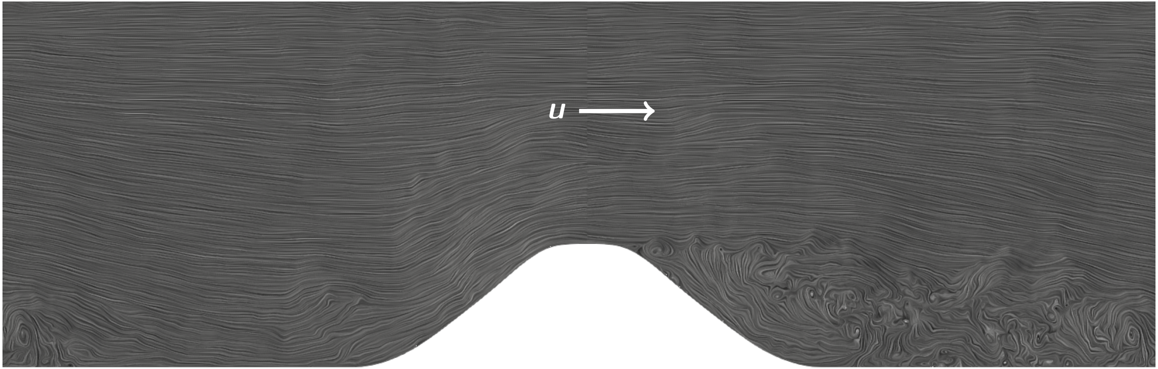
Separation phenomenon - Sketch



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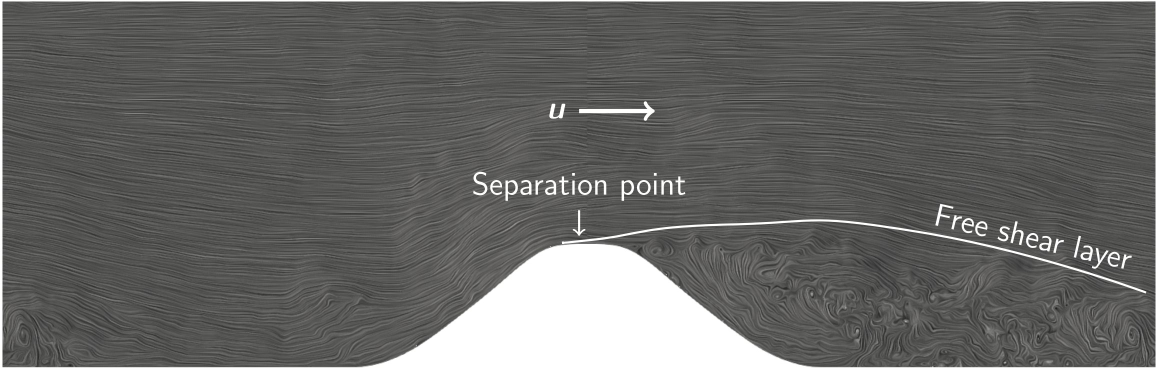

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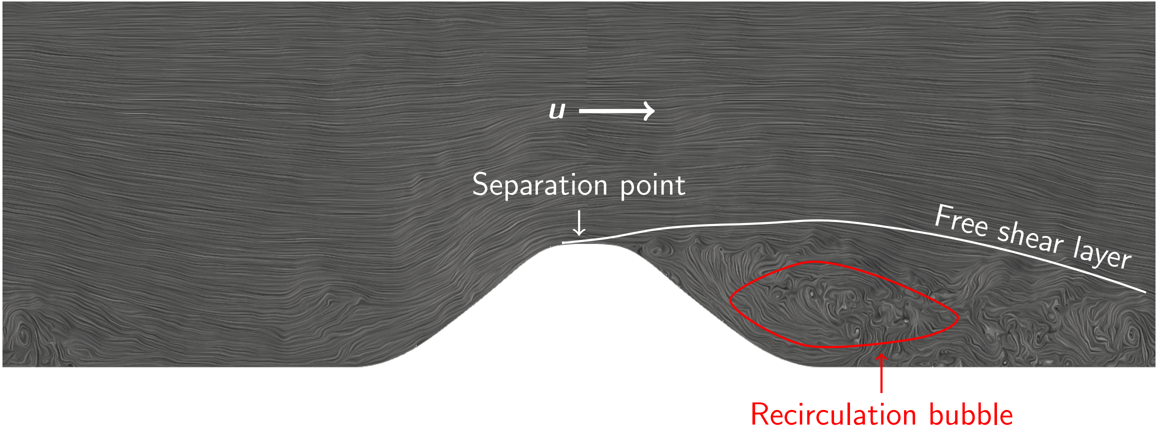

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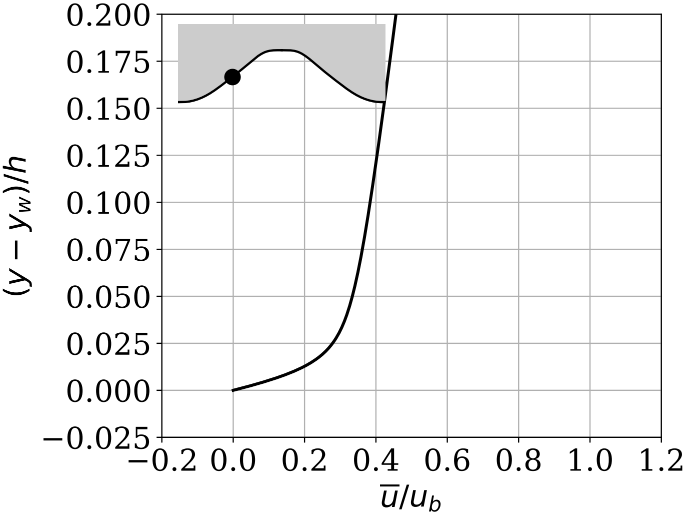
Separation phenomenon - Profiles



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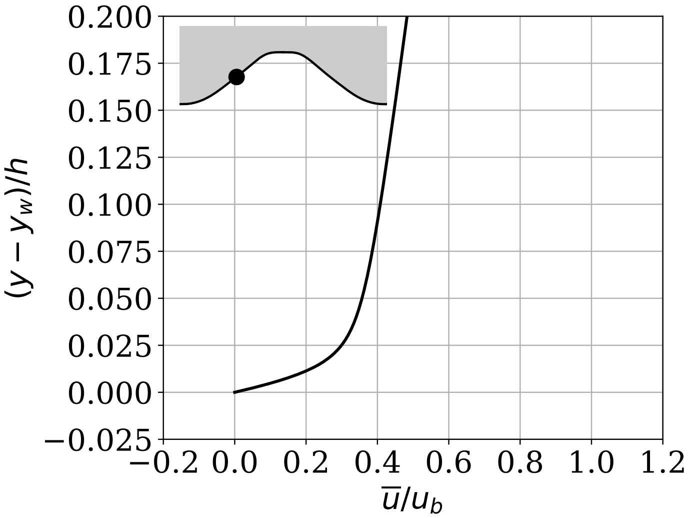
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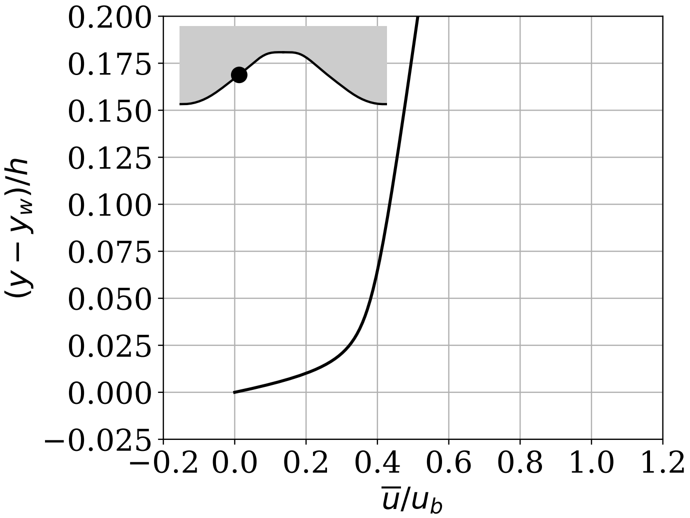

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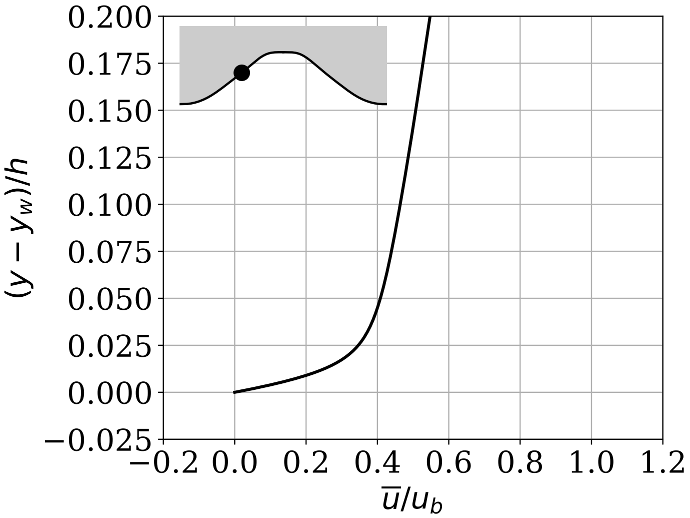

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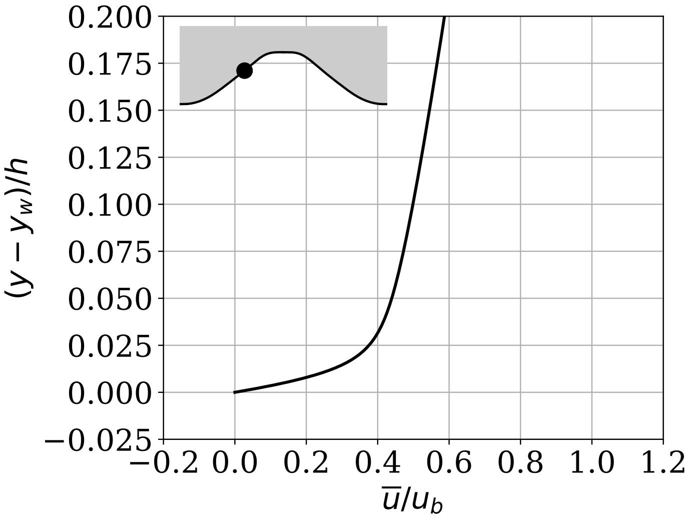

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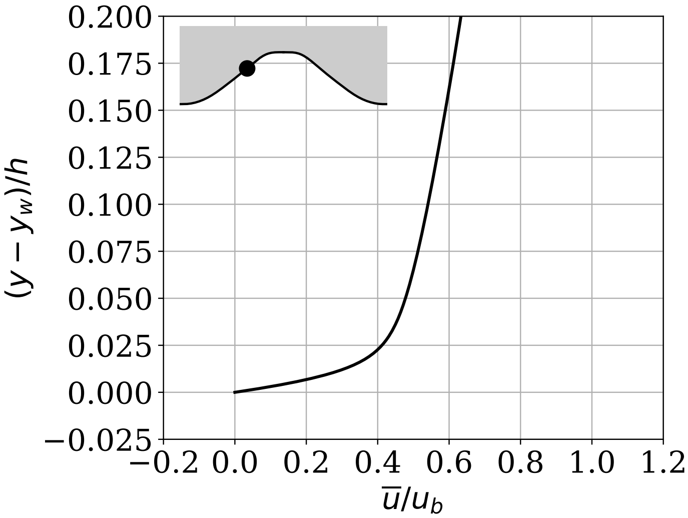

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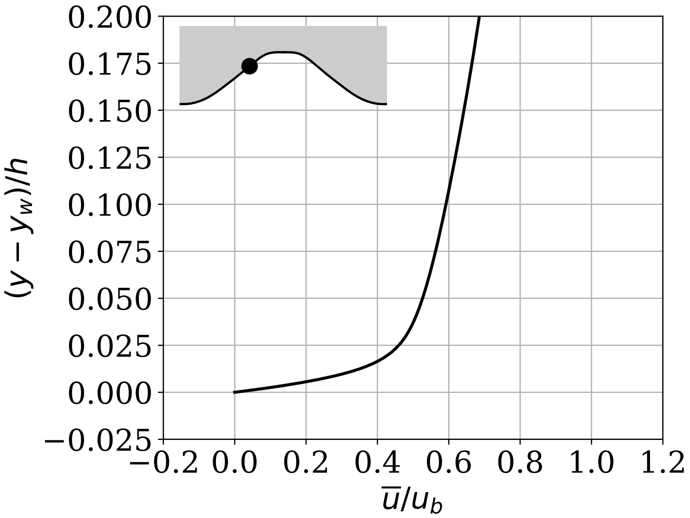

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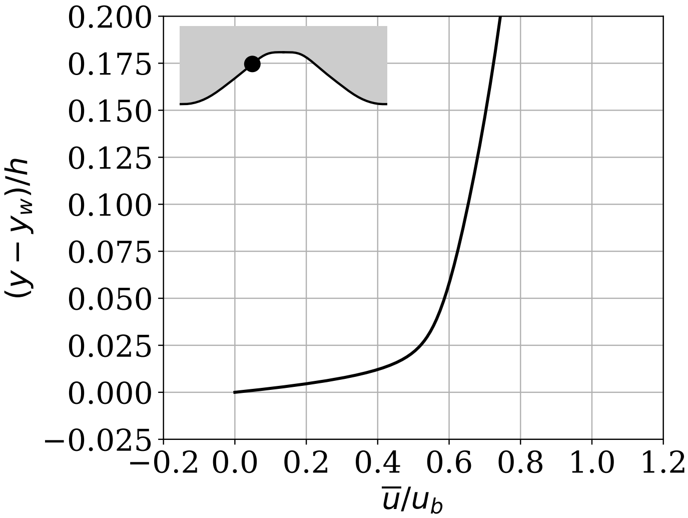

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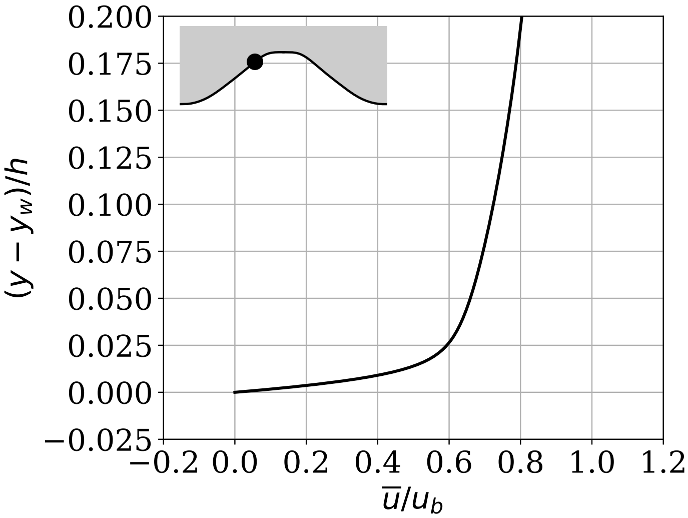
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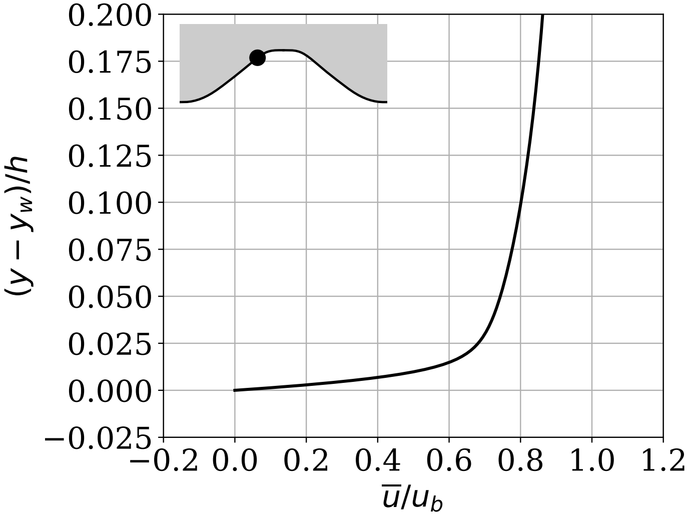

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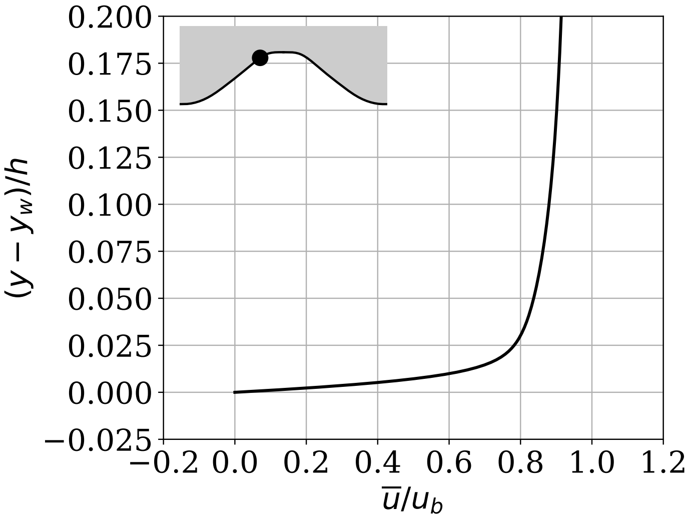
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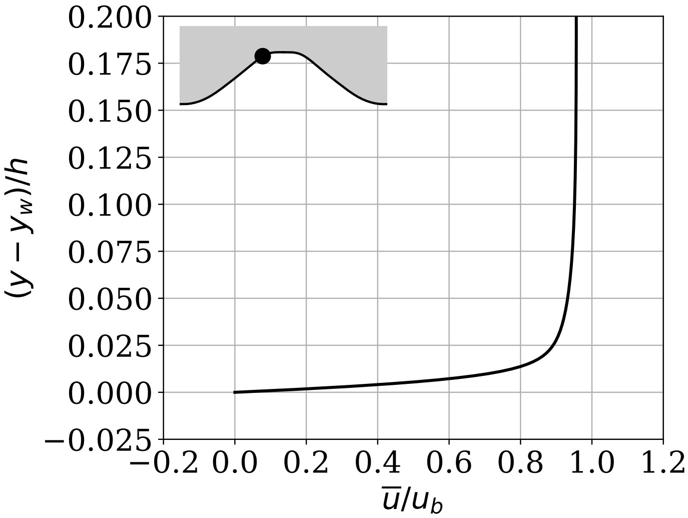

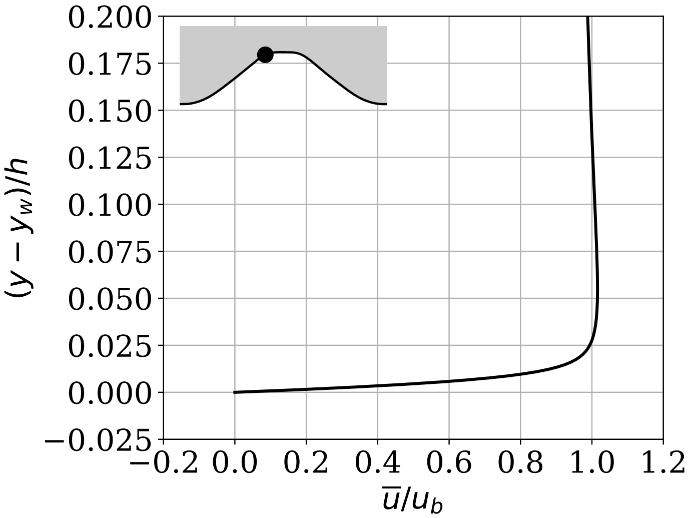

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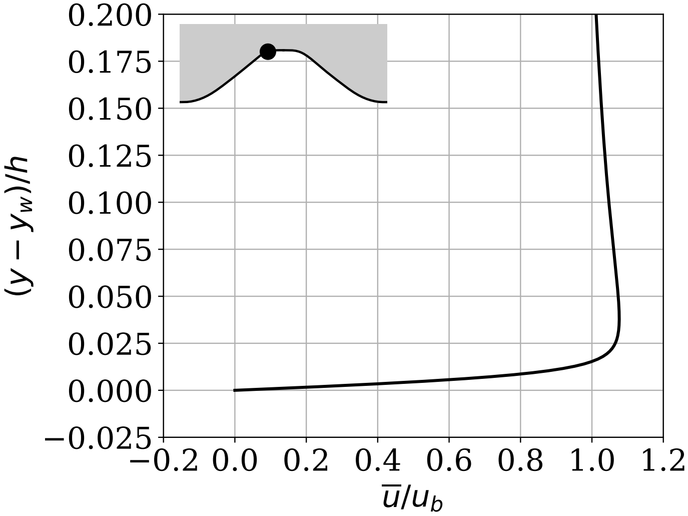

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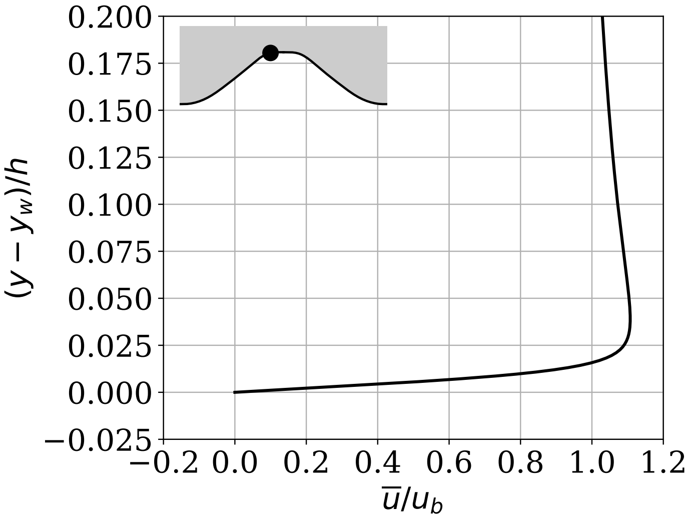

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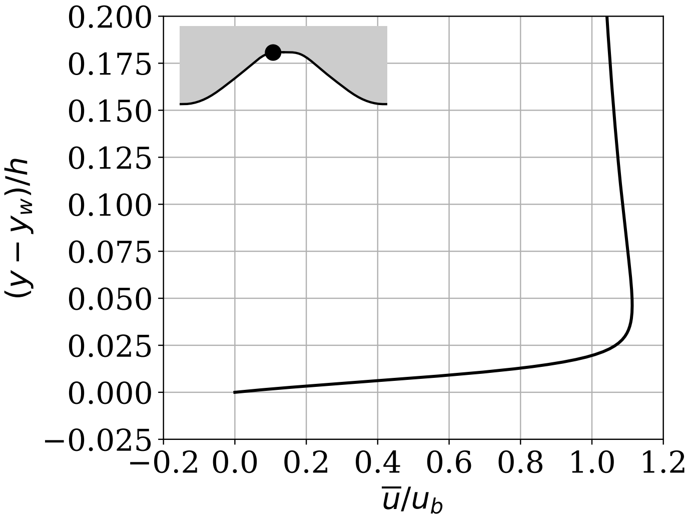

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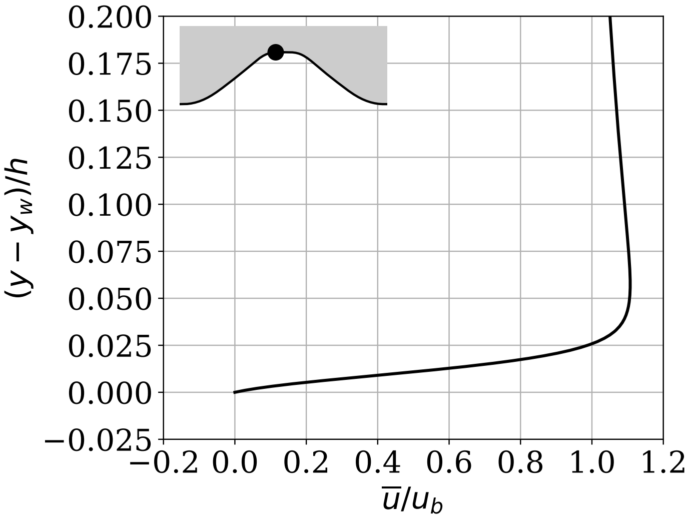
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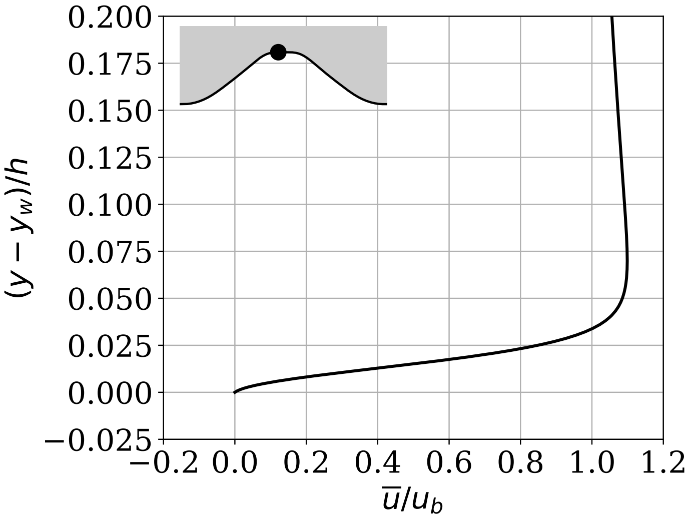
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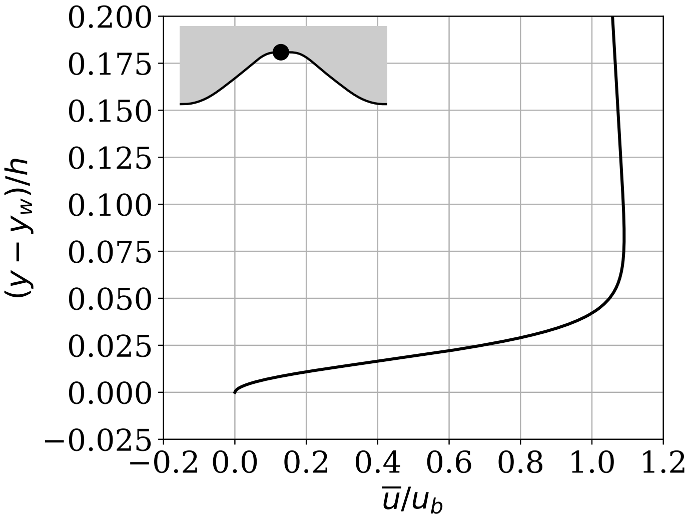
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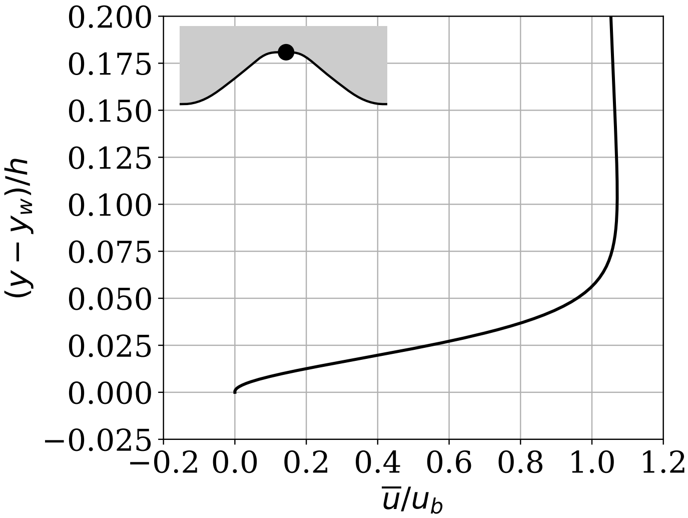
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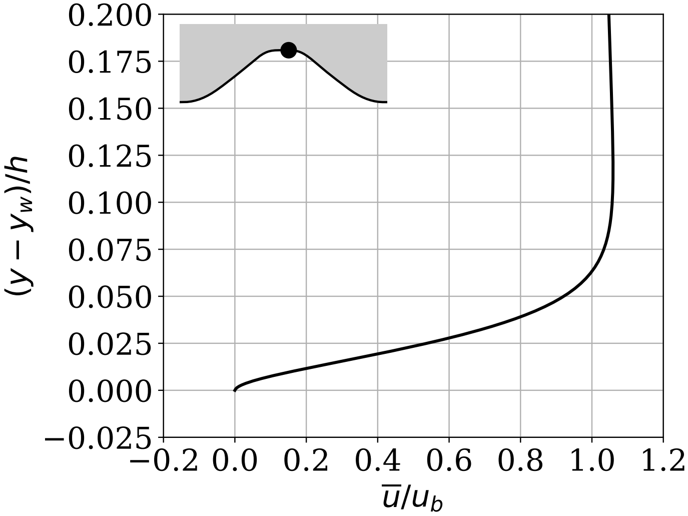
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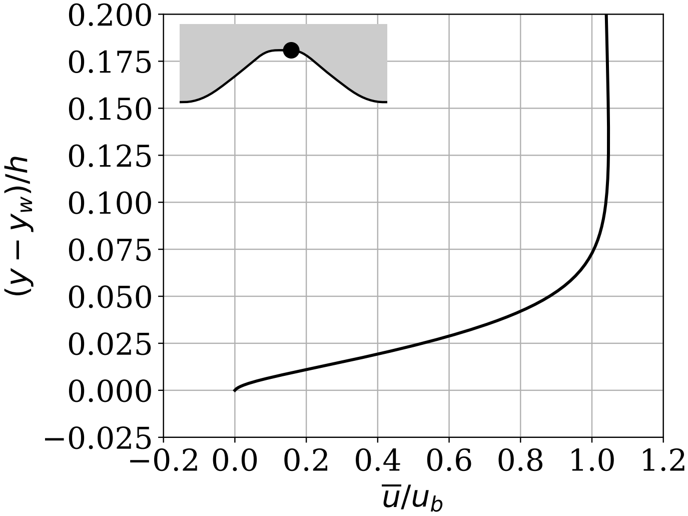
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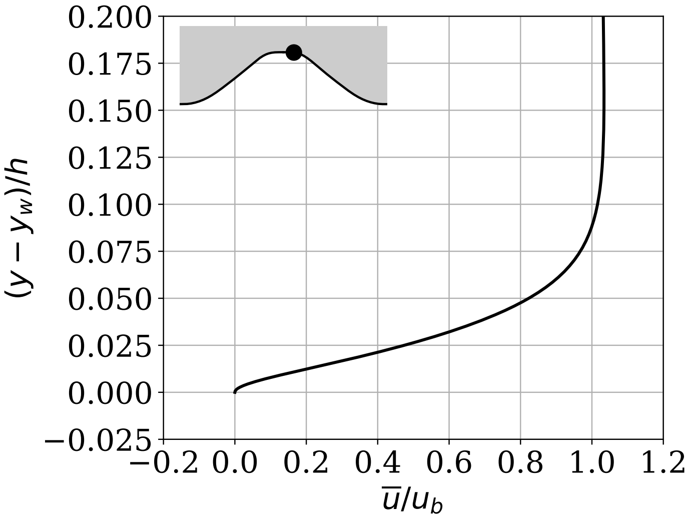

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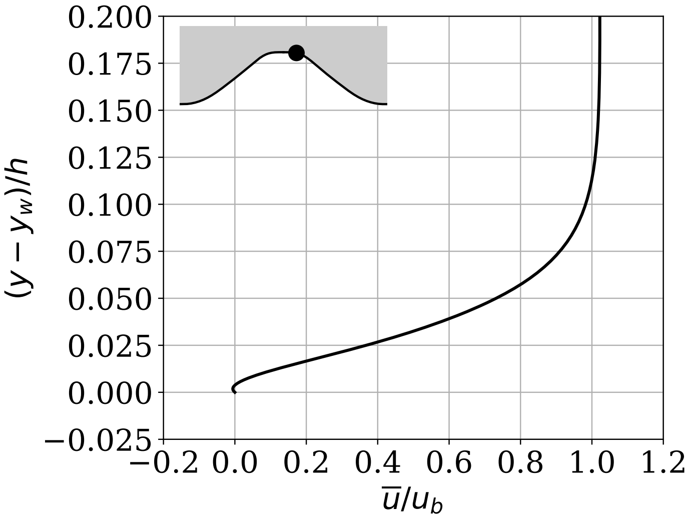

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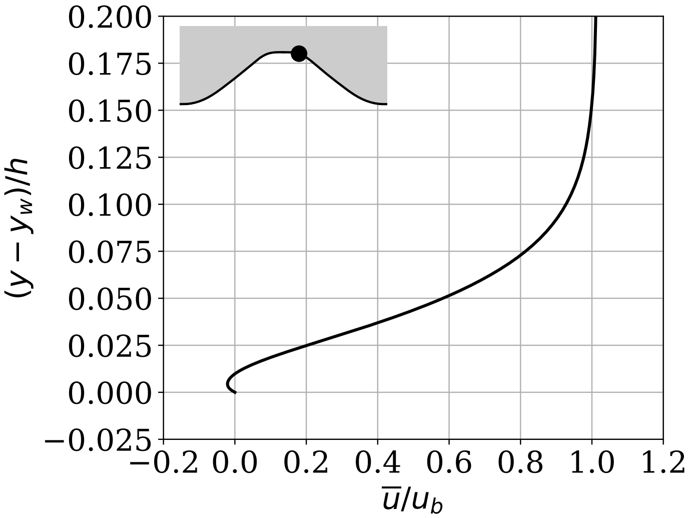

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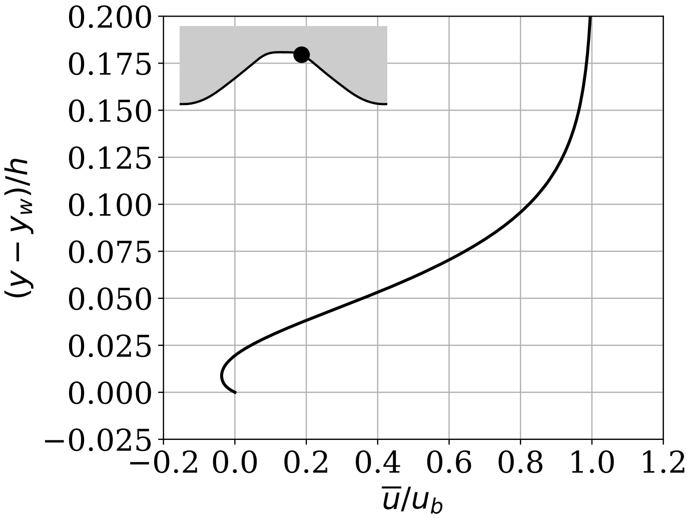

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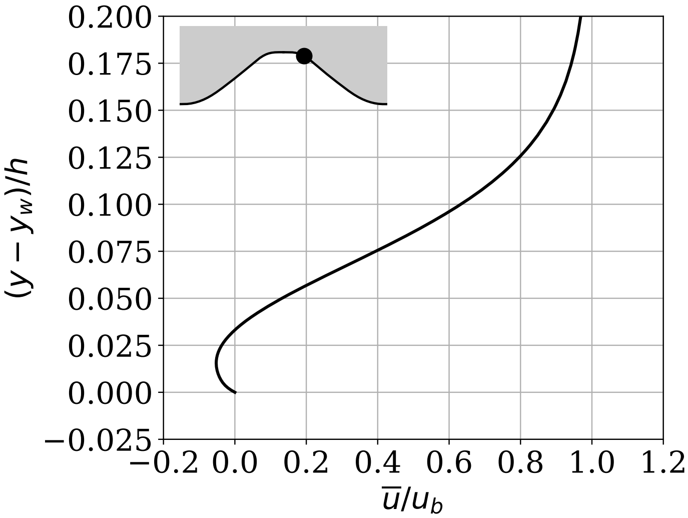

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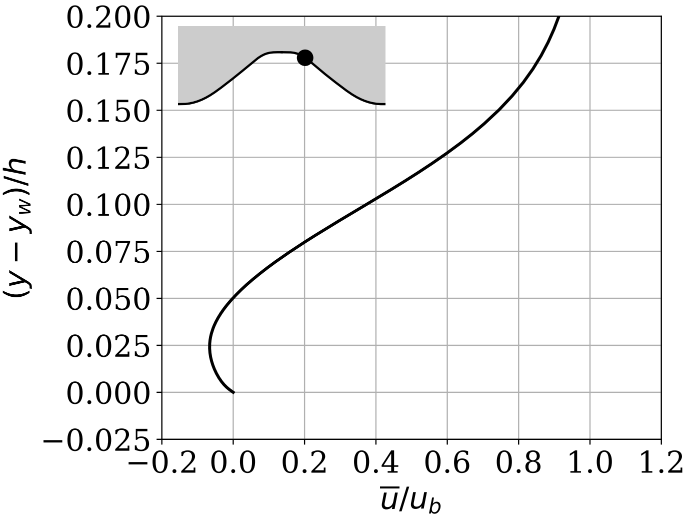
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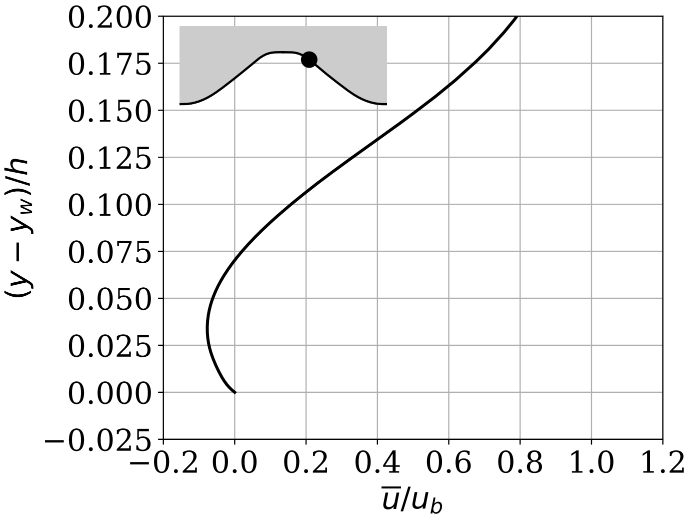

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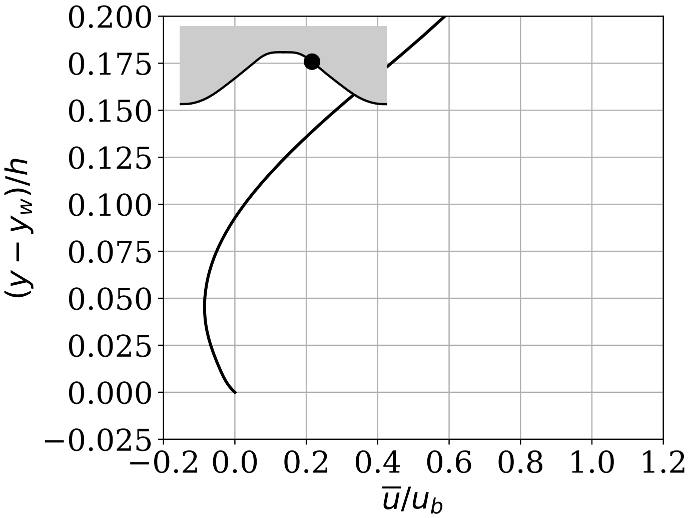

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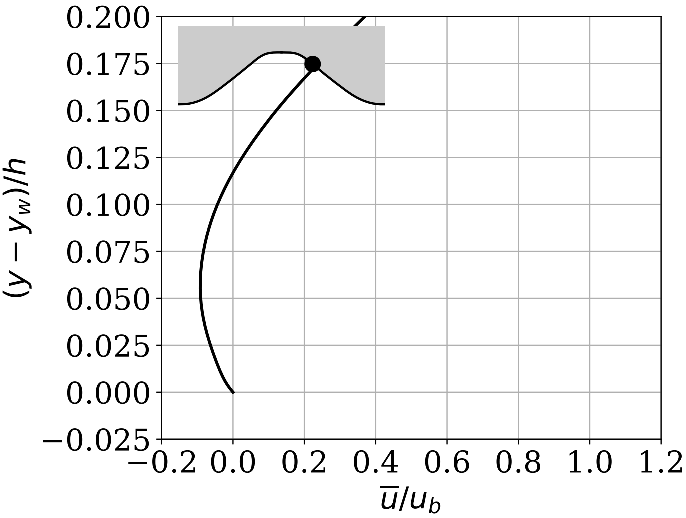

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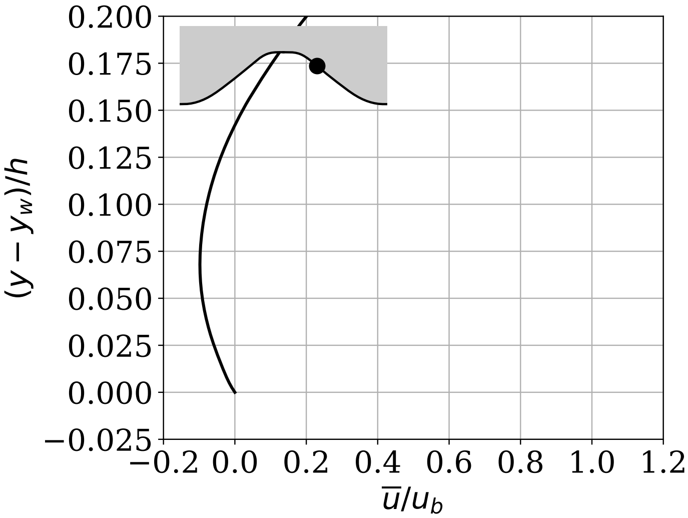

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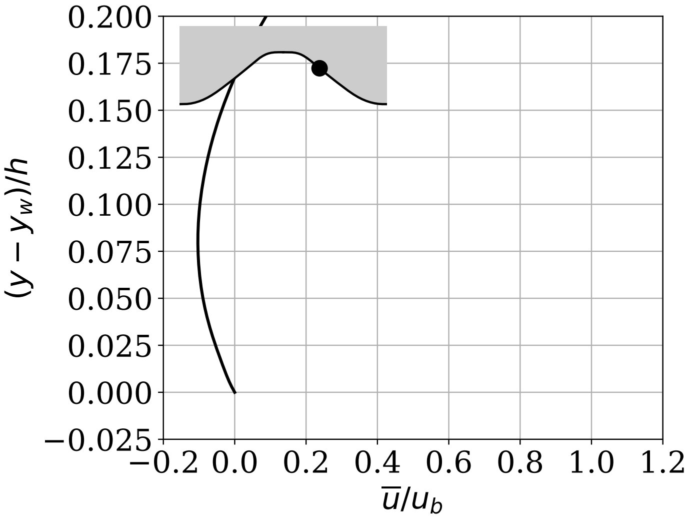

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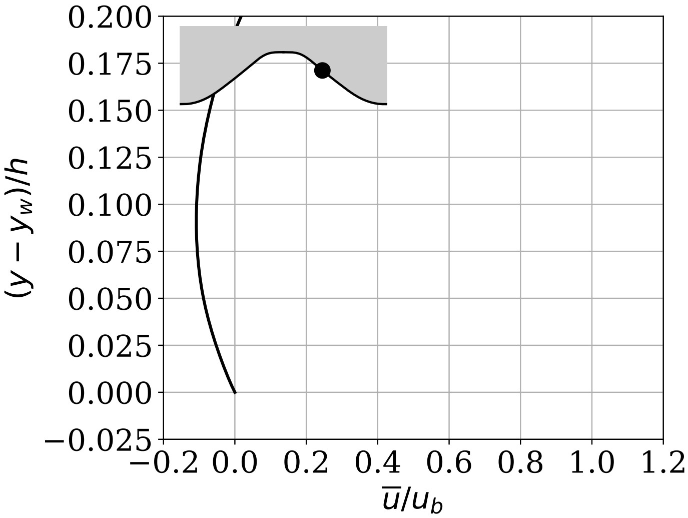

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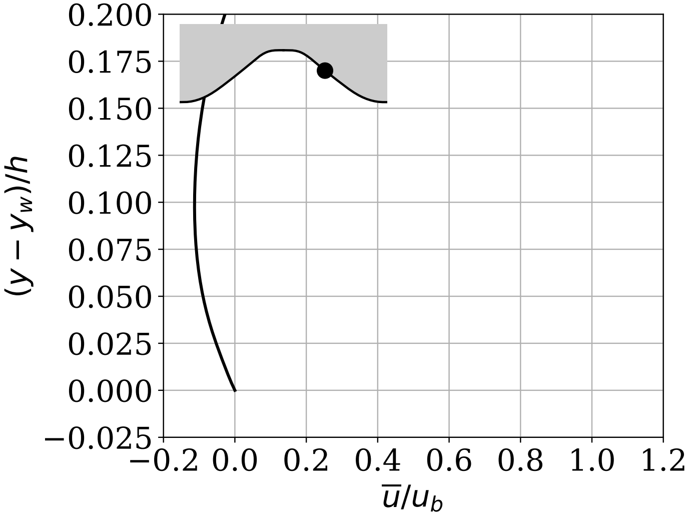
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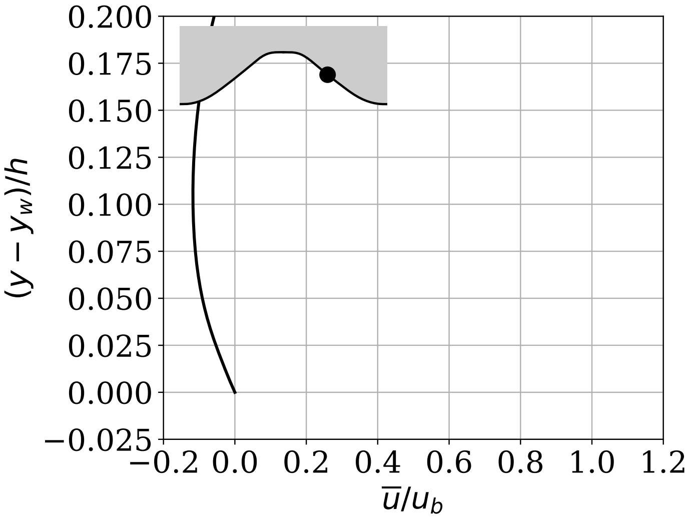
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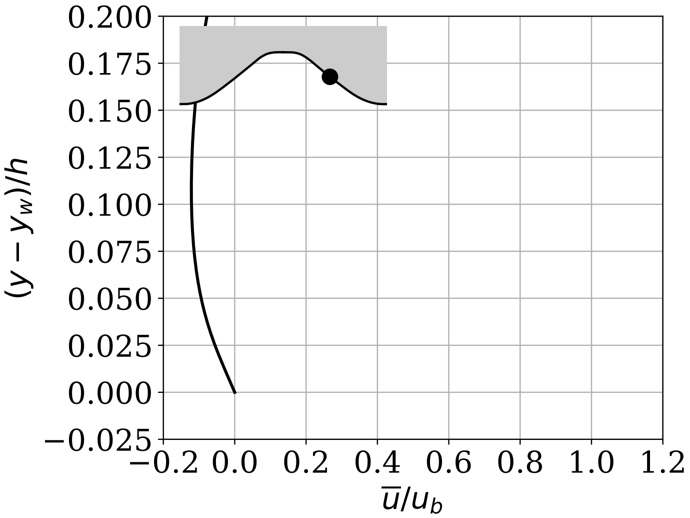

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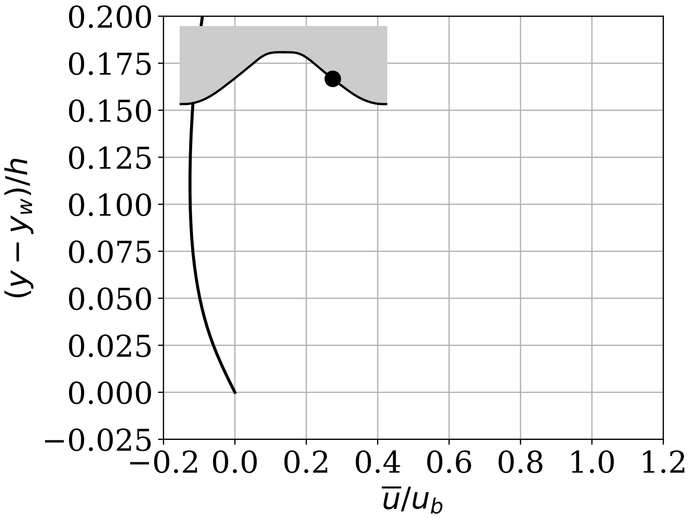
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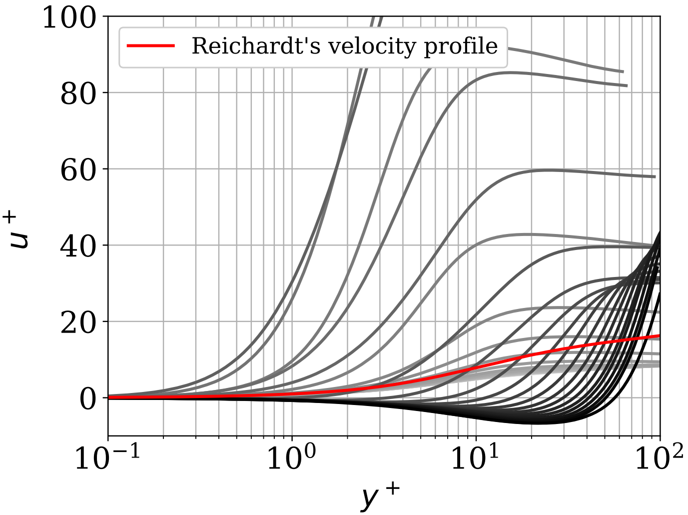
Separation phenomenon - Non-dimensional mean velocity profiles



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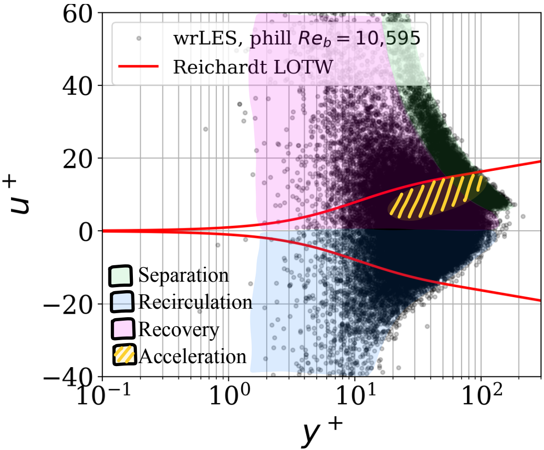
Analytic WSS model based on Reichardt's profile (a priori)



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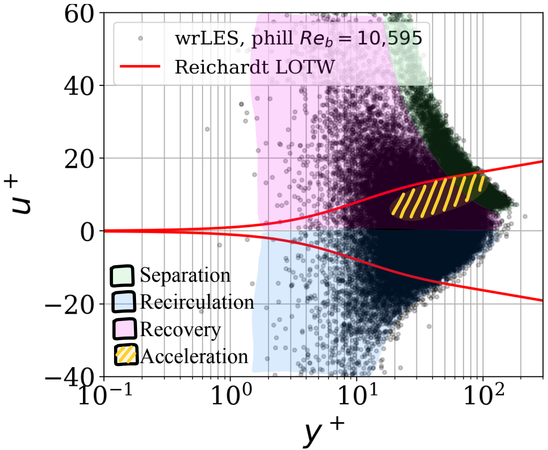

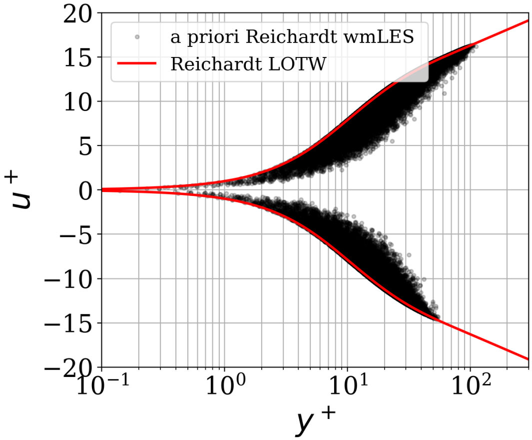

Observation: The analytical WSS model is too restrictive for the flow physics developing in the periodic hill. Nonetheless, the reverse flow is detected because the bubble is sufficiently thick.

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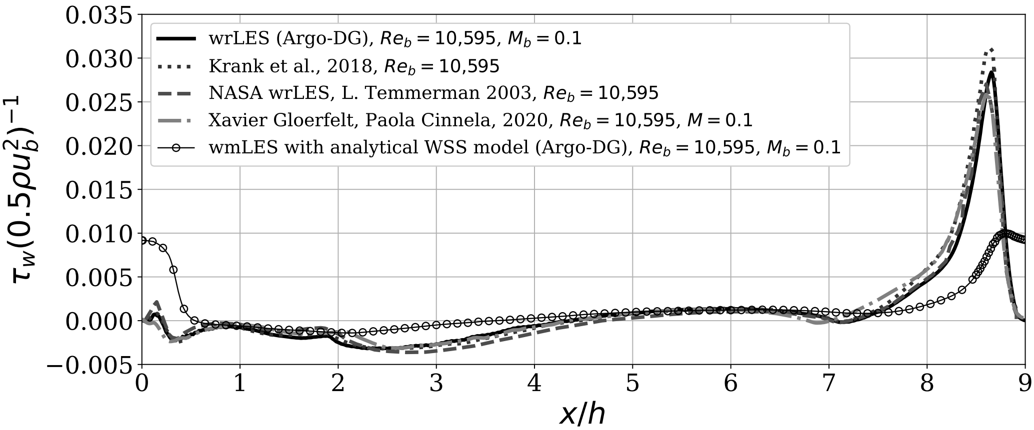
Analytic WSS model based on Reichardt's profile (a posteriori)



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Observation: Misprediction of (1) separation and (2) reattachment location, and (3) underestimation of friction peak. There is room for improvement.

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Data-driven WSS model - Motivations

Recent advances in **hardware** (mostly GPUs and now TPUs)

Exponential generation and accumulation of high-quality **data**

Now possible to train **deeper and deeper** neural networks

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Among all the possible solutions to **improve WSS model**, we selected neural networks, the core element of Deep Learning.

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Among all the possible solutions to **improve WSS model**, we selected neural networks, the core element of Deep Learning.

Better prediction of the **instantaneous** behaviors of the wall shear stress

Moreover, to cope with the lack of **variance**, the network is trained to predict a **distribution** rather than a point estimate.

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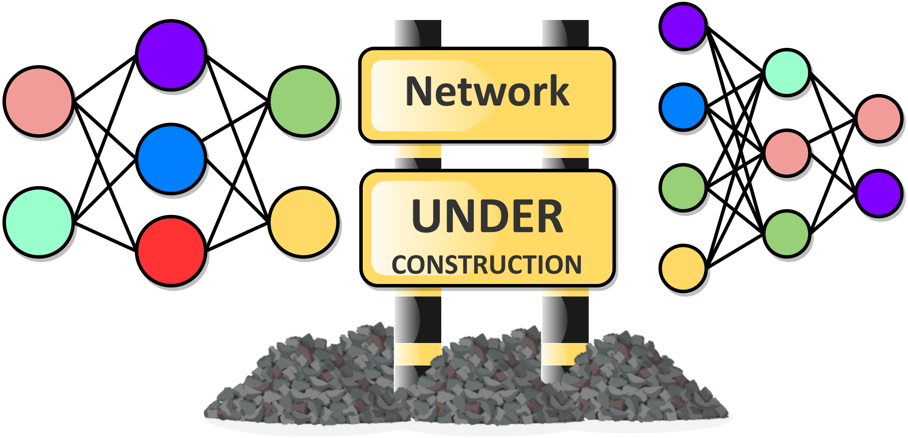
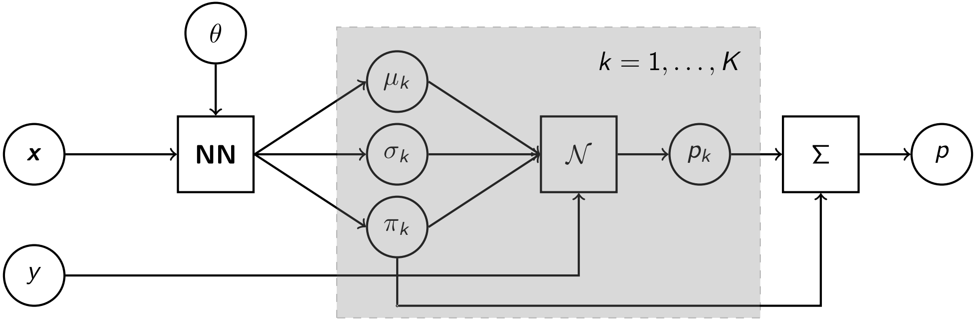

Network
UNDER
CONSTRUCTION

Gaussian Mixture Neural Networks (GMN) aim to predict the probability distribution $p(\tau_w|\mathbf{x})$ of the wall shear stress component as a linear combination of Gaussian distribution :

$$p(\tau_w|\mathbf{x}) = \sum_{k=1}^{K} \pi_k p_k = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mu_k, \sigma_k)$$



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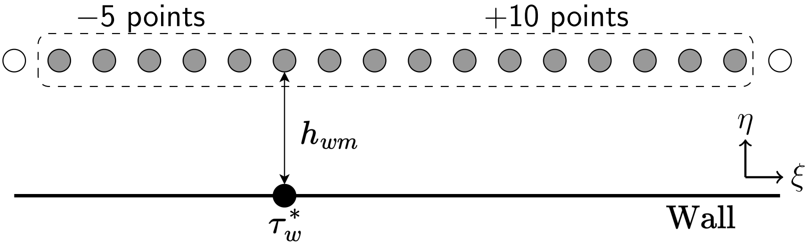
The input **stencil size**[1] is represented as follows,



Remark: Proper training requires a high correlation between input and output. Causality has nothing to do with it. Due to the large input size, **NN** is replaced by a Convolutional Neural Network (combined with residual blocks).

[1] based upon analysis of space-time correlations [1].

Inputs			Outputs
Velocity	Pressure gradients	Curvature	Wall shear stress
$\boldsymbol{u}^* = \dfrac{\boldsymbol{u}\, h_{wm}}{\nu}$	$\boldsymbol{u}_p^* = \dfrac{\boldsymbol{u}_p\, h_{wm}}{\nu}$	$\mathcal{K}^* = \mathcal{K} h_{wm}$	$\boldsymbol{\tau}_w^* = \mathrm{sign}\left(\boldsymbol{\tau}_w\right) \dfrac{y}{\nu}\sqrt{\dfrac{\lvert\boldsymbol{\tau}_w\rvert}{\rho}}$

where $\boldsymbol{u}_p = \mathrm{sign}\left(\nabla p\right)\left(\frac{\nu}{\rho}\lvert\nabla p\rvert\right)^{1/3}$ is a velocity based on the pressure gradient.

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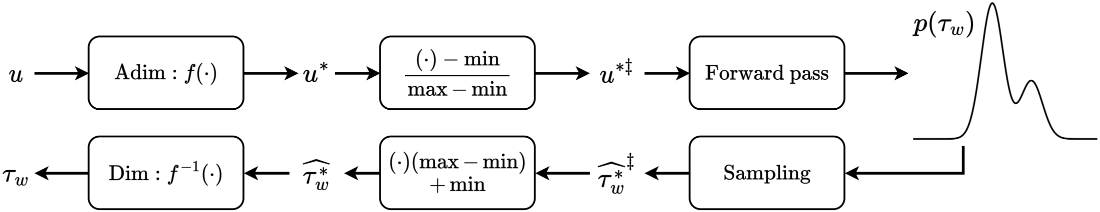
Data-driven WSS model - Complete procedure

The wall shear stress τ_w is **sampled** from the predicted distribution,



The predicted τ_w is implemented as a **boundary condition** in Argo-DG [2].

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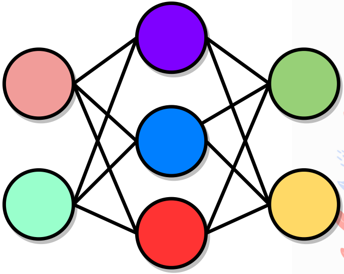


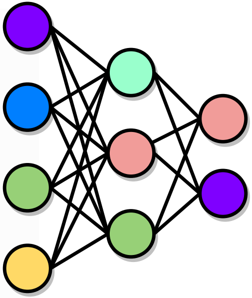
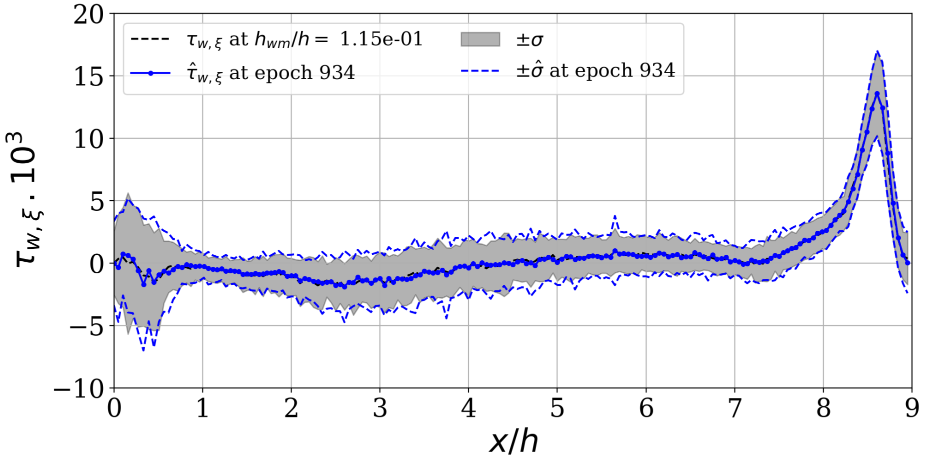


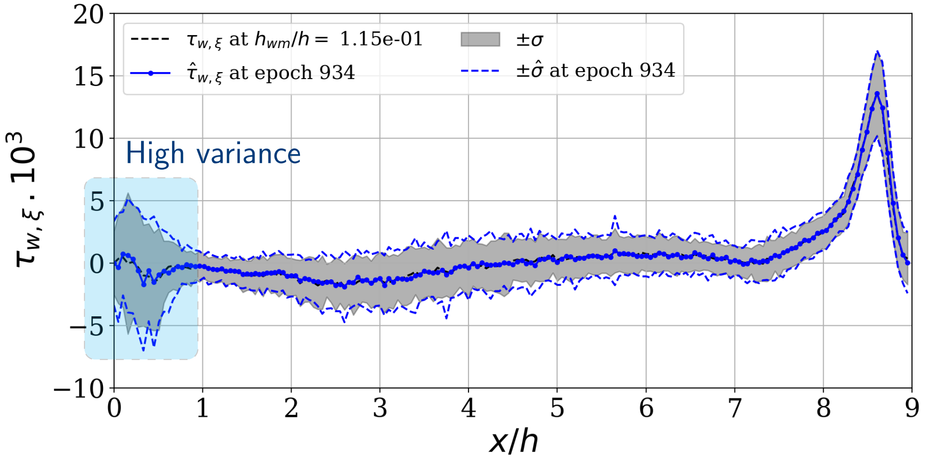
Image by pikisuperstar on Freepik

A priori prediction on the **lower wall** of the two-dimensional periodic hill,



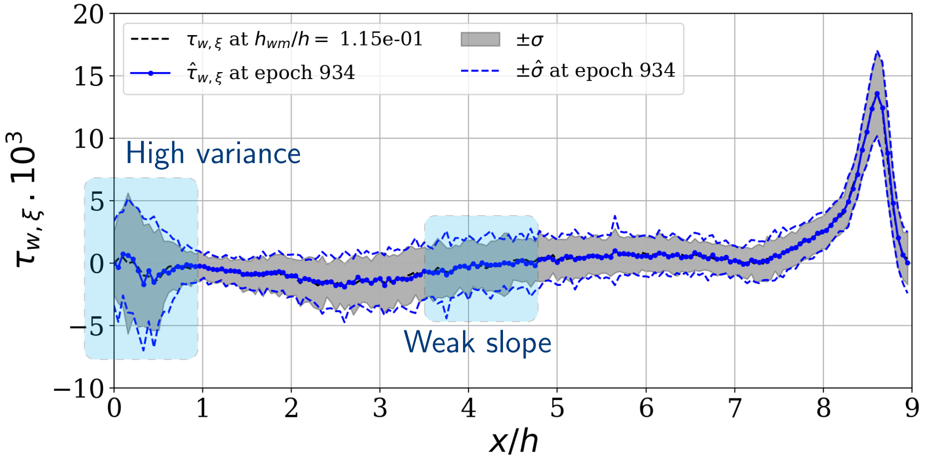
Data-driven WSS model - A priori testing

A priori prediction on the **lower wall** of the two-dimensional periodic hill,

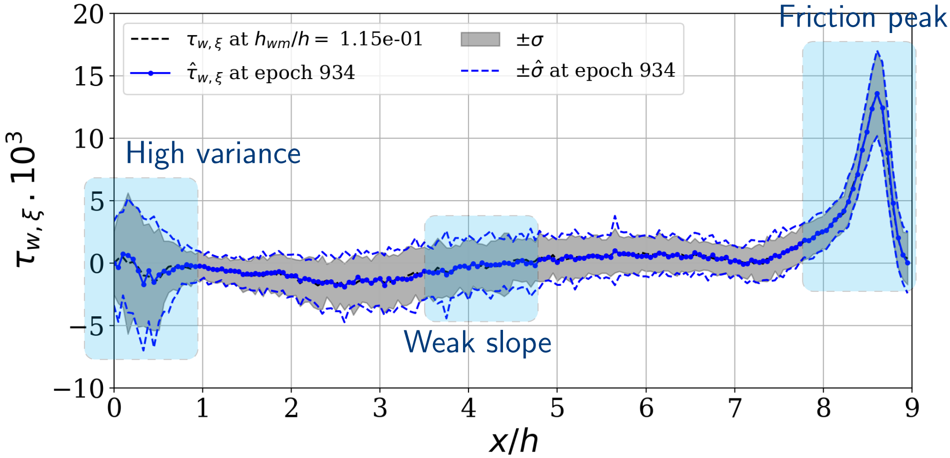


Data-driven WSS model - A priori testing

A priori prediction on the **lower wall** of the two-dimensional periodic hill,

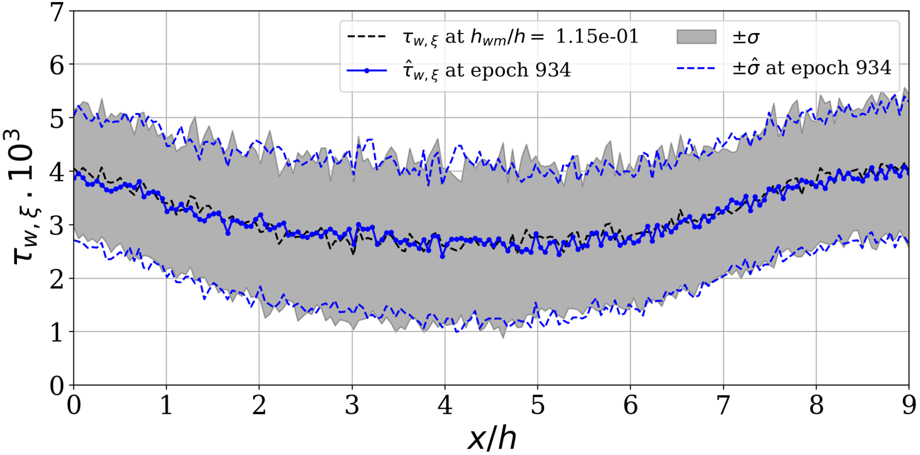


A priori prediction on the **lower wall** of the two-dimensional periodic hill,



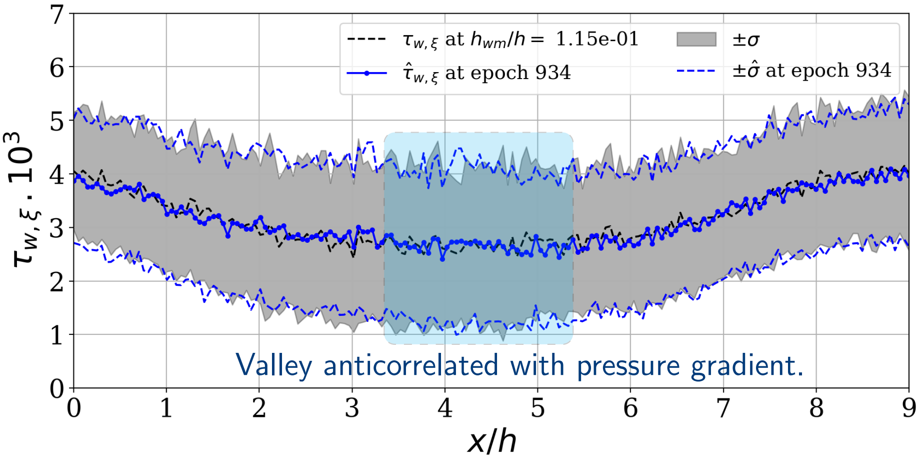
Data-driven WSS model - A priori testing

A priori prediction on the **upper wall** of the two-dimensional periodic hill,



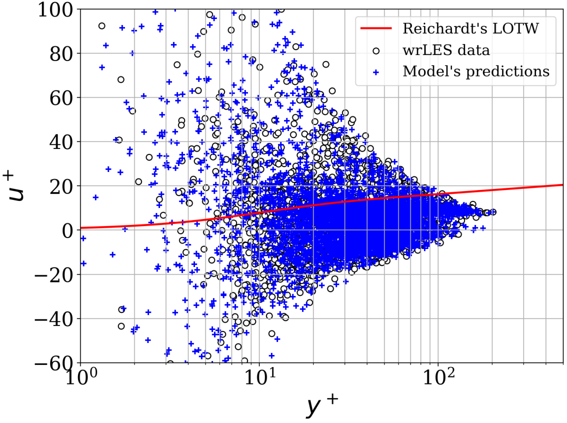
Data-driven WSS model - A priori testing

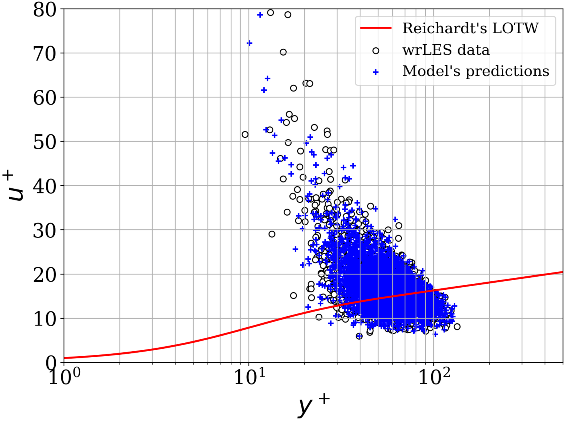
A priori prediction on the **upper wall** of the two-dimensional periodic hill,



Data-driven WSS model - A priori testing

A priori prediction observed in a (y^+, u^+) graph,





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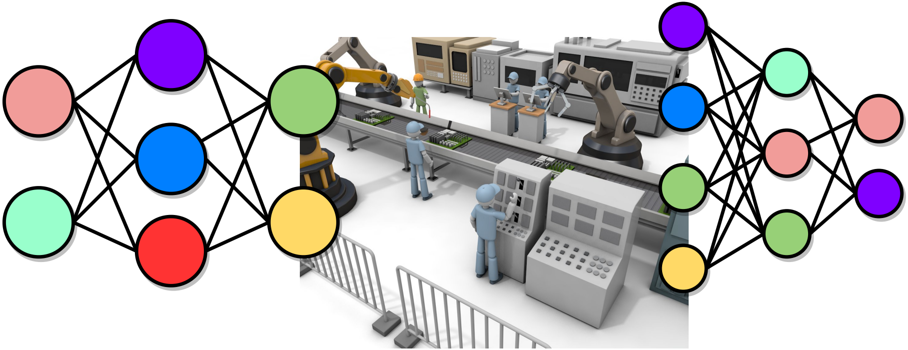
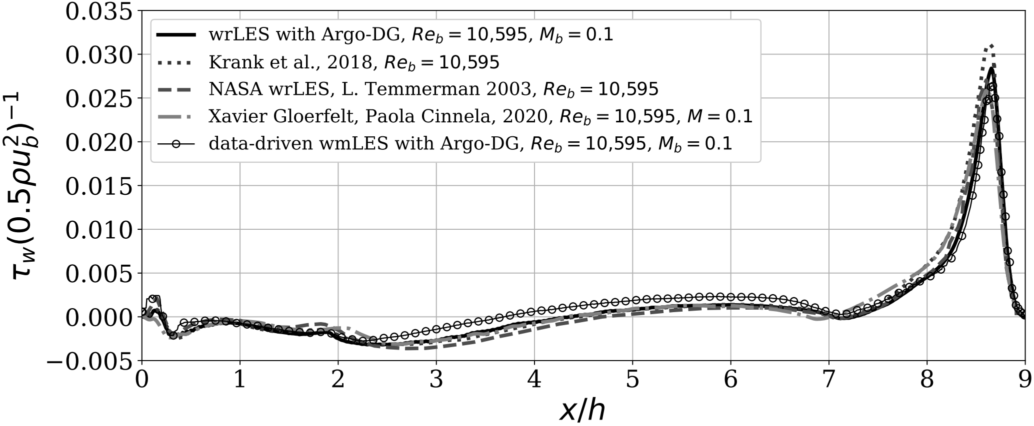
Observation: This graph illustrates the capability of the network to correctly predict the variance.



Free illustrations extracted from PictArts

Data-driven WSS model - A posteriori testing

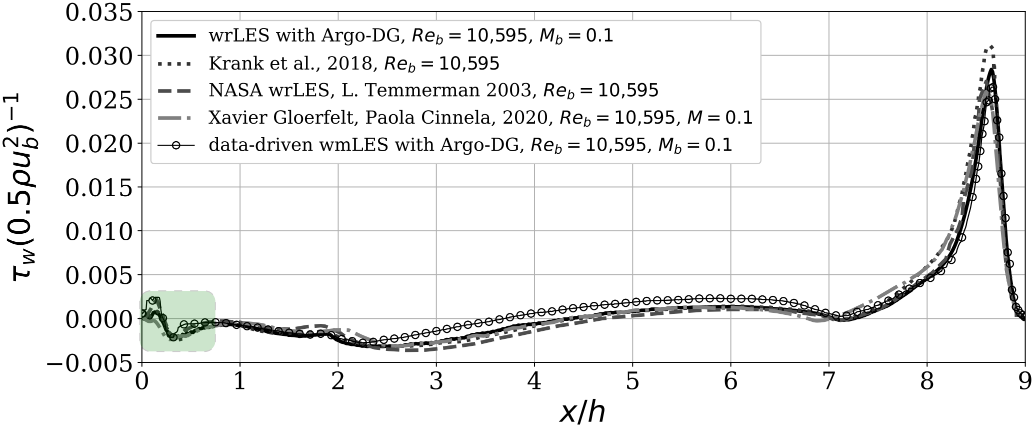
Friction coefficient on the lower wall obtained after the accumulation of statistics over about $35t_c$,



Remark: Reattachment at 3.7 instead of 4.21 experimentally, thus a relative error of 12%.

Data-driven WSS model - A posteriori testing

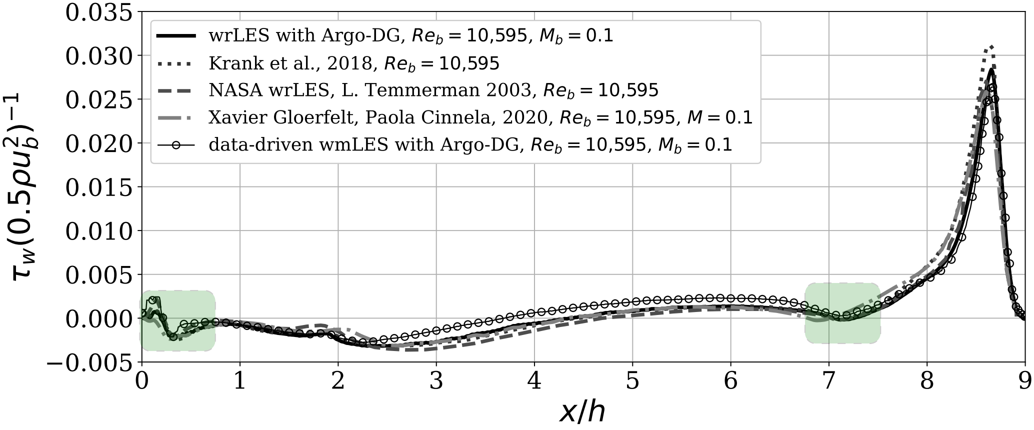
Friction coefficient on the lower wall obtained after the accumulation of statistics over about $35t_c$,



Remark: Reattachment at 3.7 instead of 4.21 experimentally, thus a relative error of 12%.

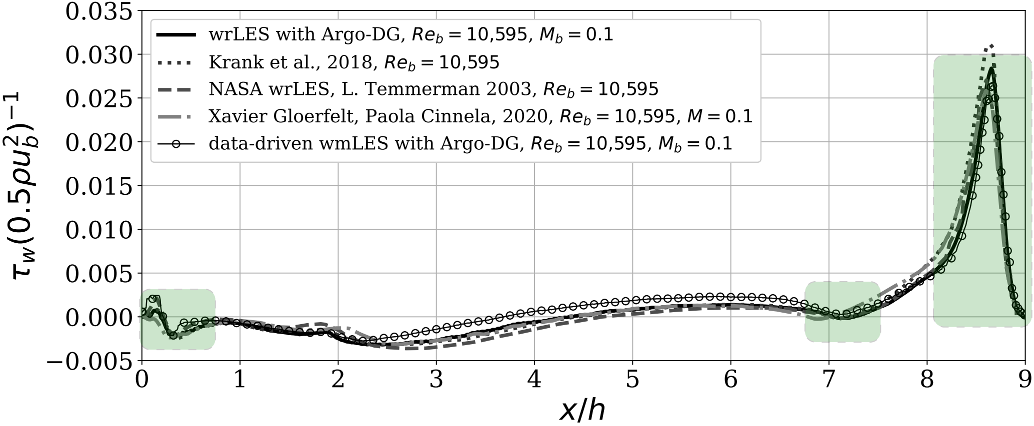
Data-driven WSS model - A posteriori testing

Friction coefficient on the lower wall obtained after the accumulation of statistics over about $35t_c$,



Remark: Reattachment at 3.7 instead of 4.21 experimentally, thus a relative error of 12%.

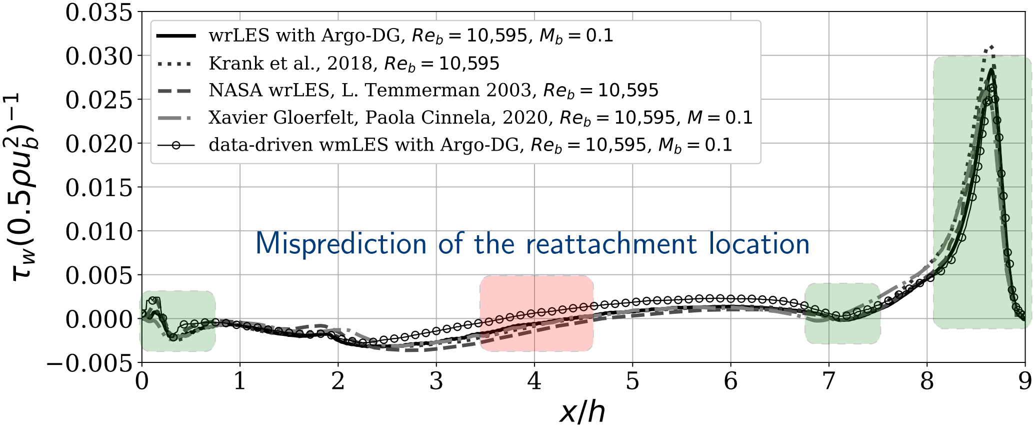
Friction coefficient on the lower wall obtained after the accumulation of statistics over about $35t_c$,



Remark: Reattachment at 3.7 instead of 4.21 experimentally, thus a relative error of 12%.

Data-driven WSS model - A posteriori testing

Friction coefficient on the lower wall obtained after the accumulation of statistics over about $35t_c$,



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Remark: Reattachment at 3.7 instead of 4.21 experimentally, thus a relative error of 12%.

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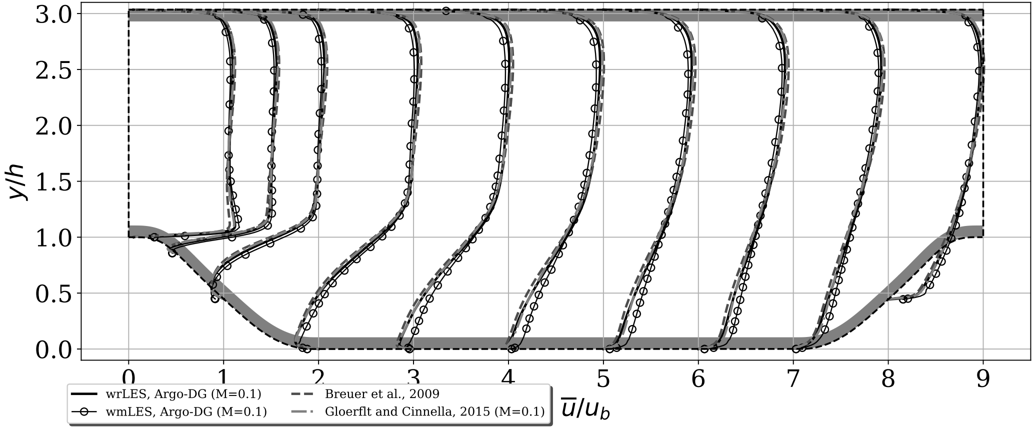
Mean velocity profile obtained after the accumulation of statistics over about $35t_c$,



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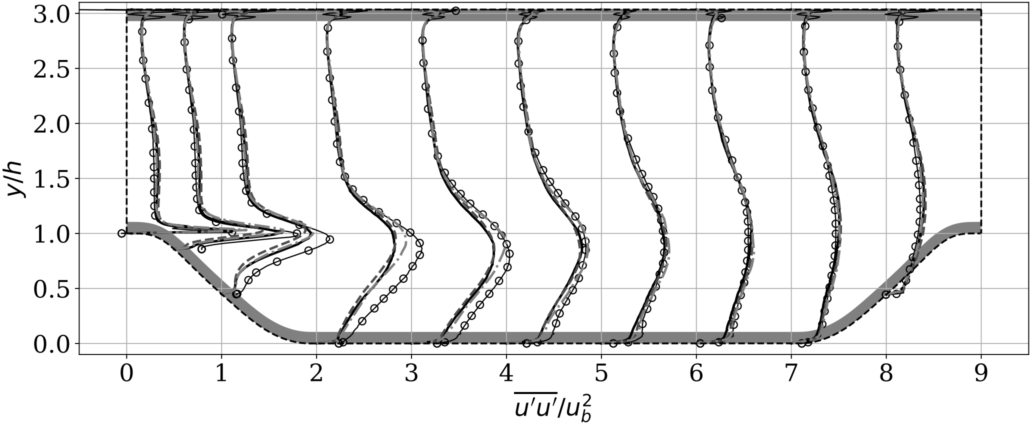
Remark: The misprediction of the recirculation bubble is visible between $x/h \in [3, 6]$.

Cenaero

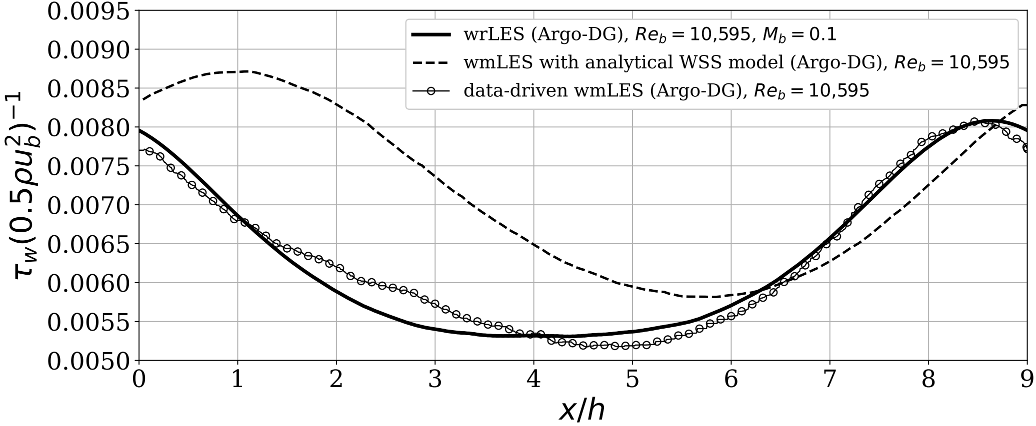
Mean Reynolds stress profile obtained after the accumulation of statistics over about $35t_c$,



Remark: Discrepancy in the recirculation bubble and at the edge with the free shear layer.

Cenaero

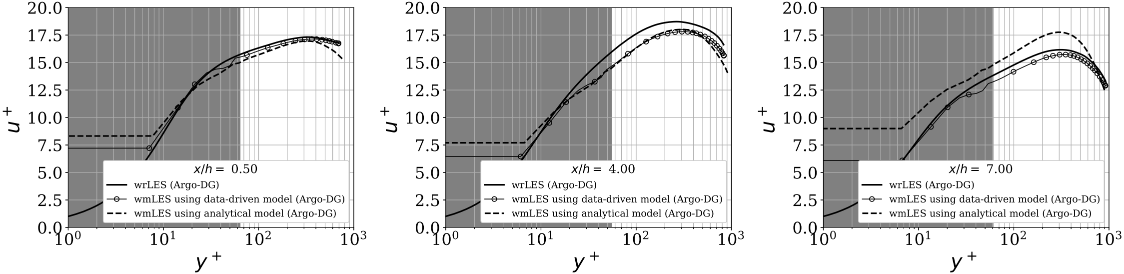
Friction coefficient on the upper wall obtained after the accumulation of statistics over about $35t_c$,



Remark: The analytical WSS model is wrong because it predicts τ_w based up $\boldsymbol{u}$ and not on ∇p.

Data-driven WSS model - A posteriori testing

Non-dimensional velocity profiles on the upper wall obtained after the accumulation of statistics over about $35t_c$,



Remark: The data-driven wmLES is always closer to the DNS profiles.

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Data-driven WSS model - Conclusion

- **Objective.** Development of a novel WSS model for the separation/reattachment phenomenon.
- **Scientific contribution.** Generate a data-driven WSS model to predict a distribution that better captures the instantaneous behaviour of wall shear stress.
- **Positive impact.** A great improvement in the WSS curve is observed on both the upper and lower walls of the two-dimensional periodic hill.
- **Points to be improved.** The reattachment location is underestimated and this affects the physics in the whole domain. Dupuy *et al.* [3, 4] have also observed this underestimation on other test cases featuring separation. The volume data may be more influenced by the direction of the wall shear stress (which is currently randomly generated) than its amplitude.

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